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Soft-Gluon Corrections to the Drell-Yan Process.

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Summary. — The 4-momentum distribution, $d^4\mathscr{P}(K)$, of the multiple soft-gluon emission is studied using a Bloch-Nordsieck approach. An asymptotic separability is found: $d^4\mathscr{P}(K) \simeq d\mathscr{P}(K_+)\, d\mathscr{P}(K_-)\, d^2\mathscr{P}(K_\perp)$, where $K_\pm = K_0 \pm K_3$. Their effect in μ pair production is investigated and explicit expressions for the double differential cross-section are obtained. The slope of the mean transverse momentum with total energy is calculated and compared with data. Our mean scaling curves are also in good agreement with experiment. Further tests are suggested.

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1. – Introduction.

The Drell-Yan (1) (DY) mechanism for production of lepton pairs in hadron-hadron scattering is by now firmly established (2,3). Several papers have been

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(2) L. M. LEDERMAN: *Proceedings of the XIX International Conference on High Energy Physics, Tokyo, 1978* (Tokyo, 1979), p. 706.
(3) G. MATTHIAE: *Riv. Nuovo Cimento*, **4**, No. 3 (1981).

devoted to computing important quantum chromodynamic (QCD) corrections to the basic DY Born amplitudes, altering the absolute value of the cross-section [4-7] as well as its dependence on the transverse momentum [8-10].

In this paper we present the effects of summed-up soft gluons on the DY spectrum. The formalism we use is based on the classic Bloch-Nordsieck method of utilizing classical currents to obtain the required summation. For this purpose a coherent-state approach was developed [11] and used successfully in a variety of problems: e^+e^- annihilation [12], $K_\perp$-distribution of jets [13], deep inelastic moments [14] and mean scaling [15]. A comparison of this formalism with the renormalization group was also made in ref. [14].

In this approach the quantity to be computed is the 4-momentum probability distribution $d^4\mathscr{P}/d^4K$, where K is the momentum of the QCD radiation. We first show that there is an asymptotic separability of this distribution: $d^4\mathscr{P}(K) \simeq d\mathscr{P}(K_+)\, d\mathscr{P}(K_-)\, d^2\mathscr{P}(K_\perp)$, where $K_\pm = K_0 \pm K_3$. This allows us to obtain a very simple expression (eq. (4.3)) for the differential cross-section $d\sigma/dx_1 dx_2$ which can be used to study scaling violations. Especially for meson-nucleon and $\bar{p}p$ initial states, where the « sea » contribution is expected to be small, it is possible to do a moment analysis reminiscent of that for deep inelastic scattering. Even if the « sea » contribution is not completely ignored, large n moments probe the infra-red (IR) region and thus our soft-gluon expressions may be useful. As a by-product we obtain a compact formula for $\langle K_\perp^2(x_1, x_2, s)\rangle$ (eq. (5.3)). Since no « intrinsic » (or « primordial ») $K_\perp$ has been included in this formula, at best we can hope to obtain the (dimensionless) derivative $\partial\langle K_\perp^2(x_1, x_2, s)\rangle/\partial s$. If we perform such an analysis for π-nucleus

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(4) G. PARISI: *Phys. Lett. B*, **90**, 295 (1980).
(5) G. ALTARELLI, R. ELLIS and G. MARTINELLI: *Nucl. Phys. B*, **143**, 521 (1978); **146**, 544 (1978); **157**, 461 (1979).
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(9) G. PARISI and R. PETRONZIO: *Nucl. Phys. B*, **154**, 427 (1979); K. KAJANTIE and J. LINDFORS: *Phys. Lett. B*, **74**, 384 (1979); J. CLEYMANS and M. KURODA: *Phys. Lett. B*, **80**, 385 (1979); J. C. COLLINS: *Phys. Rev. Lett.*, **42**, 291 (1979).
(10) F. HALZEN and D. M. SCOTT: *Phys. Rev. D*, **18**, 3378 (1978); **19**, 216 (1979).
(11) M. GRECO, F. PALUMBO, G. PANCHERI-SRIVASTAVA and Y. SRIVASTAVA: *Phys. Lett. B*, **77**, 282 (1978).
(12) G. PANCHERI-SRIVASTAVA and Y. SRIVASTAVA: *Phys. Rev. Lett.*, **43**, 11 (1979).
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scattering data obtained by the NA3 group [2,16], we find 0.0032 for this derivative, in agreement with the experimental value [16].

An approximate formula for $d^2\mathscr{P}(K_\perp)$ obtained earlier [15,17] is used to discuss mean scaling. Data from the CFS group [2,18] are compared with the theoretical expression and good agreement is found.

2. – Notation and factorization formula.

In this section we introduce kinematics for the DY process and set up the formalism through which soft gluons will be incorporated in the cross-section. Our approach rests on two conjectures which have been proved valid at least in the leading logarithmic approximation. They are that the emission of IR gluons is finite to all orders in the QCD coupling constant α_s and that it exponentiates as it does in QED, with non-Abelian effects appearing only through the energy dependence of α_s as obtained via the renormalization group equation.

To zeroth order in α_s, the cross-section for the DY process, quark + antiquark $\to \mu^+\mu^-$ is given by

$$\frac{d^4\sigma}{d^4q} = \frac{4\pi\alpha^2}{9Q^2}\, e_q^2\, \delta^4(P' - q)\,, \tag{2.1}$$

where q is the 4-momentum of the μ pair, $q_\mu q^\mu = Q^2$, and P' is the momentum of the $q\bar{q}$ system. DY cross-section is obtained from eq. (2.1) by folding in the quark and antiquark probability distributions

$$\left(\frac{d\sigma}{dQ^2}\right)_{DY} = \frac{4\pi\alpha^2}{9Q^2}\sum_i e_i^2 \int dx_1 \int dx_2\, \delta(sx_1x_2 - Q^2)\{f_i(x_1)\bar{f}_i(x_2) + \bar{f}_i(x_1)f_i(x_2)\}\,, \tag{2.2}$$

where the sum extends over all flavours and s is the square of the total c.m. energy of the incoming hadrons. The DY cross-section does not account for many features of the μ pair distribution, particularly the $K_\perp$-dependence. To lowest order in α_s, this is obtained through the process $q\bar{q} \to (\mu^+\mu^-)$ + gluon for which

(16) J. Badier, J. Boucrot, G. Burgun, O. Callot, Ph. Charpentier, M. Crozon, D. Decamp, P. Delpierre, A. Diop, R. Dubé, B. Gandois, R. Hagelberg, M. Hansroul, W. Kienzle, A. Lafontaine, P. Le Dû, J. Lefrançois, Th. Leray, G. Matthiae, A. Michelini, Ph. Miné, H. Nguyen Ngoc, O. Runolfsson, P. Siegrist, J. Timmermans, J. Valentin, R. Vanderhaghen and S. Weisz: *Phys. Lett. B*, **89**, 145 (1979); **93**, 394 (1980); presented at the *International Symposium on Lepton and Photon Interactions at High Energies*, Fermilab, August 1979.

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the differential cross-section reads [19]

$$\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}Q^2\,\mathrm{d}z} = \frac{2\alpha^2\alpha_s C_F}{9Q^2 s'} e_q^2 \left(1 - \frac{Q^2}{s'}\right)\frac{1}{1-z^2}(1 + z^2 + Q^2/k^2)\,, \tag{2.3}$$

where

$$s' = P'^2 = 4E^2\,, \qquad k = \frac{s' - Q^2}{2\sqrt{s'}}$$

is the gluon c.m. momentum and $Z = \cos\theta$, where θ is the scattering angle in the c.m. For $SU_{3,\text{color}}$, $C_F = \frac{4}{3}$.

When this cross-section is folded with the «primitive» parton densities, Q^2-dependent parton densities can be defined [19], which to first order in α_s agree with those obtained from deep inelastic scattering (DIS). In a previous paper [14] an expression was obtained for the parton densities which incorporates the leading soft-gluon corrections to all orders in α_s. We would like to show similar momentum-dependent parton densities for the Drell-Yan process as well.

It is quite useful for what follows to employ the light cone variables $k_\pm = k(1 \pm Z)$. Equation (2.3) may then be recast as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}k_+\,\mathrm{d}k_-} = \frac{2\alpha^2\alpha_s e_q^2 C_F}{9s'}\frac{1}{k_+k_-}\frac{(1 - k_+/\sqrt{s'})^2 + (1 - k_-/\sqrt{s'})^2}{1 - k_+/\sqrt{s'} - k_-/\sqrt{s'}}\,. \tag{2.4}$$

Equation (2.4) allows us to form a probability distribution in $k_\pm$:

$$\frac{\mathrm{d}^2\mathscr{P}}{\mathrm{d}k_+\,\mathrm{d}k_-} \equiv \frac{1}{\sigma_0(Q^2)}\frac{\mathrm{d}\sigma}{\mathrm{d}k_+\,\mathrm{d}k_-} = \frac{C_F\alpha_s}{\pi}\frac{1}{k_+k_-}\frac{(1 - k_+/\sqrt{s'})^2 + (1 - k_-/\sqrt{s'})^2}{2[1 - k_+/\sqrt{s'} - k_-/\sqrt{s'}]}\,. \tag{2.5}$$

In eq. (2.5), IR divergence in both k_+ and k_- variables is exposed. Also, it is seen that the distribution $\mathrm{d}^2\mathscr{P}(k_+, k_-)$ is *separable* in the IR region. This would be quite important to us later in showing the separability of the summed-up gluon spectrum.

Also, the collinear hard singularity is rather transparent in eq. (2.5). If we insert a regulator mass m and consider $Z \to \pm 1$ so that $k_\pm \to 0$, we find

$$\frac{\mathrm{d}\mathscr{P}}{\mathrm{d}k} \simeq \frac{\alpha_s C_F}{\pi}\left(\ln\frac{2E}{m}\right)\frac{1 + (1 - k/E)^2}{2k}\,, \tag{2.6}$$

if we retain only the singular part in m.

In the appendix, we consider the n-soft-gluon emission process quark $(p_1') +$ quark $(p_2') \to (\mu^+\mu^-) + (n$ soft gluons). We sum over n, color-average

[19] G. ALTARELLI, G. PARISI and R. PETRONZIO: *Phys. Lett. B*, **76**, 351 (1978).

and add virtual contributions to obtain (at the leading order) eqs. (A.12)-(A.14):

$$\left(\frac{d^4\sigma}{d^4q}\right)_{q\bar{q}\to\mu^+\mu^-+\text{«soft»}} = \int d^4\mathscr{P}(K)\,\sigma_{\text{Born}}(q)\,\delta^4(P'-K-q)\,, \tag{2.7}$$

where $\sigma_{\text{Born}}(q) = 4\pi\alpha^2/9Q^2$ and the 4-momentum distribution of the QCD radiation is given by

$$F(K) \equiv \frac{d^4\mathscr{P}(K)}{d^4K} = \int \frac{d^4x}{(2\pi)^4} \exp\left[iK\cdot x - h(x)\right]\,, \tag{2.8}$$

with

$$h(x) = \int d^3\bar{n}(k)\left(1 - \exp\left[-ik\cdot x\right]\right) \tag{2.9}$$

and $d^3\bar{n}(k)$ defined in eq. (A.11). Now, as is usual, we insert the quark/antiquark probabilities in eq. (2.7) to obtain for the observed hadronic cross-section

$$\left(\frac{d^4\sigma}{d^4q}\right)_{h_1h_2\to\mu^+\mu^-+X} = \frac{4\pi\alpha^2}{9Q^2}\sum_i e_i^2 \int dy_1 \int dy_2\,[f_i(y_1)\bar{f}_i(y_2) + \bar{f}_i(y_1)f_i(y_2)]F(P'-q)\,. \tag{2.10}$$

In writing eq. (2.10) we have neglected any intrinsic transverse momentum for the partons. We shall return to this problem later.

In the next section, we obtain approximate expressions for $F(K)$, which we then use to discuss the double differential cross-section as well as the transverse-momentum behavior.

3. – Asymptotic separability and approximate forms for $d^4\mathscr{P}(K)$.

In this section, we examine the structure of $d^4\mathscr{P}(K)$ in detail and try to exploit the consequences of taking the zero-mass limit, $m_q \to 0$, in eqs. (2.8) and (2.9). As an inspection of these equations shows, the distribution of $d^4\mathscr{P}(K)$ exhibits a mass singularity, which can be dealt with by introducing a small regulator $(\text{mass})^2\ \mu \sim m^2$. One can then show that, in the asymptotic limit $\mu \to 0$, the distribution separates into the product of independent distributions in the light-cone variables, *i.e.*

$$d^4\mathscr{P}(K) \underset{\mu\to 0}{\simeq} d\mathscr{P}(K_+)\,d\mathscr{P}(K_-)\,d^2\mathscr{P}(K_\perp)\,, \tag{3.1}$$

where $d^2\mathscr{P}(K_\perp)$ is finite as $\mu \to 0$.

The above separation is obtained by first considering the function $h(x)$, defined in eq. (2.9), in the q$\bar{\text{q}}$ c.m. frame:

$$(3.2)\quad h(x) \equiv \frac{C_F}{\pi}\int_{\mu}^{E^2}\frac{\mathrm{d}k_\perp^2}{k_\perp^2}\alpha(k_\perp^2)\int_{-\sqrt{E^2-k_\perp^2}}^{\sqrt{E^2-k_\perp^2}}\frac{\mathrm{d}k_3}{\sqrt{k_3^2+k_\perp^2}}\left\{1-\exp\left[-\frac{i}{2}(k_+t_-+k_-t_+)\right]J_0(k_\perp x_\perp)\right\},$$

where $2E$ is the c.m. energy and $t_\pm = t\pm x_3$, $k_\pm = k_0\pm k_3$. To study the behavior of $h(x)$ as $\mu\to 0$, it is convenient to introduce the functions

$$(3.3)\quad h_\mathrm{L}(t, x_3) \equiv h(t, x_3; x_\perp = 0) =$$

$$= \frac{C_F}{\pi}\int_{\mu}^{E^2}\frac{\mathrm{d}k_\perp^2}{k_\perp^2}\alpha(k_\perp^2)\int_{-\sqrt{E^2-k_\perp^2}}^{\sqrt{E^2-k_\perp^2}}\frac{\mathrm{d}k_3}{\sqrt{k_3^2+k_\perp^2}}\left[1-\exp\left[-\frac{i}{2}(k_+t_-+k_-t_+)\right]\right]$$

and

$$(3.4)\quad h_\perp(x_\perp) \equiv h(t=0, x_3=0; x_\perp) = \frac{C_F}{\pi}\int_{\mu}^{E^2}\frac{\mathrm{d}k_\perp^2}{k_\perp^2}\alpha(k_\perp^2)[1-J_0(k_\perp x_\perp)]\int_{-\sqrt{E^2-k_\perp^2}}^{\sqrt{E^2-k_\perp^2}}\frac{\mathrm{d}k_3}{\sqrt{k_3^2+k_\perp^2}},$$

such that h_L is logarithmically singular, whereas $h_\perp$ is regular and goes to a constant as $\mu\to 0$.

From eq. (3.3) we see that $h_\mathrm{L}\sim\log\mu$ as $\mu\to 0$. We now proceed to also determine the behavior of $h_\perp$ and the relationship of h to h_L and $h_\perp$.

From eqs. (3.2) and (3.3) we find

$$(3.5a)\quad -\mu\frac{\partial}{\partial\mu}[h-h_\mathrm{L}] =$$

$$= \frac{C_F}{\pi}\alpha(\mu)[1-J_0(\sqrt{\mu}x_\perp)]\int_{-\sqrt{E^2-\mu}}^{\sqrt{E^2-\mu}}\frac{\mathrm{d}k_3}{\sqrt{k_3^2+\mu}}\exp\left[-\frac{i}{2}(k'_+t_-+k'_-t_+)\right]$$

with $k'_\pm = \sqrt{k_3^2+\mu}\pm k_3$. As $\mu\to 0$ the r.h.s. of eq. (3.5a) $\to(\mu\ln\mu)f(x)$. Thus $(\partial/\partial\mu)(h-h_\mu)\sim\log\mu$ and is divergent as $\mu\to 0$. We may improve the behavior by considering $h-h_\mathrm{L}-h_\perp$. We find

$$(3.5b)\quad -\mu\frac{\partial}{\partial\mu}[h-h_\mathrm{L}-h_\perp] =$$

$$= -\frac{C_F}{\pi}\alpha(\mu)[1-J_0(\sqrt{\mu}x_\perp)]\int_{-\sqrt{E^2-\mu}}^{\sqrt{E^2-\mu}}\frac{\mathrm{d}k_3}{\sqrt{k_3^2+\mu}}\left[1-\exp\left[-\frac{i}{2}(k'_\perp t_-+k'_-t_+)\right]\right].$$

Now, as $\mu \to 0$, the r.h.s. of eq. (3.5b) $\to \mu l(x)$ and hence $h \to h_L + h_\perp + g(x_\perp, t_+, t_-) + O(\mu)$, where $g(x_\perp, t_+, t_-)$ is independent of μ, and may be neglected.

One may argue that, while the introduction of $h_\perp$ resulted in the improved behavior exhibited on the r.h.s. of eq. (3.5b), $h_\perp$ is not singular as $\mu \to 0$ and one should keep g as being potentially comparable to $h_\perp$. However, we find from eqs. (3.2)-(3.4) that $g \sim (t_+ + t_-)\, x_\perp^2$. Thus it would contribute only when t_+, t_- and $x_\perp^2$ are all nonzero. In this case the h_L term is also present, which goes as $\log\mu$ and hence dominates the behavior of h as $\mu \to 0$. On the other hand, if we integrate eq. (2.8) over $dK_+\, dK_-$ in order to get a $d^2\mathscr{P}/d^2K_\perp$ distribution we obtain a $\delta(t_+)\,\delta(t_-)$ which makes $g = 0$. However, as can be seen from eq. (3.3), h_L is also zero and, therefore, $h = h_\perp + O(\mu)$ for this distribution. Thus g *only* contributes when h_L is also present, and is thus negligible as $\mu \to 0$. In contrast, when looking at $d^2\mathscr{P}/d^2K_\perp$ we find $h_\perp$ is the leading term in h and hence must be retained.

The foregoing discussion leads us to conclude that we may set $h(x) = h_L(t, x_3) + h_\perp(x_\perp)$ as $\mu \to 0$.

Neglecting terms of order $\mu\log\mu$ in $h_L(t, x_3)$, we then obtain

$$h(x) \underset{\mu\to 0}{\simeq} \frac{C_F}{\pi}\int_\mu^{E^2}\frac{dk_\perp^2}{k_\perp^2}\,\alpha(k_\perp^2)\left\{\int_0^E\frac{dk}{k}\,(1-\exp[-ikt_+]) + \int_0^E\frac{dk}{k}\,(1-\exp[-ikt_-])\right\} + \frac{C_F}{\pi}\int_0^E\frac{dk_\perp^2}{k_\perp^2}\,\alpha(k_\perp^2)\left(\log\frac{E+\sqrt{E^2-k_\perp^2}}{E-\sqrt{E^2-k_\perp^2}}\right)[1-J_0(k_\perp x_\perp)]\,. \tag{3.6}$$

From this, the separation property (3.1) immediately follows with

$$d\mathscr{P}(K_\pm) = dK_\pm\int\frac{dt}{2\pi}\exp[iK_\pm t]\exp\left[-\frac{C_F}{\pi}\int_\mu^{E^2}\frac{dk_\perp^2}{k_\perp^2}\,\alpha(k_\perp^2)\int_0^{2E}\frac{dk}{k}\,(1-\exp[-ikt])\right] \tag{3.7}$$

and

$$d^2\mathscr{P}(K_\perp) = d^2\boldsymbol{K}_\perp\int\frac{d^2\boldsymbol{x}_\perp}{(2\pi)^2}\exp[-i\boldsymbol{K}_\perp\cdot\boldsymbol{x}_\perp - h_\perp(x_\perp)]\,. \tag{3.8}$$

For $K_\pm < 2E$, eq. (3.7) can be integrated to give

$$d\mathscr{P}(K_\pm) = \frac{dK_\pm}{2E}\,\frac{\gamma^{-\beta_L}}{\Gamma(\beta_L)}\left(\frac{K_\pm}{2E}\right)^{\beta_L-1} \tag{3.9}$$

with

$$\beta_L = \frac{C_F}{\pi}\int_\mu^{E^2}\frac{dk_\perp^2}{k_\perp^2}\,\alpha(k_\perp^2)\,.$$

The distributions defined in eqs. (3.7) and (3.8) are all normalized to 1, as is $d^4\mathscr{P}(K)$, and they all behave as δ-function distributions in the $\alpha_s \to 0$ limit. It is a check upon the approximations which led to eqs. (3.1), (3.7) and (3.8) that, if we integrate $d^4\mathscr{P}(K)$, as given by those equations, in the momentum variables K_3 and $K_\perp$, for $K_0 < E$ we obtain the well-known [20] energy distribution

$$(3.10)\qquad d\mathscr{P}(K_0) = \int \frac{d^4\mathscr{P}(K)}{d^3\boldsymbol{K}}\, d^3\boldsymbol{K} = \int \frac{d\mathscr{P}(K_+)\, d\mathscr{P}(K_-)}{dK_3} = \frac{dK_0}{E}\,\frac{\gamma^{-\beta}}{\Gamma(\beta)}\left(\frac{K_0}{E}\right)^{\beta-1}$$

with

$$\beta = 2\beta_L = \frac{2C_F}{\pi}\int_\mu^{E^2} \frac{dk_\perp^2}{k_\perp^2}\,\alpha(k_\perp^2)\,,$$

as it should be. A comparison with QED calculations when α is a constant and $C_F = 1$ then leads to a more accurate definition $\mu \simeq m^2/4$.

So far, in eq. (3.9), only the soft-gluon contribution has been incorporated. A more complete calculation must, however, also consider the collinear hard gluons which are also known to exponentiate [7]. As suggested in ref. [14], these corrections can be applied in such a way as to reproduce the first-order result while, at the same time, summing hard collinear gluons to all orders. We propose, for $d\mathscr{P}(K_+)$ and $d\mathscr{P}(K_-)$, the following expression:

$$(3.11)\qquad [d\mathscr{P}(K_\pm)]_{\text{hard and soft}} = [d\mathscr{P}(K_\pm)]_{\text{soft}} \frac{\exp\left[\frac{3}{4}\beta_L\right]}{2}\left[1 + \left(1 - \frac{K_\pm}{2E}\right)^2\right].$$

For $\beta \to 0$, this expression approximates to the first-order result, eq. (2.6). It should be noted that, whereas $d^4\mathscr{P}(K)$ is a relativistic invariant, the individual probabilities $d\mathscr{P}(K_+)$ and $d\mathscr{P}(K_-)$ are not and one has to specify the frame in which $K_\pm$ and $2E$ are calculated. We postpone this to the next section, where explicit expressions for the differential cross-sections are derived and compared with results from deep inelastic scattering.

4. – Differential cross-section and comparison with DIS.

For computing the double differential cross-section $d\sigma/dx_1 dx_2$, after the primitive quark densities are folded in, eq. (3.1) is very convenient. The variables $x_{1,2}$ are defined as

$$x_1 x_2 = \tau = \frac{Q^2}{s} \qquad \text{and} \qquad x_1 - x_2 = x_F = \frac{2q_\parallel^*}{\sqrt{s}}\,,$$

[20] E. Etim, G. Pancheri and B. Touschek: *Nuovo Cimento B*, **51**, 276 (1967).

where $q_\parallel^*$ is the longitudinal momentum of the lepton pair in the hadronic c.m. system. If we integrate over $q_\perp$, after a little algebra we find

$$(4.1)\qquad s\frac{\mathrm{d}^2\sigma}{\mathrm{d}x_1\,\mathrm{d}x_2}=\frac{4\pi\alpha^2}{9x_1x_2}\exp\left[\frac{\pi}{2}C_F\alpha_s\right]\sum_i e_i^2\int_{x_1}^{1}\mathrm{d}y_1\int_{x_2}^{1}\mathrm{d}y_2\{f_i(y_1)\bar{f}_i(y_2)+\bar{f}_i(y_1)f_i(y_2)\}\cdot$$
$$\cdot\,\mathrm{d}\mathscr{P}(u_1)\,\mathrm{d}\mathscr{P}(u_2)\,\delta(y_1-x_1-u_1)\,\delta(y_2-x_2-u_2)\,,$$

where $u_{1,2}=K_\pm/\sqrt{s}$. Choosing $E^2=(s/4)\,y_1y_2$, we get (ignoring for the moment the collinear hard term)

$$(4.2)\qquad \left[s\frac{\mathrm{d}^2\sigma}{\mathrm{d}x_1\,\mathrm{d}x_2}\right]_{\text{soft}}\simeq\frac{4\pi\alpha^2}{9x_1x_2}\exp\left[\frac{\pi}{2}C_F\alpha_s\right]\sum_i e_i^2\int_{x_1}^{1}\frac{\mathrm{d}y_1}{y_1}\int_{x_2}^{1}\frac{\mathrm{d}y_2}{y_2}\cdot$$
$$\cdot\{f_i(y_1)\bar{f}(y_2)+\bar{f}_i(y_1)f_i(y_2)\}\left[\frac{1}{\gamma^{\beta_\mathrm{L}}\Gamma(\beta_\mathrm{L})}\right]^2\left(1-\frac{x_1}{y_1}\right)^{\beta_\mathrm{L}-1}\left(1-\frac{x_2}{y_2}\right)^{\beta_\mathrm{L}-1}.$$

The spectrum β_L depends on y_1 and y_2. Since the integral is peaked at $y_i=x_i$, we shall evaluate β_L at $y_i=x_i$. β_L is then a function of Q^2 alone as in DIS [14]. We may thus write the following expression (which accounts for the collinear hard gluons as well):

$$(4.3)\qquad s\frac{\mathrm{d}\sigma}{\mathrm{d}x_1\,\mathrm{d}x_2}=\frac{4\pi\alpha^2}{9x_1x_2}\exp\left[\frac{\pi}{2}C_F\alpha_s\right]\sum_i e_i^2\cdot$$
$$\cdot\{I_i(x_1,\beta_\mathrm{L})\bar{I}_i(x_2,\beta_\mathrm{L})+I_i(x_2,\beta_\mathrm{L})\bar{I}_i(x_1,\beta_\mathrm{L})\}\,,$$

where

$$(4.4)\qquad I_i(x,\beta)=\frac{\exp\left[\frac{3}{4}\beta\right]}{\gamma^\beta\Gamma(\beta)}\int_x^1\frac{\mathrm{d}y}{y}f_i(y)\left(1-\frac{x}{y}\right)^{\beta-1}\frac{1+(x/y)^2}{2}.$$

Comparison with standard approaches leads to the obvious identification of $I_i(x,\beta)$ as the momentum-dependent quark densities. Also, it agrees with an earlier paper [14] where the same expression was obtained for (spacelike) DIS.

If DIS parton densities are to be inserted in eq. (4.3) for the DY process, the extra factor $\exp[(\pi/2)\,C_F\,\alpha_s]$ survives. As has been discussed by various authors [21], it helps much to alleviate the problem of the absolute value of the cross-section.

As a consistency check, if we let $\alpha_s\to 0$, so does β and we obtain from

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([21]) G. Parisi: *Phys. Lett. B*, **90**, 295 (1980).

eq. (4.4)

$$I_i(x,\beta)\xrightarrow[\beta\to 0]{} f_i(x)\,. \tag{4.5}$$

Thus, the DY result is recovered in this limit.

Before proceeding further, we want to stress that the above expressions are valid for small energy losses, *i.e.* for $x\simeq 1$ and small $q_\perp$. For large $1-x$ values and large $q_\perp$, the hard, wide bremsstrahlung contributions may become dominant and must be added to this calculation. It now appears possible to do a moment analysis similar to that employed in DIS. It is especially suited for meson-nucleon or $p\bar{p}$-initiated lepton pair processes from which the « sea » contribution can either by completely eliminated (by suitable linear combinations) or at least minimized.

Consider, for example, the difference

$$L^{(-)}(x_1,x_2,\beta)\equiv s\left[\frac{d\sigma^{\pi^-p}}{dx_1\,dx_2}-\frac{d\sigma^{\pi^+p}}{dx_1\,dx_2}\right]\equiv F_\pi^{(-)}(x_1,\beta)\,G^{(-)}(x_2,\beta) \tag{4.6}$$

from which the sea contribution cancels out. The N-th moment in the variable x_1 (for fixed x_2) then gives

$$K(N,x_2,\beta)=\int_0^1\frac{dx_1}{x_1}x_1^N F_\pi^{(-)}(x_1,\beta)\,G^{(-)}(x_2,\beta)\equiv C(N)\langle x_1^N(\beta)\rangle G^{(-)}(x_2,\beta)\,, \tag{4.7}$$

where

$$C(N)=\int_0^1\frac{dy}{y}y^N F_\pi^{(-)}(y)$$

and

$$\langle x_1^N(\beta)\rangle=\frac{\exp[\frac{3}{4}\beta]\Gamma(N)}{2\gamma^\beta\Gamma(N+\beta)}\left[1+\frac{N(N+1)}{(N+\beta)(N+\beta+1)}\right]. \tag{4.8}$$

Here x_2 is a redundant variable and it may be useful to integrate over some fixed range in x_2 to increase the statistics. In fact, let

$$\bar{K}(N,\beta)=\int_{x_{20}}^{x_{21}}dx_2\,K(N,x_2,\beta)\equiv C(N)\,\hat{G}(\beta)\langle x_1^N(\beta)\rangle\,. \tag{4.9}$$

Now consider the ratio of 2 different moments:

$$R(N, M; \beta) \equiv \frac{\int_0^1 \mathrm{d}x_1 x_1^N \int_{x_{20}}^{x_{21}} \mathrm{d}x_2 \{s(\mathrm{d}\sigma^{\pi^- p}/\mathrm{d}x_1 \mathrm{d}x_2) - s(\mathrm{d}\sigma^{\pi^+ p}/\mathrm{d}x_1 \mathrm{d}x_2)\}}{\int_0^1 \mathrm{d}x_1 x_1^M \int_{x_{20}}^{x_{21}} \mathrm{d}x_2 \{s(\mathrm{d}\sigma^{\pi^- p}/\mathrm{d}x_1 \mathrm{d}x_2) - s(\mathrm{d}\sigma^{\pi^+ p}/\mathrm{d}x_1 \mathrm{d}x_2)\}} \equiv \equiv \frac{\bar{K}(N, \beta)}{\bar{K}(M, \beta)} = \frac{C(N)}{C(M)} \frac{\langle x_1^N(\beta)\rangle}{\langle x_1^M(\beta)\rangle} . \tag{4.10}$$

Variations in $\ln R(N, M\beta)$ *vs.* β are given by our model, independent of the « primitive » quark densities. Larger values of N, M sample larger x and thus the IR region for which our formulae should be reliable.

5. – Energy dependence of $\langle q_\perp^2 \rangle$.

In this section we derive an expression for the mean of the squared transverse momentum, $\langle q_\perp^2 \rangle$, of the lepton pair. Let us first assume that the partons inside the hadron carry no intrinsic transverse momentum. Then, eq. (2.10) gives

$$\langle q_\perp^2(x_1, x_2, s)\rangle \frac{\mathrm{d}\sigma}{\mathrm{d}x_1 \mathrm{d}x_2} = \frac{4\pi\alpha^2}{9Q^2} \int \mathrm{d}y_1 \int \mathrm{d}y_2 \, \mathscr{G}(y_1, y_2) \int q_\perp^2 \, \mathrm{d}^2\boldsymbol{q}_\perp \frac{\mathrm{d}^4\mathscr{P}(P' - q)}{\mathrm{d}^4 q} , \tag{5.1}$$

where

$$\mathscr{G}(y_1, y_2) = \sum_i e_i^2 \{f_i(y_1) \bar{f}_i(y_2) + (y_1 \leftrightarrow y_2)\} .$$

It can be shown that the probability distribution $\mathrm{d}^4\mathscr{P}(K)$ satisfies the following general relation:

$$\int K_\perp^2 \, \mathrm{d}^2\boldsymbol{K}_\perp \frac{\mathrm{d}^4\mathscr{P}(K)}{\mathrm{d}^4 K} = \int \mathrm{d}^3\bar{n}(k) \, k_\perp^2 \frac{\mathrm{d}^2\mathscr{P}(K_0 - k, K_3 - k_3)}{\mathrm{d}K_0 \, \mathrm{d}K_3} , \tag{5.2}$$

where

$$\mathrm{d}^2\mathscr{P}(K_0, K_3) = \int \mathrm{d}^2\boldsymbol{K}_\perp \frac{\mathrm{d}^4\mathscr{P}(K)}{\mathrm{d}^4 K} .$$

Inserting eq. (5.2) in (5.1), we obtain

$$\langle q_\perp^2(x_1, x_2, s)\rangle \frac{\mathrm{d}\sigma(s, q)}{\mathrm{d}x_1 \mathrm{d}x_2} = = \frac{4\pi\alpha^2}{9Q^2} \int \mathrm{d}y_1 \int \mathrm{d}y_2 \, \mathscr{G}(y_1, y_2) \int \mathrm{d}^3\bar{n}(k) \, k_\perp^2 \, \mathrm{d}^2\mathscr{P}(P'_0 - q_0 - k, P'_3 - q_3 - k_3) .$$

For x_1, $x_2 > \frac{1}{2}$ one can exchange the limits of integration and obtain the sum rule

$$\langle q_\perp^2(x_1, x_2, s)\rangle \frac{d\sigma(s, q)}{dx_1\, dx_2} = \int d^3\bar{n}(k)\, k_\perp^2 \frac{d\sigma(s, q+k)}{dx_1\, dx_2}\,. \tag{5.3}$$

Equation (5.3) is quite general. The only assumptions made in deriving eq. (5.3) are that i) the collinear hard- and soft-gluon contributions can be factorized from the « hard » cross-section and that ii) the parton densities have no (intrinsic) transverse momentum. Thus, given the experimental differential cross-section alone, eq. (5.3) allows one to test the above factorization hypothesis quite clearly. If there is an energy-independent, intrinsic transverse momentum, eq. (5.3) is no longer correct. However, it can be used to compute the energy-dependent terms in $\langle q_\perp^2\rangle$, *i.e.* to predict the slope $(\partial/\partial s)\langle q_\perp^2\rangle$. A detailed numerical test based on the above is in preparation and will be presented elsewhere. Here we present a simple application to the π-nucleus data.

In the soft limit

$$d^3\bar{n}(k)\, k_\perp^2 \simeq \frac{C_F \alpha_s}{\pi}\, dk_+\, dk_-\,.$$

Then eqs. (4.3) and (5.3), for x_1, $x_2 \geqslant \frac{1}{2}$, give

$$\langle q_\perp^2(x_1, x_2, s)\rangle = \frac{4\alpha_s}{3\pi}\, s\, \frac{\sum_i e_i^2 \{\int_{x_1}^1 dy_1\, I_i(y_1, \beta_L) \int_{x_2}^1 dy_2\, \bar{I}_i(y_2, \beta_L) + (x_1 \leftrightarrow x_2)\}}{\sum_i e_i^2 \{I_i(x_1, \beta_L)\, \bar{I}_i(x_2, \beta_L) + (x_1 \leftrightarrow x_2)\}}\,, \tag{5.4}$$

where α_s has been assumed to be a constant, for simplicity.

For π-nucleus case, valence quarks alone seem to work very well, at least for $x_{1,2} \geqslant 0.2$ [3,16]. In this approximation then we have

$$\langle q_\perp^2(x_1, x_2, s)\rangle \simeq \frac{4\alpha_s}{3\pi}\, s\, \frac{\int_{x_1}^1 dy_1\, I_\pi(y_1) \int_{x_2}^1 dy_2\, I_N(y_2)}{I_\pi(x_1)\, I_N(x_2)}\,. \tag{5.5}$$

Using the parameterization presented in ref. [16], we obtain for the slope at $x_1 = x_2 = \sqrt{\tau} = 0.5$

$$\left.\frac{\partial \langle q_\perp^2\rangle}{\partial s}\right|_{x_1 = x_2 = 0.5} = 0.01\alpha_s\,. \tag{5.6}$$

We require that the model also provide a consistent value for the absolute cross-section. The factor K, which is experimentally known [16] to be $\simeq 2$, in the model is given by $K \simeq \exp[\frac{2}{3}\pi\alpha_s]$. By requiring $K \simeq 2$, we obtain

$\alpha_s \simeq 0.33$ and

$$\left.\frac{\partial\langle q_\perp^2\rangle}{\partial s}\right|_{x_1=x_2=0.5} \simeq 0.0033 . \tag{5.7}$$

This value is in good agreement with the slope 0.0029 measured in π-nucleus scattering at $x_1 = x_2 = 0.275$. In consideration of the gross approximations made in obtaining eq. (5.7), this result is quite encouraging, as it is of the correct order of magnitude. A more refined analysis is in progress.

6. – Mean scaling and the transverse-momentum distribution.

In this section, we shall present a phenomenological analysis of the transverse-momentum distribution of μ pairs produced in pp collisions. We shall show that the data exhibit scaling in the mean and that they can all be fitted by the same universal curve derived from the soft-gluon emission formula. We analyze the data by KAPLAN *et al.* [22], at 400 GeV/c incident proton energy, for two different values of the invariant mass, including the Υ region. The remarkable fact is that, although the mean transverse-momentum squared $\langle q_\perp^2\rangle$ changes in the resonance region, the normalized transverse momentum distribution for the two different mass regions is fitted by the same dimensionless function of the dimensionless variable $z_\perp = q_\perp/\langle q_\perp\rangle$. This follows from the separability of the cross-section into a « soft » and a « hard » part and from the asymptotic factorization of the BN function $d^4\mathscr{P}(K)$ intod $\mathscr{P}_+ d\mathscr{P}_- d^2\mathscr{P}(K_\perp)$.

As shown in sect. **2** and **3**, one can write for the normalized μ pair distribution the expression

$$\frac{d^2\mathscr{P}(K_\perp)}{d^2\boldsymbol{K}_\perp} = \int \frac{d^2\boldsymbol{x}_\perp}{(2\pi)^2} \exp\left[-i\boldsymbol{K}_\perp\cdot\boldsymbol{x}_\perp - h(x_\perp, E)\right], \tag{6.1}$$

where $\boldsymbol{q}_\perp = -\boldsymbol{K}_\perp$ is the transverse momentum of the μ pair and

$$h(x_\perp, E) = \int d^3\bar{n}(k)\left(1 - \exp\left[+i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp\right]\right). \tag{6.2}$$

An approximate closed-form solution of eq. (6.1), discussed in ref. [15], is given by

$$\frac{d^2\mathscr{P}(K_\perp)}{d^2\boldsymbol{K}_\perp} = \frac{(2\pi)^{-1}}{2E^2A}\frac{\beta_\perp}{\Gamma(1+\beta_\perp/2)}\left(\frac{K_\perp}{2E\sqrt{A}}\right)^{-1+\beta_\perp/2}\mathscr{K}_{(\beta_\perp/2)-1}\left(\frac{K_\perp}{E\sqrt{A}}\right), \tag{6.3}$$

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[22] D. M. KAPLAN, R. FISK, A. ITO, H. JÖSTLEIN, J. APPEL, B. BROWN, C. BROWN, W. INNES, R. KEPHART, K. UENO, T. YAMANOUCHI, S. HERB, D. HOM, L. LEDERMAN, J. SENS, H. SNYDER and J. YOH: *Phys. Rev. Lett.*, **40**, 435 (1978).

where $\mathscr{K}_\nu$ is a modified Bessel function and where $\beta_\perp$ and $E\sqrt{A}$ are defined through the large- and small-$x_\perp$ behavior of $h(x_\perp, E)$, *i.e.*

$$h(x_\perp, E) \sim \begin{cases} \beta_\perp \ln iEx_\perp\,, & Ex_\perp \to \infty\,, \\ \frac{1}{4}x_\perp^2\int d^3\bar{n}(k)\,k_\perp^2 \equiv \frac{1}{2}\beta_\perp E^2 A x_\perp^2\,, & Ex_\perp \to 0\,, \end{cases}$$

and we have used the leading logarithmic approximation to define the function $\beta_\perp$, which contains the (integrated) dependence upon α_s. Notice that $\beta_\perp$ is the transverse-momentum analogue of β_L. In the above approximation scheme, one does not really need to know the details of the single soft-gluon distribution $d^3\bar{n}(k)$, excepting that it has a logarithmic singularity as $k \to 0$.

The appearance in eq. (6.3) of the quantity $E\sqrt{A}$ is what ensures that $d^2\mathscr{P}(K_\perp)$ exhibits scaling in the mean. In fact, from eq. (6.1), one obtains

$$\langle K_\perp^2\rangle = \int d^3\bar{n}(k)\,k_\perp^2 = 2E^2 A\beta_\perp\,. \tag{6.4}$$

Both $\beta_\perp$ and A can be separately calculated in QED ([17]), where the soft-photon spectrum is known. In QCD, there are some yet unsolved questions which make a computation of A and $\beta_\perp$ problematic, *i.e.* the behavior of α_s in the deep infra-red region and the folding of parton scattering processes with intrinsic momentum dependences inside the hadrons.

The inclusion of an intrinsic transverse momentum in our picture can be phenomenologically accomplished by considering $\beta_\perp$ and $\beta_\perp A$ as empiric parameters to be determined from the data. Thus we use experimental values in expressions obtained from eq. (6.3), as in the following dispersion formula:

$$\Delta(\beta_\perp) \equiv \frac{\langle q_\perp\rangle}{\sqrt{\langle q_\perp^2\rangle - \langle q_\perp\rangle^2}} = \frac{1}{\sqrt{\dfrac{8}{\pi\beta_\perp}\left\{\dfrac{\Gamma(\beta_\perp/2+1)}{\Gamma(\beta_\perp/2+\frac{1}{2})}\right\}^2 - 1}}\,. \tag{6.5}$$

In fig. 1 we show the renormalized transverse-momentum distribution as obtained from the data of ref. ([22]), plotted in terms of the variables $z_\perp$. We have taken $\langle q_\perp\rangle = 1.2$ GeV for the mass variable $M_{\mu\mu} = (5\div 6)$ GeV. For the Υ region, we use $\langle q_\perp\rangle = 1.35$ GeV ([18]). The two sets of data indeed fall on the same curve and they can all be fitted by the function $(1/z_\perp)(d\mathscr{P}/dz_\perp)$ with

$$\frac{d\mathscr{P}}{dz_\perp} = \frac{\sqrt{\pi}\,\Gamma(\beta_\perp/2+\frac{1}{2})}{2^{(\beta_\perp/2)-1}[\Gamma(\beta_\perp/2)]^2}\, z_T^{\beta_\perp/2}\,\mathscr{K}_{(\beta_\perp/2)-1}(z_T)\,,$$

where

$$z_T = \frac{\sqrt{\pi}\,\Gamma(\beta_\perp/2+\frac{1}{2})}{\Gamma(\beta_\perp/2)}\,z_\perp\,.$$

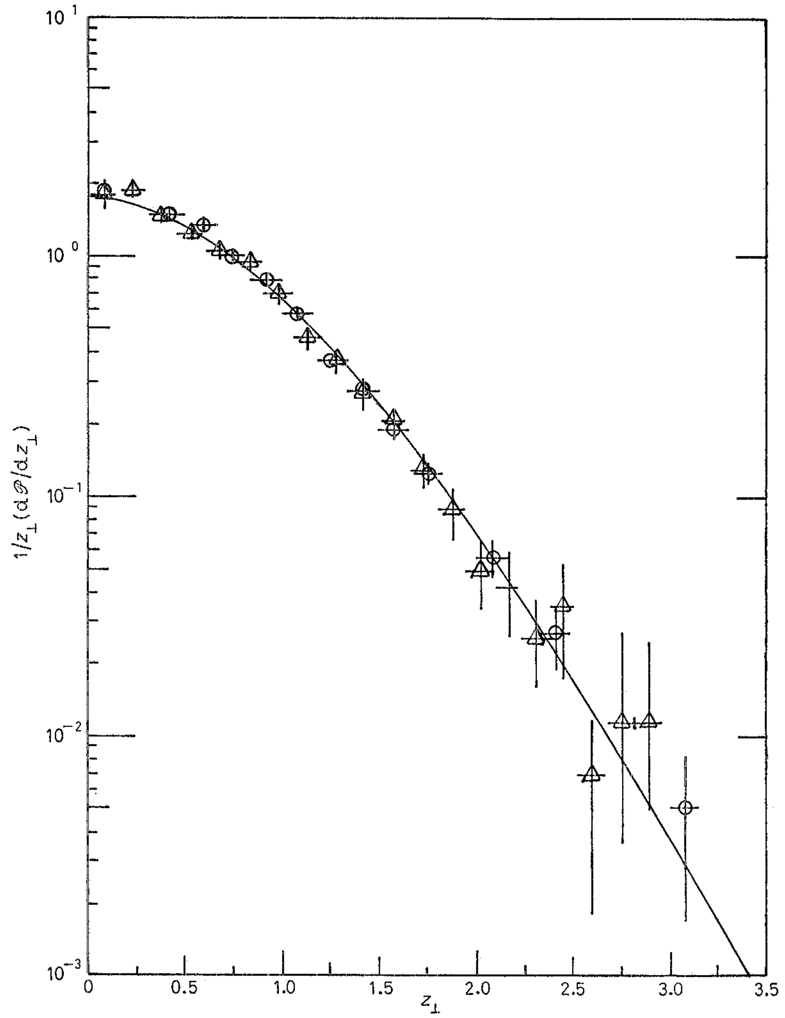

Fig. 1. – A plot of the normalized transverse-momentum distribution in terms of the variable $z_\perp = q_\perp/\langle q_\perp \rangle$ for the reaction $pp \to \mu^+\mu^- + X$ at 400 GeV/c: ○ $M_{\mu\mu} = (5 \div 6)$ GeV, △ $M_{\mu\mu} = (9 \div 10)$ GeV. The data were obtained from ref. [22]. We have chosen $\langle q_\perp \rangle = 1.2$ GeV for the mass variable $M_{\mu\mu} = (5 \div 6)$ GeV. In the Υ region, we choose $\langle q_\perp \rangle = 1.35$ GeV.

The data for $\langle q_\perp \rangle$ and $\langle q_\perp^2 \rangle$ in the two mass regions suggest $\beta_\perp \simeq 13$. With this choice, one obtains the solid curve in excellent agreement with data.

In a previous publication [15], it was shown that distribution (6.3), expressed as a function of $z_\perp$, fits the transverse momentum of pions in

$$e^+e^- \to \pi + X \quad \text{and} \quad pp \to \pi + X .$$

It was also shown that eq. (6.3) was consistent with an experimental fit to μ-pair production in a wide x_F range. Recently, new evidence has been presented for mean scaling in the processes ([23])

$$\mathrm{pp} \to \mathrm{K}_s^0 + \mathrm{X}\,, \quad \mathrm{pp} \to \pi^0 + \mathrm{X}\,, \quad \pi\mathrm{p} \to \pi + \mathrm{X}\,.$$

Thus mean scaling appears to be characteristic of most inclusive hadronic and semi-hadronic processes. Our analysis strongly suggests it to be a consequence of a factorizable collective effect like soft-gluon emission. This hypothesis is further confirmed by the fact that mean scaling is known experimentally to occur for almost all ([24]) one-dimensional distributions, and not just for the transverse momentum. Such an occurrence is indeed a natural consequence of the asymptotic factorization of $\mathrm{d}^4\mathscr{P}(K)$, eq. (3.1).

7. – Conclusion.

We have presented here a detailed account of soft-gluon corrections to the DY processes. The formalism is borrowed from the Bloch-Nordsieck approach in QED. This approach is direct, intuitive and physically appealing. Based as it is in terms of probability distributions, it meshes nicely with the underlying probabilistic parton picture of the DY process.

As far as we are aware, for the first time the complete soft-gluon distribution $\mathrm{d}^4\mathscr{P}(K)$ has been discussed. The asymptotic separability in terms of the light-cone variables $K_\pm$ and $K_\perp$ is a new result. It allowed us to obtain an expression for $\langle q_\perp^2 \rangle$ as an integral over the measured energy and longitudinal distributions alone. This remarkable property can be used to test factorization itself. A partial test was attempted here with positive results.

The details of $q_\perp$-spectrum were examined using an approximation to $\mathrm{d}^2\mathscr{P}(K_\perp)$ which showed mean scaling. Even though $\langle q_\perp^2 \rangle$ changes appreciably in the Υ-resonance region, we found that the same mean scaling curves work

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([23]) B. BATYUNYA, I. BOGUSLAVSKY, I. GRAMENITSKY, R. LEDINCKY, L. TIKHONOVA, A. VALVAROVA, V. VRBA, D. JERMILOVA, V. PHILIPPOVA, V. SAMOJLOV, T. TEMIRALIEV, S. DUMBRAJS, J. ERVANNE, E. HANNULA, P. VILLANEN, R. DEMENTIEV, I. KHORZAVINA, E. LEIKIN, A. PAVLOVA, V. RUD, I. HERYNEK, V. KOL, J. ŘIDKÝ, V. ŠIMÁK, L. ROB, M. SUK, J. PATOČKA, G. KURATASHVILI, A. HUDZHADZE, V. TSINTSADZE, T. TOPURIYA and S. LEVONJAN: *Scaling in the mean and associative multiplicities for the inclusive reactions* $\bar{\mathrm{p}}\mathrm{p} \to \mathrm{K}_s^0 + \mathrm{X}$ *and* $\bar{\mathrm{p}}\mathrm{p} \to \Lambda\mathrm{X}$ *at* 22.4 GeV/c, JINR preprint, Dubna E1-80-316 (1980).

([24]) The one exception is the stange-baryon distribution. W. DUNWOODIE: private communication.

within and without this Q^2 range. This attests to the basic soundness of the approach.

More work needs to be done to include the variation of α and some calculable (nonsingular) hard corrections.

APPENDIX

In this appendix we wish to exhibit the origins of the exponential form for the soft-gluon emission. Let us consider a quark of momentum p_1 and an antiquark of momentum p_2. We are interested in the process

$$\text{(A.1)} \qquad \mathrm{q}(p_1) + \bar{\mathrm{q}}(p_2) \to \mu^+\mu^- + ng$$

where ng means n gluons. The q and $\bar{\mathrm{q}}$ come from two different hadrons. One calculates the transition amplitude for the process, squares and sums over final states (of color) and averages over the spin and color of the incident quarks. One can show that the leading logarithmic terms are then the same as would have been obtained from an Abelian theory with a classical current $j_\mu(k)$, where k is the momentum of the emitted gluon. This is so because the non-Abelian graphs with three and four gluons are nonleading and thus do not appear in the soft limit. Thus the cross-section for the process of eq. (A.1) may be written as

$$\text{(A.2)} \qquad \mathrm{d}\sigma_{\text{brems}} = \mathrm{d}\sigma(\mathrm{q}\bar{\mathrm{q}} \to \mu^+\mu^- + n \text{ soft gluons}) = \\ = \frac{1}{2s'}\frac{1}{n!}\prod_{i=1}^{n}\left[\frac{\mathrm{d}^3\boldsymbol{k}_i}{(2\pi)^3 2k_i}|j_\mu(k_i)|^2\right]\frac{1}{4}\operatorname{Tr}\left|M_e\left(P' - \sum_{i=1}^{n} k_i\right)\right|^2 \cdot \\ \cdot \frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^3\omega_q}(2\pi)^4\delta^4\left(P' - \sum_{i=1}^{n} k_i - q\right).$$

In eq. (A.2) s' is the total energy squared in the center of mass of the two quarks, $M_e\left(P' - \sum_i k_i\right)$ is the «elastic» amplitude with its argument shifted to allow for the momentum carried off by the gluons, q is the total momentum of the μ pair.

To get the cross-section for the emission of any number of gluons, we wish to integrate over each of the variables k_i and then sum from $n = 0$ to ∞ It is convenient to do this by performing the integration subject to the condition that the total gluon 4-momentum is K and then performing $\sum_n$.

Thus eq. (A.2) becomes

$$\text{(A.3)} \qquad \mathrm{d}\sigma_{\text{brems}} = \frac{1}{2s'}\sum_n \frac{1}{n}\int \mathrm{d}^4K \prod_{i=1}^{n}\left[\frac{\mathrm{d}^3\boldsymbol{k}_i}{(2\pi)^3 2k_i}|j_\mu(k_i)|^2\right]\delta^4\left(K - \sum_{i=1}^{n} k_i\right) \cdot \\ \cdot \frac{1}{4}\operatorname{Tr}|M_e(P' - K)|^2 \frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^3 2\omega_q}(2\pi)^4\delta^4(P' - K - q),$$

i.e.

$$\text{(A.4)}\qquad \mathrm{d}\sigma_{\text{brems}} = \frac{1}{2s'}\int \frac{\mathrm{d}^4x}{(2\pi)^4}\,\mathrm{d}^4K \exp[iK\cdot x]\sum_n \frac{1}{n!}\left[\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3 2k}\,|j_\mu(k)|^2 \exp[-ik\cdot x]\right]^n\cdot$$
$$\cdot\frac{1}{4}\,\mathrm{Tr}\,|M_{\mathrm{e}}(P'-K)|^2\,\frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^3 2\omega_{\mathrm{q}}}\,(2\pi)^4\delta^4(P'-K-q)$$

and

$$\text{(A.5)}\qquad \mathrm{d}\sigma_{\text{brems}} = \frac{1}{2s'}\int \frac{\mathrm{d}^4x}{(2\pi)^4}\,\mathrm{d}^4K \exp[iK\cdot x]\exp[-h_{\mathrm{B}}(x)]\cdot$$
$$\cdot\frac{1}{4}\,\mathrm{Tr}\,|M_{\mathrm{e}}(P'-K)|^2\,\frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^3 2\omega_{\mathrm{q}}}\,(2\pi)^4\delta^4(P'-K-q)\,,$$

where

$$\text{(A.6)}\qquad h_{\mathrm{B}}(x) = -\int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3 2k}\,|j_\mu(k)|^2 \exp[-ik\cdot x]\,.$$

Equation (A.3) is the product of two parts. One describes the scattering without soft-gluon emission, while the other describes the soft-gluon emission. In fact, the latter can be used to define the probability that there will be an emission of gluons with 4-momentum K. This distribution is given by

$$\text{(A.7)}\qquad \frac{\mathrm{d}^4\mathscr{P}(K)}{\mathrm{d}^4K} = C\sum_{n=0}^{\infty}\frac{1}{n!}\int\prod_{i=1}^{n}\frac{\mathrm{d}^3\boldsymbol{k}_i}{(2\pi)^3 2k_i}\,|j_\mu(k_i)|^2\,\delta^4\!\left(K-\sum_{i=1}^{n}k_i\right),$$

where C will be determined below. For this to be a probability distribution we must require

$$\int\frac{\mathrm{d}^4\mathscr{P}(K)}{\mathrm{d}^4K}\,\mathrm{d}^4K = 1\,.$$

This gives

$$\text{(A.8)}\qquad C = \exp\left[-\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3 2k}\,|j_\mu(k)|^2\right] \equiv \exp[-h_v]\,.$$

The factor C is missing in eq. (A.3) and one has to multiply by it. Its origins are, of course, due to the virtual soft gluons, which when included will just have the effect of multiplying eq. (A.3) by C. They are thus contained in the scattering term and one has

$$\text{(A.9)}\qquad \tfrac{1}{4}\,\mathrm{Tr}\,|M(P'-K)|^2 = \exp[-h_v]\tfrac{1}{4}\,\mathrm{Tr}\,|M_{\mathrm{e}}(P'-K)|^2\,.$$

Thus in eq. (A.5) we must replace $h_{\mathrm{B}}(x)$ by $h(x) = h_{\mathrm{B}} + h_v$. We then have

$$\text{(A.10)}\qquad h(x) = \int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^3 2k}\,|j_\mu(k)|^2\,[1-\exp[-ik\cdot x]].$$

It will be convenient to define $d^3\bar{n}(k)$, where

$$d^3\bar{n}(k) = \frac{d^3k}{(2\pi)^3 2k}\,|j_\mu(k)|^2 . \tag{A.11}$$

$\bar{n}(k)$ has the physical significance of the average number of soft gluons of momentum k.

We note that $h(x)$ of eq. (A.9) has *no* IR divergence, the virtual soft-gluon exchange term h_v of eq. (A.8) providing the cancellation which is missing in $h_B(x)$ of eq. (A.6). Thus we have, including both virtual and real soft-gluon emission, for the cross-section

$$\frac{d^4\sigma}{d^4q} = \int d^4\mathscr{P}(K)\,\sigma_{Born}(q)\,\delta^4(P' - K - q) , \tag{A.12}$$

$$\sigma_{Born} = \frac{4\pi\alpha^2}{9Q^2}\,e_q^2 , \tag{A.13}$$

where e_q is the charge of the quark in units of the proton charge, and

$$\frac{d^4\mathscr{P}(K)}{d^4K} = \int \frac{d^4x}{(2\pi)^4} \exp[iK\cdot x - h(x)] . \tag{A.14}$$

● RIASSUNTO (*)

Si studia la distribuzione del quadrimpulso $d^4\mathscr{P}(K)$ dell'emissione multipla di gluoni morbidi usando un approccio di Bloch-Nordsieck. Si trova una separabilità asintotica: $d^4\mathscr{P}(K) \simeq d\mathscr{P}(K_+)\,d\mathscr{P}(K_-)\,d^2\mathscr{P}(K_\perp)$, dove $K_\pm = K_0 \pm K_3$. Si studia il loro effetto nella produzione di coppie di μ e si ottengono espressioni esplicite per la sezione d'urto doppio differenziale. Si calcola la pendenza dell'impulso medio trasversale con l'energia totale e si confronta con i dati. Le curve medie di scala sono in buon accordo con l'esperimento. Si suggeriscono altri controlli.

(*) *Traduzione a cura della Redazione.*

Поправки, связанные с испусканием мягких глюонов, к процессу Дрелла-Яана.

Резюме (*). — Используя подход Блоха-Нордсика, исследуется распределение четырех-импульса, $d^4\mathscr{P}(K)$, для множественного испускания мягких глюонов. Обнаружена асимптотическая разделяемость: $d^4\mathscr{P}(K) \simeq d\mathscr{P}(K_+)\,d\mathscr{P}(K_-)\,d^2\mathscr{P}(K_\perp)$, где $K_\pm = K_0 \pm K_3$. Исследуется влияние испускания мягких глюонов на рождение мюонных пар. Получаются точные выражения для двойного дифференциального поперечного сечения. Вычисляется наклон среднего поперечного импульса с полной энергией. Полученный результат сравнивается с экспериментальными данными. Наши средние кривые скейлинга также хорошо согласуются с экспериментом. Предлагаются дополнительные исследования.

(*) *Переведено редакцией.*