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SPIN-ISOSPIN CORRELATIONS IN LIGHT DEFORMED NUCLEI AT NORMAL AND HIGH DENSITY

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ABSTRACT

It is shown that a σ-τ phase can be realized only in deformed nuclei with broken isospin. The occurrence of deformation is intuitively related to the laminated structure in nuclear matter. σ-τ correlations are considered in light deformed nuclei at normal and high density. At normal density such correlations give rise to M2 isovector collective states whose properties depend on the oblate or prolate nuclear shape and on the longitudinal (w.r. to the symmetry axis) or transverse character of the mode. At high density the signature of a σ-τ phase in a specific model is discussed and conjecturally related to anomalons.

1. Why a σ-τ phase has a laminated structure in nuclear matter

The normal state of nuclear matter becomes unstable when the pion propagator acquires a pole at zero energy. At this point it costs no energy to produce pions and the pion field develops a nonvanishing vacuum expectation value (Ref. 1) which obeys the equation

$$(\nabla^2+\mu^2)\langle\Phi_3\rangle = \frac{f}{\mu}\,\partial_k\,\langle S_{3k}\rangle \tag{1}$$

where S_{ik} is the spin-isospin density operator $S_{ik}=\bar{\psi}\tau_i\sigma_k\psi$. Detailed calculations justify the minimal assumption

$$\partial_k\langle S_{3k}\rangle = \partial_3\langle S_{33}\rangle \tag{2}$$

x_3 being the axis of spin quantization. The order parameter $\langle S_{33}\rangle$ can be expressed in terms of the one-body density matrix $\varrho_{\sigma_3\tau_3}$ of nucleons of isospin τ_3 and spin σ_3

$$\langle S_{33}\rangle = \sum_{\tau_3\sigma_3} \tau_3\,\sigma_3\,\varrho_{\tau_3\sigma_3} \,. \tag{3}$$

A nonvanishing $\langle \Phi_3 \rangle$ implies one-dimensional σ-τ density fluctuations.

If the phase transition is of second order it is not necessary to know the properties of the ordered phase. All one needs to know is at which density the normal phase becomes unstable. At present most people accept that the critical density derived in this way is more than twice the normal density or that the phase transition does not take place at all (Ref. 3).

If on the other hand the phase transition is of first order, its critical density can only be established by comparing the energies of the ordered and disordered phase between themselves.

Now the ordered phase can be realized in many ways and therefore one needs a specific model to make such a comparison.

A number of models of the ordered phase has been invented independently of pion condensation (Ref. 4). These models were based on the observation that if the OPEP were strong enough it could distort the structure of normal nuclear matter in order to get a nonvashing expectation value in Hartree approximation

$$\langle V_\pi \rangle_d = \frac{1}{2} \int d\vec{r}_1 \int d\vec{r}_2 \langle S_{33}(\vec{r}_1) \rangle \langle S_{33}(\vec{r}_2) \rangle \left[V_c + (2z^2 - r_\perp^2) V_T \right] \qquad (4)$$

We see that also in this approach $\langle S_{33} \rangle$ cannot be constant in space because otherwise the tensor contribution would average to zero. Such a contribution is actually attractive for the laminated structure shown in Fig. 1 because in such a configuration $\langle z^2 \rangle \ll \langle r_\perp^2 \rangle$. In a variational calculation one finds that at the critical density there is a discontinuous jump from homogeneous nuclear matter to one-dimensional crystallization (Fig. 2, the curve AB refers to normal matter, the curve CD to σ-τ matter), i.e. the onset of σ-τ density fluctuations is not smooth.

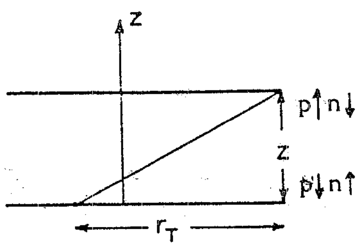

FIG. 1

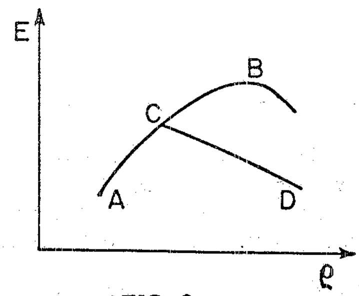

FIG. 2

This property, later fully exploited by Tamagaki and coworkers (Ref. 4), qualifies the phase transition as a first order one.

A realistic evaluation of the crytical density is much more difficult in a first order phase transition, because one must compare the energy of the disordered phase to that of the ordered one, and this latter requires complicated s.p.w.f., a cylindrical Fermi surface and anisotropic and σ-τ dependent short range correlations (Ref. 5).

The critical density might result different from that evaluated in the framework of pion condensation with a procedure which is appropriate only to a second order phase transition.

An example is provided by the liquid-vapor phase diagram, Fig. 3. The

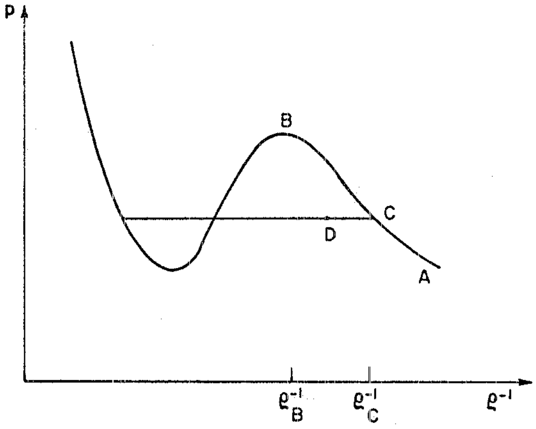

FIG. 3

critical density at C is determined by comparing the free energy of liquid and vapor. Actually the vapor (normal nuclear matter) can be made to follow the curve AB beyond the critical point C by means of an adiabatic compression. The resulting phase of supersaturated vapor (metastable normal nuclear matter) can also be described theoretically by means of the RPA. The point B will appear as a second order phase transition occurring at $\varrho_B > \varrho_C$. A RPA would completely ignore the crossing at ϱ_C. It is perhaps worth noticing that the order of the phase transition is relevant to phenomenology: i) if it is first order, and if the experimental conditions in heavy ion collisions give rise to an adiabatic compression, nuclear matter would remain in the normal metastable phase even beyound the critical density; ii) if it is second order, below the critical density there must be percursor phenomena (Ref. 6) which could be observed in nuclei if the critical density is not too higher than normal density. We will see that in nuclei, due to their finite size, phenomena analogous to the precursor ones are possible even if the phase transition is of first order in nuclear matter.

2. Why a σ-τ phase implies deformation in nuclei

We will consider only nuclei with N=Z and we will show that if a nucleus is spherical or has a definite isospin, the order parameter vanishes.

The wavefunction of a spherical nucleus can be written

$$|\psi\rangle = \sum_{\substack{S S_z \\ L L_z}} |\Phi_s\rangle |LL_z\rangle\, |S S_z\rangle \langle S S_z\, LL_z |\, 00\rangle \qquad (5)$$

Since S_{ik} does not act on $|LL_z\rangle$,

$$\langle\psi|S_{ik}|\psi\rangle = \delta_{k3} \sum_{SS_z} \frac{1}{2S+1}\, S_z \langle \phi_s|\mathcal{T}_i(S)|\,\phi_s\rangle = 0 \qquad (6)$$

where we made use of the Wigner-Eckart theorem:

$$\langle SS_z|S_{ik}|SS_z\rangle = \delta_{k3}\, S_z\, \mathcal{T}_i(S)$$

In analogous way we can show that $\langle S_{ik} \rangle = 0$ in a state of definite isospin. A σ-τ phase can therefore be realized only in a deformed nucleus with broken isospin. At normal density, in view of the high degree of isospin conservation, nuclei can have at most very small σ-τ density fluctuations.

In a heavy nucleus the density fluctuations could be very similar to those of nuclear matter as illustrated in Fig. 4a. In a small nucleus one can only accomodate two layers (Fig. 4b) or one (Ref. 7) (Fig. 4c). In this latter case the fact that the σ-τ-density fluctuations must be small means that there can only be a small excess of nucleons with $\sigma_3 \tau_3 = 1$ ($\sigma_3 \tau_3 = -1$).

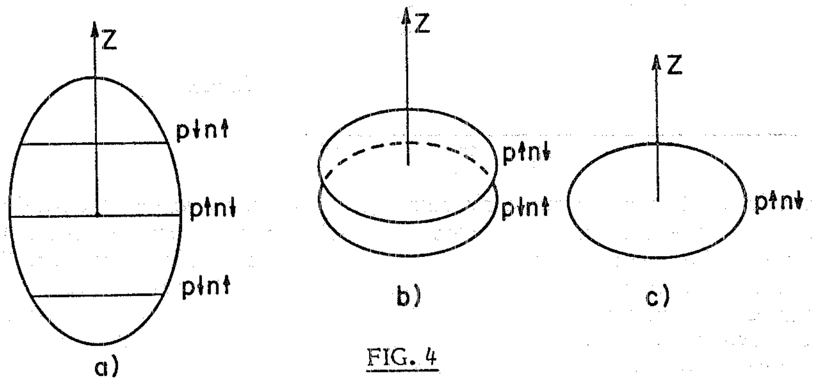

FIG. 4

The analogy with nuclear matter suggests that oblate deformation should be favoured in light nuclei by the OPEP.

We will confine ourselves to light nuclei.

The study of a σ-τ phase requires the evaluation of the energy in such a phase as a function of the density, the deformation and other parameters. These are the displacement in the two layers configuration and the $\sigma_3 \tau_3 = +1$ ($\sigma_3 \tau_3 = -1$) nucleon excess in the one layer configuration.

In the next section we will study the energy of the two layers configuration at normal density for the experimental value of the deformation and small displacement d.

At normal density the two layers configuration cannot be realized statically, even with small σ-τ density fluctuations, because it breaks parity. It can however occur dynamically, as zero-point σ-τ oscillation. If the related σ-τ configuration energy is repulsive, it will give rise to a hardened collective state as predicted (Ref. 8) by Ikeda et al. and recently found (Ref. 9) in ^{208}Pb. Such prediction does not take into account the effect of deformation that we are going to study. If such effect makes the σ-τ correlations attractive, there will be a softened collective state. The softening of such a state and the enhancement of its amplitude correspond to the signature for precussor phenomena (Ref. 6). Note however that strictly speaking it could not be interpreted as a precritical phenomenon, because it would disappear in the limit of large volume.

The study of σ-τ correlations in light deformed nuclei at normal density provides the first piece of information on σ-τ phases.

3 σ-τ correlations in light deformed nuclei at normal density

We will describe small σ-τ displacements by a collective model (Ref. 10).The total nuclear w.f. is assumed to be

$$\Psi_{\nu_3 \nu_T K} = \Phi_{\nu_3 \nu_T K}(\vec{d})\, \Lambda(\vec{d}), \qquad (7)$$

where $\Lambda(\vec{d})$ is a Slater determinant of displaced s.p.w.f.

$$\lambda_{n_z n_T m \sigma_3 \tau_3} = \varphi_{n_z n_T m}(\vec{r} - \tfrac{1}{2}\vec{d}\,\sigma_3\,\tau_3)\,\chi_{\sigma_3 \tau_3}, \qquad (8)$$

$\varphi_{n_z n_T m}$ being h.o.w.f. in a cylindical basis and $\chi_{\sigma_3 \tau_3}$ spin-isospin w.f.

The z-axis is taken parallel to the nuclear symmetry axis. If axis-3 of spin space is taken parallel to the symmetry axis, we will talk of longitudinal polarization, if it is taken orthogonal we will talk of transverse polarization. In analogous way we will talk of longitudinal and transverse oscillations.

The collective Hamiltonian is

$$H = \frac{p^2}{2M} + W(\vec{d}), \qquad (9)$$

where $M = \frac{1}{4}$ Am. For small oscillations we can take a harmonic approximation to W,

$$W(\vec{d}) \simeq \tfrac{1}{2}(C_z + K_z)\, d_z^2 + (C_T + K_T)\, \vec{d}_T^{\,2}, \qquad (10)$$

C_z and C_T being the σ-τ independent restoring constants and K_z and K_T the dependent ones. These latter are determined by the equation

$$\langle \Lambda(d) \mid V_{\sigma\tau} \mid \Lambda(d) \rangle \simeq \tfrac{1}{2} K_z\, d_z^2 + \tfrac{1}{2} K_T\, \vec{d}_T^{\,2}. \qquad (11)$$

$V_{\sigma\tau}$ is usually approximated by the regularized π- and ϱ-exchange potentials plus a contact term which is assumed to account for short range and exchange effects, whose strength is parametrized by the Landau-Migdal parameter g'.

Following the prescription of the unified theory of nuclear vibrations, the σ-τ independent restoring constants are determined by the equations

$$C_z = \frac{A}{4}\, m\, \bar{\omega}_z^2 \qquad C_T = \frac{A}{4}\, m\, \bar{\omega}_T^2, \qquad (12)$$

$\bar{\omega}_z$ and $\bar{\omega}_T$ being s.p.h.o. frequencies.

The numerical evaluation of K_z and K_T confirms, according to the analogy between Figs. 1 and 4b that longitudinal polarization is favored in oblate nuclei for both longitudinal and transverse oscillations.

In prolate nuclei the contribution of the OPEP gets larger for transverse polarization. Such a choice breaks axial symmetry in the excited states. We will neglect such a small breaking.

The first collective excited states are characterized by the quantum number K and the M2 transition strength

$$B(M2, 0 \to k)\uparrow = \frac{5}{32\pi}\, \frac{\hbar^2}{m}\, A\left(\frac{1}{\hbar\omega_z}\,\delta_{k0} + \frac{1}{\hbar\omega_T}\,\delta_{k1}\right)(g_p - g_n)^2 \mu^2 \mathrm{fm}^2 \qquad (13)$$

for longitudinal polarization and

$$B(M2,0\rightarrow K)\uparrow = \frac{15}{128\pi}\frac{\hbar^2}{m} A\left(\frac{1}{3}\frac{1}{\hbar\omega_x}\delta_{k0}+\frac{1}{\hbar\omega_z}\delta_{k1}+\frac{1}{2}\left(\frac{1}{\hbar\omega_x}+\frac{1}{\hbar\omega_y}\right)\delta_{k2}\right)\cdot$$
$$\cdot (g_p - g_n)^2 \mu^2 \text{ fm}^2 \qquad (14)$$

for transverse polarization (assuming the axis of spin quantization parallel to the x-axis). Note that neglecting the breaking of axial symmetry $\omega_x \sim \omega_y \sim \omega_T$.

The collective model is consistent only when K is not negligible w.r. to C. When it is not so, the corresponding level cannot be assumed to be collective and will therefore be ignored. It will turn out that $|K_x| \ll |C_x|$ in prolate nuclei and therefore we will put $K_y = K_T$.

The anisotropic character of the response should be stressed. Longitudinal excitations have magnetic quantum number K=0,1, while transverse excitations have K=1,2. This is closely related to the σ-τ response in nuclear matter (see Alberico's talk in this Workshop) which has magnetic quantum numbers K=0 in the longitudinal (w.r. momentum transfer) channel and K=1 in the transverse one.

The values of K_z and K_T have been evaluated for g'=0.33, 0.5, 0.7. While K_T turns out to be always positive for both oblate and prolate nuclei, K_z results positive for prolate nuclei, but for the oblate ones $K_z > 0$ for g'=0.7, $K_z \sim 0$ for g'$\sim$ 0.5÷0.6 and $K_z < 0$ for g'=0.33. We have therefore the distinctive prediction that for g'$\sim$ 0.5÷0.6 there should be two collective states in prolate nuclei and only one in oblate nuclei as displayed in Fig. 5. This characterization seems far from trivial because one would expect in general a splitting of M2 collective states in analogy with the E1 resonance. The predicted position and strength of the collective levels (indicated by their quantum numbers K^{π}) are reported in Figs. 6 and 7 for ^{12}C and ^{28}Si for different values of g' as compared to the experimental results (without quantum number).

The experimental M2 levels of ^{28}Si are closely centered (Ref. 11) around an energy value of 14.5 MeV with an integrated strength $\Sigma B(M2)\uparrow \simeq 400\ \mu^2\ \text{fm}^2$. In ^{12}C the M2 strength is practically all concentrated (Ref.12) in one signle level at 19.3 MeV and has the value $B(M2)\uparrow \simeq 700\ \mu^2\ \text{fm}^2$. This value is not a result of a direct measure but follows from a theoretical analysis of the data carried out using R.P.A. with spherical s.p.w.f.s'.

These experimental results are not in contradiction with our prediction for g'$\sim$ 0.5÷0.6.

Our schematic model cannot account for the detailed properties of the nuclei considered here. Only a R.P.A. calculation in a deformed basis, which includes the effect of the Δ isobar, the spin-orbit terms etc., would allow a complete description. Such a microscopic calculation could possibly account for the fragmentation of the M2 states observed in ^{28}Si and would allow a more realistic fit of the Landau-Migdal parameter g' for light deformed nuclei.

We would like to emphasize the importance of using a deformed rather than a spherical basis. With a deformed basis one has to expect in general two collective states, and it is not trivial to get only one. With a spherical basis, on the other hand, one gets naturally a single collective level. Such a basis, however, can hardly give a consistent description of the properties of deformed nuclei. It cannot account, for instance, in a simple way for the possible splitting of M2 resonances in prolate nuclei.

Let us relate the above analysis to σ-τ phases. Let us assume that at some fixed density the potential energy W has a secondary minimum at some σ-τ

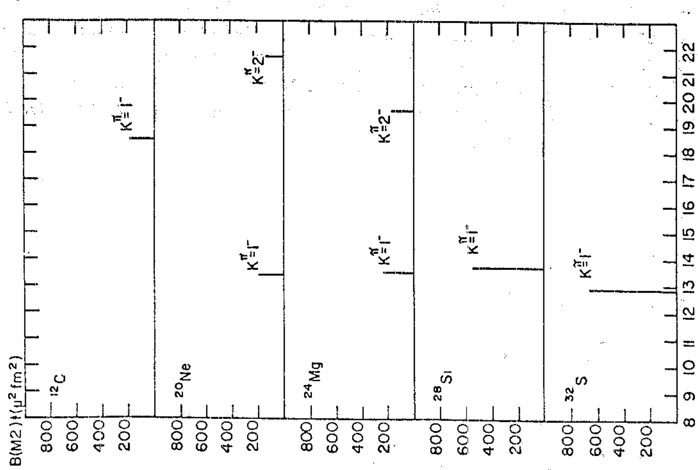

FIG. 5

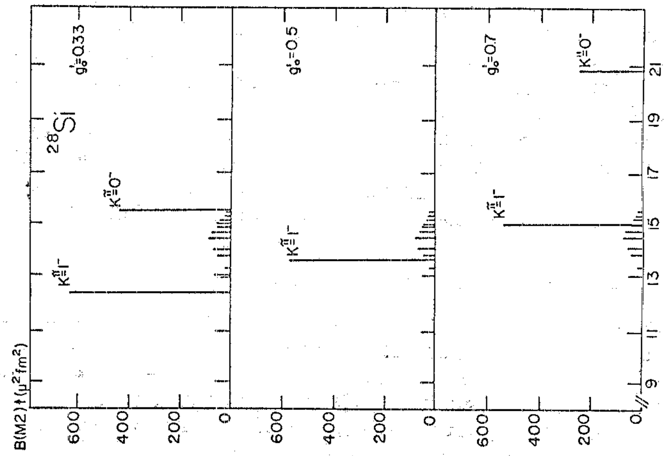

FIG. 6

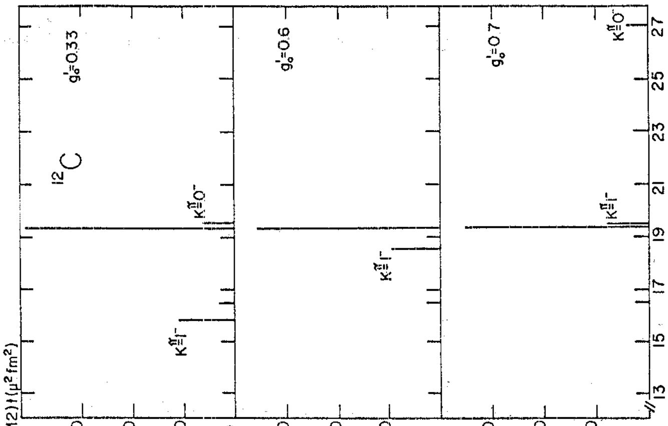

FIG. 7

displacement d_c, as depicted in Fig. 8. In this figure different possibilities arising

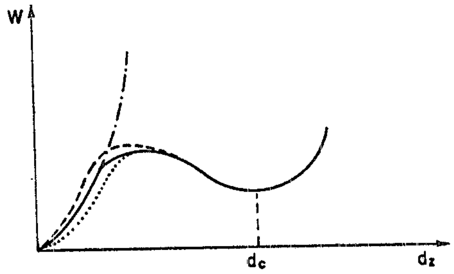

FIG. 8

according that $\langle V_{\sigma\tau} \rangle$ is positive, null or negative for small displacements are also schematically reported. What we learn if our predictions are confirmed is that the most favorable situation occurs for longitudinal displacements in oblate nuclei, for which the σ-τ correlation energy vanishes. The vanishing of K_z arises in such a case from the cancellation of a large attractive term due to the OPEP and a large repulsive term due to short range effects. It is then conceivable that at high momentum transfer the nuclear response be softnened and enhanced, as in a precursor to pion condensation.

We finally observe that in the present two-fluid scheme it is possible to consider for light deformed nuclei all kinds of σ-τ oscillations giving rise to the pion-like excitations studied in relation to precursor phenomena. In particular a M1 unidimensional σ-τ breathing oscillation mode has been studied (Ref. 13).

4 Signature of a σ-τ phase in light nuclei at large density

A realistic evaluation of W at high density is beyond the present theoretical possibilities. For the one-layer configuration there are some preliminary investigations of the role of short range correlations, (Ref. 14) Δ-N mixing and shell effects (Ref. 15) which will be presented in separate contributions.

Here we will only discuss the signature (Ref. ⁶) of the static two layers configuration which would result if there were an energy minimum at $d_c \neq 0$.

If the potential energy W has a local minimum at density ϱ_c and separation d_c, and if such a minimum is not destroyed by the kinetic energy, there is a metastable state in which the $\sigma_3\tau_3=1$ nucleons oscillate w.r. to the $\sigma_3\tau_3=-1$ ones around the average separation d_c. Such oscillation will generate a dynamical choherent pion field according to the equation

$$(\Box + m_\pi^2)\langle \Phi_3 \rangle = \frac{f}{m\pi}\, \partial_3 \langle S_{33} \rangle \, . \qquad (15)$$

If the oscillation has frequency

$$\langle S_{33}(\vec{r}, t)\rangle = \langle S_{33}(\vec{r}, 0)\rangle\, e^{i\omega t}$$
$$\langle \Phi_3(\vec{r}, t)\rangle = \langle \Phi_3(\vec{r}, 0)\rangle\, e^{i\omega t} \qquad (16)$$

and

$$\left[\Delta-(m_\pi^2-\omega^2)\right] \langle \Phi_3(\vec{r},0)\rangle = \frac{f}{m_\pi}\, \partial_3 \langle S_{33}(\vec{r}, 0)\rangle \, . \qquad (17)$$

The pion outside the nucleus has therefore an effective Compton wavelength $\hbar/\sqrt{m_\pi^2 - \omega^2}$, which (combined with the displacement d_c) will make the nuclear cross section larger than the geometrical one. It is tempting to relate this signature to anomalous nuclei (Ref. 17).

A furhter characterization of such a metastable state can be obtained by evaluating the number of pions emitted in its decay. This evaluation can be done by means of the Bloch-Nordsiek theorem.

References

1) A.B. Migdal, Z. Eksp. Teor. Fiz. 61, 2210 (1971 (English translation: Sov. Phys. JETP 34, 1185 (1972); R.F. Sawyer, Phy. Rev. Letters 29, 382 (1972); D.J. Scalapino, Phys. Rev. Letters 29, 386 (1972); for a reviw see A.B. Migdal, Rev. Mod. Phys. 50, 107 (1978).
2) F. Calogero, in "The Nuclear Many-Body Problem", Vol. 2, edited by F. Calogero and C. Ciofi degli Atti (Roma, 1972), p. 535; F. Calogero and F. Palumbo, Lett. Nuovo Cimento 6, 663 (1973); F. Calogero, F. Palumbo and O. Ragnisco, Nuovo Cimento 29A, 509 (1975).
3) A. Faessler, in "Mesons, Isobars, Quarks and Nuclear Excitations", Erice, April 6-18, 1983.
4) T. Takatsuka and R. Tamagaki, Prog. Theor. Phys. 58, 694 (1977); R. Tamagaki, Nuclear Phys. A328, 352 (1979).
5) D.J. Sandler and J.W. Clark, Phys. Letters 100B, 213 (1981); K. Tamiya and R. Tamagaki, Prog. Theor. Phys. 66, 948 (1981); ibidem 66, 1361 (1981); O. Benhar, Phys. Letters 106B, 375 (1981); ibidem 124B, 305 (1983); O. Benhar, C. Ciofi and F. Cleri, invited talk to the Intern. Conf. on the Many-Body Problem, Altenberg, August 1983 and Phys. Letters B, submitted to; E. Pace and F. Palumbo, Phys. Letters 113B, (1982).
6) S.A. Fayans, E.E. Sapershtein and S.V. Tolokonnikov, J. Phys. G3, L5 (1977); M. Gyulassi and W. Greiner, Ann. Phys. (N.Y.) 109, 485 (1977); M. Ericson and J. Delorme, Phys. Letters B76, 192 (1978). For a review see M. Ericson in "Spin-isospin Response Functions in Nuclei". Invited talk at the "Intern. Conf. on Spin Excitations in Nuclei", Telluride, Colorado, USA, March 25-27 (1982).
7) G. Do DAng, Phys. Rev. Letters 43, 1708 (1979); R.K. Tripathi, A. Faessler and K. Shimizu, Z. Phys. A297, 275 (1980); G. Giberti, N. Lo Iudice and G. Varcaccio, Lett. Nuovo Cimento 34, 391 (1982); Phys. Rev. C28, 888 (1983).
8) K. Ikeda, S. Fujii and J.J. Fujita, Phys. Letters 3, 271 (1963); K. Ikeda, Progr. Theor. Phys. 31, 434 (1964).
9) D.J. Horen, C.D. Goodman, C.C. Foster, C.A. Goulding, M.B. Greenfield, J. Rapaport, D.E. Bainum, E. Sugarbaker, T.G. Masterson, F. Petrovich and W.G. Lowe, Phys. Letters 95B, 27 (1980).
10) N. Lo Iudice and F. Palumbo, Phys. Rev. Letters 46, 1054 (1981); Lett. Nuovo Cimento 36, 91 (1983); Phys. Rev. C, submitted to.
11) A. Friebel, M.D. Gräf, W. Knüpfer, A. Ritcher, E. Spamer and O. Titze, Phys. Rev. Letters 48, 567 (1982).
12) T. Terasawa, K. Nakahara and T. Torizuka, Phys. Rev. C3, 1750 (1971); E. Grecksch, W. Knüpfer and M. Dillig, Z. Phys. A302, 165 (1981).
13) N. Lo Iudice, Phys. Rev. C24, 2770 (1981).
14) O. Behar, C. Ciofi and F. Cleri, invited talk to this Workshop.
15) G. Giberti and N. Lo Iudice, contribution to this Workshop.
16) F. Palumbo, Lett. Nuovo Cimento 34, 305 (1982).
17) E.M. Friedlander, R.W. Gimpel, H.H. Heckman, Y.J. Karant, B. Judek and E. Ganssauge, Phys. Rev. Letters 45, 1084 (1980).