To be submitted to
Phys. Letters B

ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-83/62(P)
23 Settembre 1983

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RADIATIVE CORRECTIONS FOR Z^o MASS AND WIDTH DETERMINATION AT THE $\bar{p}p$ COLLIDER

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ABSTRACT

We discuss the effects of final state bremsstrahlung from the electrons produced in the decay of the neutral vector boson Z^o at the CERN collider. We find that soft and hard collinear photon emission introduce significant effects which must be taken into account in precision experiments. We present the radiatively corrected mass spectrum and discuss the width determination, comparing traditional momentum measurements and calorimeter techniques.

Recent experimental discoveries at the CERN SPS Collider have opened the way to a systematic study of the properties of the charged and neutral vector bosons[1,2]. Precise values of their masses are required for an accurate determination of the weak angle and of the ϱ parameter. These parameters are as important today for the Electro-weak theory, as the Schwinger value $\alpha/2\pi$ for the anomalous magnetic moment, was in the early days of quantum electrodynamics. Furthermore, precise values of the Z^o width will give us valuable information, unreachable otherwise, regarding the number of neutrino species, open flavours, and much more[3,4]. To obtain accurate values of these parameters, radiative effects must be considered.

In this note we present the radiatively corrected lepton pair mass spectrum arising from the Z^o decay, relevant to the determination of the Z^o-mass and total width. In what follows, we do not consider the Drell-Yan background, nor finite (ultra-violet) radiative corrections, both of which can be neglected within the limits of present experimental sensitivity. Radiative effects for the present setup at the collider are different from those for $e^+ e^- \rightarrow$ Resonance $\rightarrow e^+ e^-$ in the following respects. For $e^+ e^-$ collider beam experiments, for which the beam energy is precisely defined, the mass and width of the resonant state are obtained from the Breit-Wigner spectrum in the <u>initial-state</u> energy variable. The width provides a natural cut-off for initial state emission[5-8] with radiative corrections giving a substantial (radiative) tail to the right of the resonance. On the other hand, for hadronic collisions, the initial state hadrons have a continuous spectrum of energy which allows them to radiate γ's or gluons without imperilling the formation of a resonant state: the hadrons provide an energy-momentum reservoir so that a parton may shed a large amount of radiation ($>\Gamma$) and still produce the resonance. It is only the <u>final state</u> radiation which influences mass and width determination.

At the proton antiproton collider the mass of the $Z^o \to e^+e^-$ is measured through the total energy deposition in adjacent modules of electromagnetic calorimeters, as schematically shown in Fig. 1. In

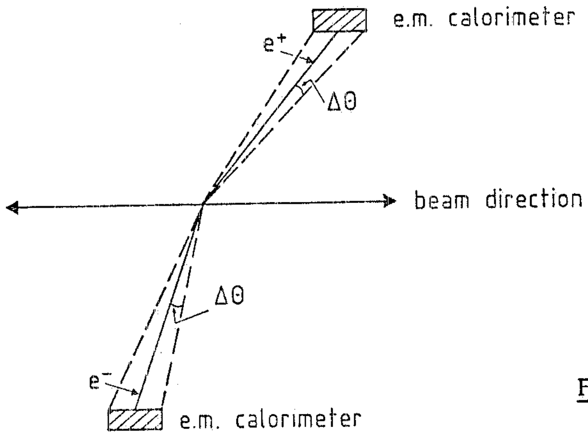

FIG. 1 - Schematic view of the apparatus.

what follows we shall estimate the amount of electromagnetic radiation (emitted by the final electrons) which escapes detection, thus introducing an uncertainty in the mass determination. This is accomplished by computing the fraction of internal bremsstrahlung which lies outside the cones subtended by the modules where the electrons are pointing (Fig. 1). It is then possible to compute the mass spectrum of the lepton pair, which has a Breit-Wigner form accompanied by a radiative tail prior to the resonance. For the UA1 collaboration[1], the present determination is based on a reconstruction of the electron momentum from the charged track as measured in the central detector (CD) and from the energy deposition (E_T) in the electromagnetic calorimeters. The CD provides the direction measurement, while the e.m. calorimeters give information on the energy carried by and along the charged track within a cone of opening angle $\Delta\theta=5°$. The use of calorimeters in the determination of the invariant Z^o-mass minimizes the effects of electromagnetic energy losses, since it is equally sensitive to the electron energy and to soft and hard collinear photons. To incorporate this flood of soft photons accompaning the process

$$q\bar{q} \longrightarrow Z^o \longrightarrow e^+e^- \tag{1}$$

we consider the 4-dimensional distribution[9]

$$d^4P(k) = \beta N^{-1}\left(\frac{d\omega}{\omega}\right)\left(\frac{\omega}{E}\right)^{\beta}\left(1-\frac{\omega}{E}+\frac{1}{2}\left(\frac{\omega}{E}\right)^2\right) f(\hat{k})\, d^3k\,\delta(|k|-1) \tag{2}$$

where

$$\beta(E) = \frac{4\alpha}{\pi} \left(\log \frac{2E}{m} - \frac{1}{2}\right)$$

with m the electron mass, E its c.m. energy, and $f(\hat{k})$ the single soft photon angular distribution given by:

$$f(\hat{k}) = - \frac{e^2}{\beta(2\pi)^3} \left[\frac{m^2}{(p_1 \cdot k)^2} + \frac{m^2}{(p_2 \cdot k)^2} - \frac{2(p_1 \cdot p_2)}{(p_1 \cdot k)(p_2 \cdot k)} \right] k^2 \qquad (3)$$

Eq. (2) gives the differential probability of a radiation loss in d^4K. In Eq. (2), the angular distribution of the four-momentum carried away by the electromagnetic radiation has been approximated by the single photon distribution $f(\hat{k})$, Eq. (3), up to terms of order β. This implies the identity of the angular distributions of classical radiation and the four-momentum loss and also, that to a good approximation, most of the energy is always carried away by a single photon. A full discussion of this approximation can be found in Ref. (9). We have modified the soft photon spectrum $(\frac{\omega}{E})^{\beta-1}$ so as to include hard collinear radiation as well. Since $d^4P(K)$ separates into an angular and an energy dependent part, we can independently evaluate the probability that either electron has lost more than a fraction x of its energy, call it $P(x)^{(10)}$, and the probability that the radiation is emitted at an angle larger than $\Delta\theta$ relative to the electron direction. We shall call the latter $F(\Delta\theta)$. For simplicity we assume that there is no appreciable acollinearity between the two electrons in the azimuthal plane. This approximation makes the integration over the photon azimuthal angle very simple. In Fig. 2, we show

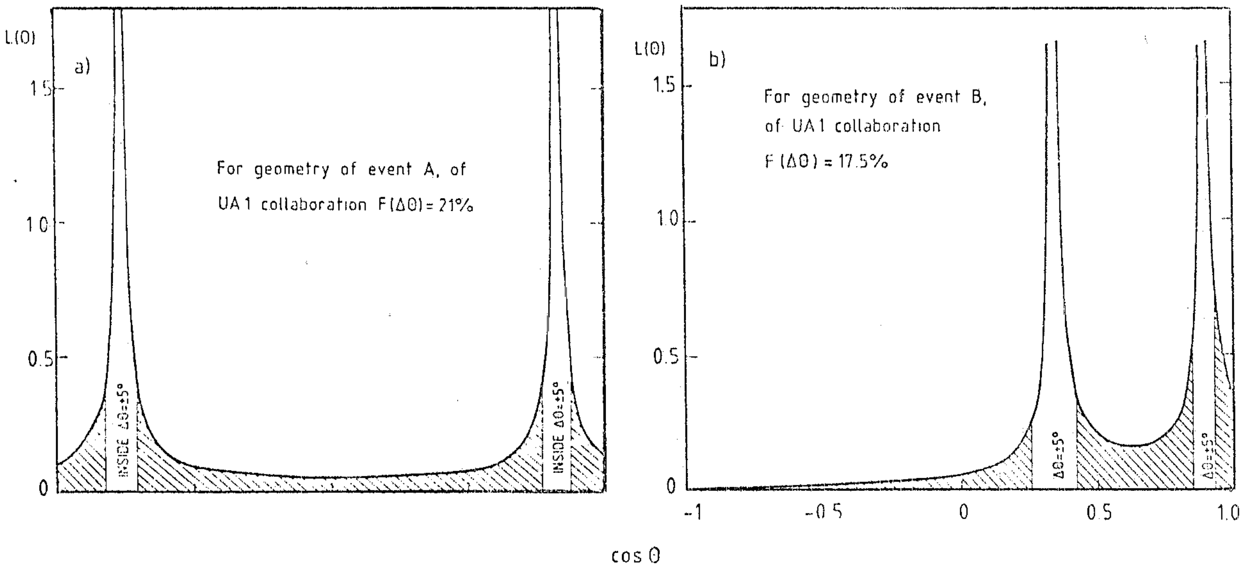

FIG. 2 - Angular distribution $L(\theta)$ of emitted soft radiation for events A and B of UA1 experiment (second of Ref. (1)).

the angular distribution of the classical radiation

$$L(\theta) = \int d\varphi \; f(\hat{k}) \qquad (4)$$

for the geometry of two of the four UA1 electron events. One notices that, although most of the radiation is along the direction of the emitting particle (within a cone of opening (m/E)) a non negligible fraction does lie outside the two cones. This isotropic background escapes detection and gives rise to an uncertainty in the Z^o-mass. In Fig. 3, we show the probability $F(\Delta\theta)$, which is obtained

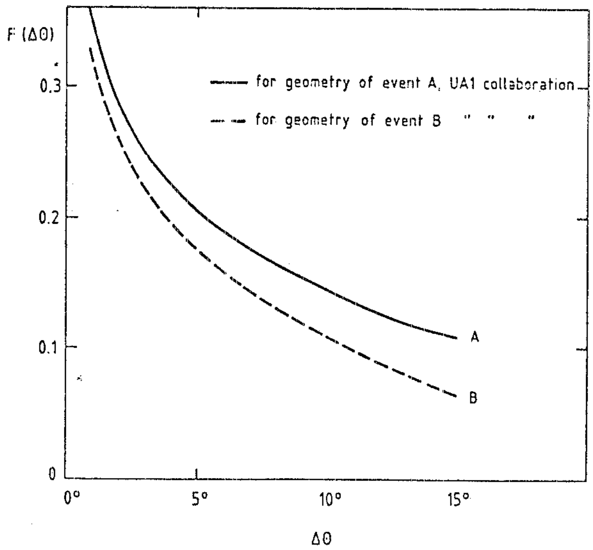

FIG. 3 - Fractional probability $F(\Delta\theta)$ that the emitted radiation lies outside a cone with resolution $\Delta\theta$ - for the geometry of events A and B of UA1 experiment (Ref. (1)).

when we integrate Eq. (4) for all values of θ outside the two cones characterized by the resolution angle $\Delta\theta$. To be definite, we have chosen the geometry of events A and B of UA1 experiment. Other configurations give very similar results, i.e. that (15÷20)% of the radiation escapes detection.

We can now estimate the average uncertainty in the mass determination. We calculate the mean energy lost through soft and hard collinear radiation in process (1), integrating over all directions. Neglecting terms of order β^2, we find

$$< \omega > = \frac{2}{3}\,\beta(E)\,E \simeq 3.2 \text{ GeV} \tag{5}$$

for a 45 GeV electron (in the c.m.) and $\beta(E)\simeq 0.108$. From the previous calculation, we have seen that roughly 20% of this radiation is not recorded in the typical UA1 geometry. This implies that, on the average,

$$<\Delta E>_{\text{outside}} = <\omega> F(\Delta\theta) \simeq 0.6 \text{ GeV}$$

which is also the uncertainty on the mass due to soft and hard collinear final state emission. This quantity is of order αM, as expected.

To obtain the radiatively corrected lepton pair mass spectrum, we fold the probability distribution $d^4P(K)$ with the resonant cross-section for Z^o-production. For definiteness, let $\sigma_o \simeq 0.7\times10^{-34}$ cm^2 be the integrated Z^o-production cross-section at the collider[11]. The resonance cross-section reads:

$$\frac{d\sigma}{dQ} \simeq \sigma_o\left(\frac{2Q}{\pi}\right) \frac{M\Gamma}{(M^2-Q^2)^2+(M\Gamma)^2} \tag{6}$$

where Γ is the total width and Q the mass of the lepton pair as measured by the calorimeters. The radiatively corrected cross-section then becomes

$$\frac{d\sigma_{rad}}{dQ} = \frac{2\sigma_o Q M\Gamma}{\pi N(\beta)} \beta \int_o^E \left(\frac{dk}{k}\right)\left(\frac{k}{E}\right)^{\beta}\left(1-\frac{k}{E}+\frac{1}{2}\left(\frac{k}{E}\right)^2\right) \chi(k, Q) \qquad (7a)$$

with

$$\chi = x_1 + x_2 \qquad (7b)$$

$$x_1(k,Q) = \frac{1-F(\Delta\theta)}{(M^2-Q^2)^2+(M\Gamma)^2} \qquad (8a)$$

$$x_2(k,Q) = \frac{F(\Delta\theta)\left(1+\frac{k}{\sqrt{k^2+Q^2}}\right)}{\left(M^2-\left(\sqrt{Q^2+k^2}+k\right)^2\right)^2+(M\Gamma)^2} \qquad (8b)$$

In the above expression, the Breit-Wigner of the unshifted argument is multiplied by the fraction of the radiation inside the angle $\Delta\theta$ while the Breit-Wigner of unshifted argument is multiplied by the fraction of the radiation outside the cones. The reason is that in a calorimeter experiment, the observed lepton pair mass already includes the radiation inside the modules, as we schematically indicate in Fig. 4. It is only the fraction of escaping electromagnetic radiation which introduces a shift and a broadening in the mass spectrum. The normalization factor

$$N(\beta) = 1-\frac{\beta(E)}{(1+\beta(E))} + \frac{\beta(E)}{2(2+\beta(E))}$$

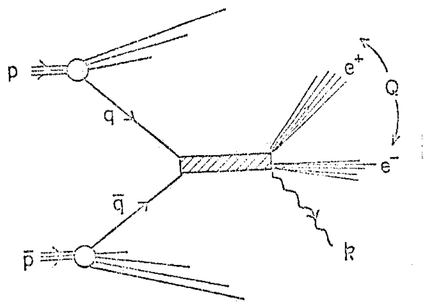

FIG. 4 - Schematic view of the measurement.

ensures that the integrated Z^o cross-section, with or without radiative corrections, is given by σ_o. In Fig. 5 we show the normalized spectrum $\frac{dP}{dQ} = \frac{1}{\sigma_o}\left(\frac{d\sigma_{rad}}{dQ}\right)$ versus Q, for Γ=3 GeV, M=93 GeV, for the three cases, $F(\Delta\theta)$=0,.2 and 1. Case (a) corresponds to the uncorrected Breit-Wigner, (b) to a typical calorimer application where ≃ 20% of the radiation escapes and case (c) is typical of a magnetic curvature determination where all the radiation escapes detection. We see that for the calorimeter case the radiative tail is very much reduced. In a practical experiment, electrons must transverse a sizeable material (vacuum chamber etc.) and ordinary bremsstrahlung losses have to be added. Since, however, these are highly collinear with the electrons[12], they are easily collected by the calorimeters.

A parallel calculation for the decay $Z^o \rightarrow \mu^+ \mu^-$ shows that radiative losses are about a factor 2 smaller than in the case of electronic decay. Therefore also for this decay mode an appropriate recovery of the radiated energy might be necessary in order to achieve high accuracy.

To conclude, a computation of radiative energy loss by the final electrons shows that the Z^o mass determination via calorimeter experiments at the collider, suffers small but significant uncertainty of

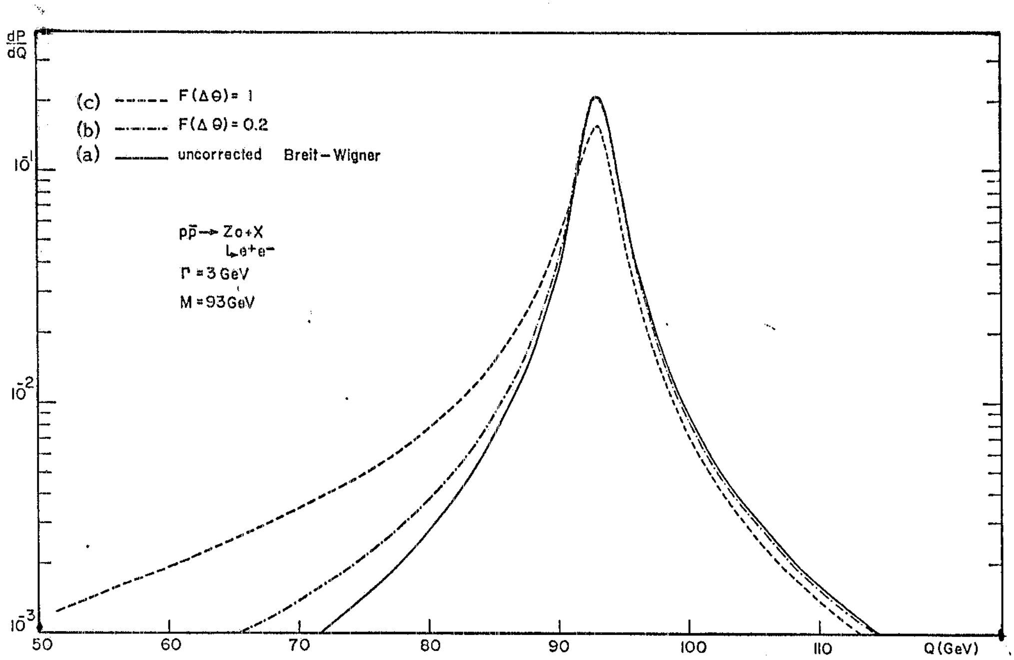

FIG. 5 - Normalized mass distribution $\frac{1}{\sigma_0}\left(\frac{d\sigma_{rad}}{dQ}\right)$: (a) uncorrected Breit-Wigner (BW), Eq. (6) with $\Gamma=3$ GeV, M=93 GeV, (b) radiatively corrected, Eqs. (7), with F($\Delta\theta$)=0.2, (c) radiatively corrected with F($\Delta\theta$)=1.

order αM. The corrected mass spectrum shows that for calorimeter experiments the radiative tail to the left of the resonance is small thus permitting accurate measurements of mass and width of Z^0 particle.

ACKNOWLEDGEMENTS

We thank Prof. C. Rubbia for suggesting us to investigate this problem, and for many useful conversations.

REFERENCES

(1) UA1 Collaboration, G. Arnison et al., Phys. Letters 122B, 103 (1983); Phys. Letters 126B, 398 (1983).
(2) UA2 Collaboration, G. Banner et al., Phys. Letters 122B, 476 (1983).
(3) UA1 Collaboration Proposal, CERN/SPSC/83/48; SPSC/P92 Add. 3 (1983).
(4) N. Cabibbo, III Topical Workshop on P$\bar{P}$ Collider Physics, Ed. by C. Bacci and G. Salvini, CERN 83-04 (May 1983).
(5) G. Pancheri, Nuovo Cimento 60, 521 (1969).
(6) M. Greco, G. Pancheri and Y.N. Srivastava, Nuclear Physics B171, 118 (1980).
(7) A. Kraemmer and B. Lautrup, Nuclear Physics B97, 380 (1975).
(8) F.A. Berends, K. Gaemer and R. Gastmans; Nuclear Physics B57, 381 (1973); F.A. Berends et al., Nuclear Physics B202, 63 (1982).
(9) E. Etim, G. Pancheri and B. Toushek, Nuovo Cimento 51B, 276 (1967).
(10) J. Rohlf and R. Wilson, CERN UA1/TN103-13 (1983); A. Nandi, private communication.
(11) R. Horgan and M. Jacob, CERN 81-04 (1981).
(12) B. Rossi, "Cosmic Rays", Allen and Unwin, London (1966).