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GALILEAN APPROXIMATION OF MASSLESS SUPERSYMMETRIC THEORIES

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There is no systematic way of studying dynamical symmetry breaking. In supersymmetry in particular, while it has been shown that fermion condensation can provide dynamical breaking[1], no model has been shown to give rise to fermion condensation.

Recently arguments have been given that Galilean approximations to relativistic theories can provide reliable information[2] on spontaneous symmetry breaking. These arguments are supported by the analysis of the Goldstone model[3], the Higgs model[4] and the Schwinger mechanism[5], whose essential features are conserved[2] in the Galilean approximation.

We have therefore studied the Galilean limit in supersymmetry. Here the situation is somewhat different from that of the above-mentioned models. The key point in those cases was to obtain the Galilean approximation by contraction (velocity of light $c \to \infty$) of the Poincaré group, while conserving all the other symmetries of the Lagrangian (charge symmetry, chiral invariance, gauge invariance,...). But now the Poincaré algebra is a subalgebra of the supersymmetry algebra, and its contraction implies the contraction of the whole supersymmetry algebra. This contraction is presented here in the simplest case, in which all the particles are massless. In such a case the relativistic theory undergoes a dimensional reduction to 0 + 1 dimensions[2], i.e., to quantum mechanics. We have supersymmetric quantum mechanics with chiral invariance for the pure Wess-Zumino model, and with non-Abelian local gauge invariance for Yang-Mills theories. An additional internal O(3) invariance arises from dimensional reduction.

The 0 + 1 dimensional field theory that we have obtained is not trivial, but it can obviously be solved, possibly by numerical methods. We will confine ourselves here to showing that spontaneous mass generation in the Wess-Zumino model[6] and gauge or supersymmetry breaking in the Fayet model[7] are reproduced in the limit. This adds confidence to the possibility of obtaining reliable information on symmetry breaking by this method.

Finally it should be mentioned that a quantum mechanical model of supersymmetry, exhibiting dynamical breaking, has been constructed by Witten[8] and further investigated by Salomonson and Van Holten[9]. As we will see, however, this model cannot be obtained as the limit of a relativistic model, and therefore its mechanism need not be relevant to relativistic supersymmetry.

THE LIMIT FOR THE WESS-ZUMINO MODEL

The Wess-Zumino Lagrangian reads

$$\mathcal{L} = -|\partial_\mu\phi|^2 - ic\bar{\psi}_L \not{\partial}\psi_L + |H|^2 + \frac{1}{\sqrt{2}}(H+H^*)X + \frac{g}{c}(\phi^2 H + \phi^{*2}H^*) - ig(\phi\bar{\psi}_L\psi_R + \phi^*\bar{\psi}_R\psi_L), \tag{1}$$

where

$$\psi_L = \frac{1}{2}(1 + i\gamma_5)\psi \qquad \psi_R = \frac{1}{2}(1 - i\gamma_5)\psi \tag{2}$$

and ψ is a Majorana spinor.

This Lagrangian is invariant under the supertransformations

$$\delta\phi = \sqrt{2}\, i\, \bar{\varepsilon}\, \psi_R \qquad \delta\psi_R = \frac{1}{c}\not{\partial}\phi\frac{1}{\sqrt{2}}(1+i\gamma_5)\varepsilon + \frac{1}{c}H\frac{1}{\sqrt{2}}(1-i\gamma_5)\varepsilon \qquad \delta H = \sqrt{2}\, i\, \bar{\varepsilon}\not{\partial}\psi_R \tag{3}$$

and, for X = 0, under the chiral transformations

$$\delta\phi = -i\alpha\phi \qquad \delta\psi_R = -i\,\frac{\alpha}{2}\,\psi_R \qquad \delta H = 2i\alpha H \tag{4}$$

If we perform the limit $c \to \infty$ leaving the fields finite we break supersymmetry. This is conserved if we rescale ϕ according to

$$\phi = c\varphi \tag{5}$$

This yields

$$\begin{aligned} \mathcal{L} \underset{c\to\infty}{\to} \mathcal{L}^{(o)} = & |\partial_t\varphi|^2 + \psi_L^* i\partial_t\psi_L + |H|^2 + \frac{1}{\sqrt{2}}(H+H^*)X + \\ & + gc(\varphi^2 H + \varphi^{*2}H^*) - igc(\varphi\bar{\psi}_L\psi_R + \varphi^*\bar{\psi}_R\psi_L) \end{aligned} \tag{6}$$

with the constraints

$$\partial_K\varphi = \partial_K\psi_L = 0 \tag{7}$$

These constraints are Galilei-invariant and under these constraints so is $\mathcal{L}^{(o)}$. It is convenient to introduce Pauli spinors by the definition

$$\begin{aligned} \psi_L &= \frac{1}{\sqrt{2}}\begin{bmatrix} \chi \\ -\chi \end{bmatrix} \\ \psi_R &= \frac{1}{\sqrt{2}}\begin{bmatrix} \sigma_2\chi^* \\ \sigma_2\chi^* \end{bmatrix} \end{aligned} \tag{8}$$

In terms of χ $\mathcal{L}^{(o)}$ becomes

$$\begin{aligned} \mathcal{L}^{(o)} = & |\partial_t\varphi|^2 + \chi^* i\partial_t\chi + |H|^2 + \frac{1}{\sqrt{2}}(H+H^*)X + \\ & + gc(\varphi^2 H + \varphi^{*2}H^*) + gc(\varphi\chi\sigma_2\chi + \varphi^*\chi^*\sigma_2\chi^*) \end{aligned} \tag{9}$$

We can now perform the limit on the supertransformations (3). Expressing the Majorana parameter ε by the Pauli spinor η according to

$$\varepsilon = \begin{bmatrix} \eta \\ \sigma_2\eta^* \end{bmatrix} \tag{10}$$

and using Eqs. (5), (7) and (8) we obtain

$$\begin{aligned} c\delta\varphi &= \eta\chi^* - \eta^*\sigma_2\chi^* \\ c\delta\chi &= i\partial_t\varphi(\eta - \sigma_2\eta^*) + H(\eta + \sigma_2\eta^*) \\ c\delta H &= -i\partial_t(\eta^*\sigma_2\chi^* + \eta\chi^*) \end{aligned} \tag{11}$$

It is easy to check that these transformations close, giving time translations on the fields, and that they leave $\mathcal{L}^{(o)}$ invariant.

The limit on the chiral transformations is obvious and gives

$$\delta\varphi = -i\alpha\varphi$$
$$\delta\chi = i\frac{\alpha}{2}\chi \qquad (12)$$
$$\delta H = 2i\alpha H$$

The form of the limiting supertransformations (11) shows that supersymmetry breaking can be discussed in terms of the auxiliary fields or of fermion bilinears as in the relativistic case. As in the relativistic case it can also be shown that for $X \neq 0$ the chiral multiplet acquires a mass. This can be seen in the following way. Since the fields in the $c \to \infty$ limit are space independent they do not carry momentum, so their energy (being at zero momentum) should be interpreted as a mass. Now, while for $X = 0$ the excitation energy starts from zero, for $X \neq 0$ it acquires a gap equal for bosons and fermions.

The above can be more clearly seen by quantizing $\mathcal{L}^{(o)}$. The canonical procedure gives rise to the usual ambiguities due to the possibility of different ordering of non-commuting operators. These ambiguities can be avoided if the Hamiltonian is given as the anticommutator of the spinorial generators, whose relativistic expression is

$$J^o_\alpha = -\ell^{\frac{3}{2}}\{\partial_0\phi\psi_L + \partial_0\phi^*\psi_R - \gamma^0(H\psi_L + H^*\psi_R)\} \qquad (13)$$

where ℓ is an arbitrary parameter with the dimension of a length that we introduce in order that J^o_α has the dimension of (energy density)$^{1/2}$.

For $c \to \infty$

$$J^o_\alpha \to Q_\alpha = -\frac{\ell^{\frac{3}{2}}}{\sqrt{2}}\left[\chi\partial_t\varphi + \sigma_2\chi^*\partial_t\varphi^* - iH\chi - iH^*\sigma_2\chi^*\right], \qquad (14)$$

and eliminating the auxiliary fields and introducing the canonical momenta

$$\pi = \partial_t\varphi^*$$
$$\pi^* = \partial_t\varphi, \qquad (15)$$

$$Q_\alpha = -\frac{\ell^{\frac{3}{2}}}{\sqrt{2}}\left[\pi^*\chi + \pi\sigma_2\chi^* + i\left(\frac{X}{\sqrt{2}} + gc\varphi^{*2}\right)\chi + \right. \left. + i\left(\frac{X}{\sqrt{2}} + gc\varphi^2\right)\sigma_2\chi^*\right] \tag{16}$$

It is now convenient to introduce dimensionless scalar and spinor fields φ_i and ξ

$$\varphi = \sqrt{\frac{Z}{\ell^3\Lambda}}\,\frac{1}{\sqrt{2}}(\varphi_1 - i\varphi_2)$$
$$\chi = \frac{1}{\ell^{\frac{3}{2}}}\,\frac{1}{\sqrt{2}}(\xi + \sigma_2\xi^*) \tag{17}$$

which satisfy canonical commutation relations

$$[\varphi_i, \pi_j] = i\delta_{ij} \;;\; \{\xi^*, \xi\} = 1 \tag{18}$$

In Eqs. (17) Z is a dimensionless parameter while Λ has the dimension of an energy.

The Hamiltonian density is

$$\mathcal{H} = \{Q^*_\alpha, Q_\alpha\} = \frac{1}{\ell^3}\,\frac{\Lambda}{Z}\,\{\frac{1}{2}\pi_1^2 + \frac{1}{2}\pi_2^2 + \frac{1}{2}g_r^2\left[(\varphi_1^2 - \varphi_2^2 + X_0)^2 + \right. \left. + 4\varphi_1^2\varphi_2^2\right] + 2g_r\varphi_1 - 2g_r\varphi_1\xi^*\xi + g_r\varphi_2 i(\xi^*\sigma_2\xi^* - \xi\sigma_2\xi)\} \tag{19}$$

where

$$g_r = g_0 Z^{\frac{3}{2}} \;,\; g_0 = \frac{g}{\sqrt{c}} \tag{20}$$

$$X_0 = \frac{\ell^{\frac{3}{2}}}{\sqrt{\Lambda}}\,\frac{X}{g_r} \tag{21}$$

By performing the shift

$$\varphi_1 = \varphi_0 + \varphi_1^1 \tag{22}$$

with

$$\varphi_0^2 = -X_0 \tag{23}$$

(note that X_0 can always be taken negative) we get

$$\mathcal{H} = \frac{1}{\ell^3}\frac{\Lambda}{Z}\{\frac{1}{2}\pi_1^2 + \frac{1}{2}\pi_2^2 + \frac{1}{2}m\varphi_1^{1^2} + \frac{1}{2}m\varphi_2^2 + m\dot{\xi}^*\dot{\xi} + \frac{1}{2}g_r(\varphi_1^{1^2} + \varphi_2^2)^2 + 2g_r^2\sqrt{-X_0}\,\varphi_1^1(\varphi_1^{1^2} + \varphi_2^2) + 2g_r\varphi_0 + 2g_r\varphi_1^1 - 2g_r\varphi_1^1\dot{\xi}^*\dot{\xi} + 2g_r\varphi_2 i(\xi^*\sigma_2\dot{\xi}^* - \dot{\xi}\sigma_2\dot{\xi})\} \qquad (24)$$

with

$$m = 2g_r\sqrt{-X_0} \qquad (25)$$

We see that for $X_0 \neq 0$ bosons and fermions have an energy spectrum with a gap m. The chiral multiplet has become massive.

We finally comment on the Witten model[8)], whose Hamiltonian is

$$H = \frac{1}{2}\left(p^2 + W^2(x) + \sigma_3\frac{dW(x)}{dx}\right)$$

where σ_3 is the Pauli matrix and W(x) an arbitrary function.

Witten has shown that if W(x) has an even number of zeros dynamical symmetry breaking occurs.

In order to compare the above Hamiltonian with the Hamiltonian (19) let us observe that the two spin states can be put into one-to-one correspondence with states of 0 or two fermions. Witten's Hamiltonian however contains a single boson field rather than two, and requires a particular relation between the quartic bosonic term and the Yukawa coupling which is not realized in the Hamiltonian (19).

THE LIMIT FOR YANG-MILLS THEORIES

It is most convenient to use the formulation of supersymmetric Yang-Mills theories in the Wess-Zumino gauge[10]. In component fields the gauge degrees of freedom of the vector multiplet are eliminated from the start.

We will not report on the relativistic formulae which can be found for instance in the work by de Wit and Freedman[11], but we will give directly their Galilean limit. For the Lagrangian we have

$$\mathcal{L}^{(o)}_{G} = \frac{1}{2}(\mathcal{D}_t A_k)^a (\mathcal{D}_t A_k)^a + \lambda^* i \mathcal{D}_t \lambda + \frac{1}{2} D^a D^a + - \frac{1}{4}(gc)^2 (f^a_{bc} A^b_i A^c_i)^2 - \frac{i}{2} gc f^a_{bc} A^b_k (\lambda^{a*} \sigma_k \sigma_2 \lambda^{c*} + + \lambda^a \sigma_2 \sigma_k \lambda^c) \tag{26}$$

Here A^a_k are the magnetic potentials and λ^a is a Pauli spinor. The covariant derivative is

$$\mathcal{D}^a_{tb} = \delta^a_b \partial_t - gc f^a_{bc} V^c \tag{27}$$

with V^c the electric potentials. All the fields V^a, A^a_k, λ^a and D^a are of course in the adjoint representation of the internal symmetry group. The spatial index k has become the internal index of an additional O(3) invariance.

We must mention, however, one ambiguity in the derivation of Eq. (26). This is related to the possibility of choosing that the product gc should remain finite for $c \to \infty$, or that g should be finite. This second alternative is the correct one in the coupling of Yang-Mills fields with massive matter fields in non-supersymmetric theories where the fields are x dependent, while the first one appears to be "natural" in the present case, although the possibility of keeping an x dependence in this case should also be investigated.

The limit for the Lagrangian for the matter fields gives

$$\mathcal{L}^{(o)}_{M} = (\mathcal{D}_t \varphi)^+ (\mathcal{D}_t \varphi) + \chi^* i \mathcal{D}_t \chi + H^{a*} H^a + + (gc)^2 (L_b A^b_k \varphi)^+ (L_a A^a_k \varphi) + gc A^b_k \chi^* \sigma_k L_b \chi + + gc \varphi^+ L_b \varphi D^b - igc(\lambda^{a*} \varphi^+ L_a \chi + \lambda^a \varphi^+ \sigma_2 L_a \chi - - \chi^* L_a \varphi \sigma_2 \lambda^{a*} - \chi^* L_a \varphi \lambda^a) \tag{28}$$

In the above equation L_a are the generators of the internal symmetry group in the representation of the matter fields φ, χ and H, so that the covariant derivative is

$$\mathcal{D}_t = \partial_t + igc L_a V^a$$

The supertransformations are

$$\begin{aligned}
c\delta V^a &= -i(\eta^*\lambda^a + \eta\lambda^{a^*}) \\
c\delta A_k^a &= -i(\eta^*\sigma_k\sigma_2\lambda^{a^*} + \eta\sigma_2\sigma_k\lambda^a) \\
c\delta\lambda^a &= (\mathcal{D}_t A_k)^a\sigma_k\sigma_2\eta^* + gc\ f^a_{bc}A_i^bA_j^c\sigma_i\sigma_j\eta + iD^a\sigma_2\eta^* \\
c\delta D^a &= -[\eta^*\sigma_2(\mathcal{D}_t\lambda^*)^a + \eta\sigma_2(\mathcal{D}_t\lambda)^a] - \\
&\quad - gc\ f^a_{bc}A_k^b(\eta^*\sigma_k\lambda^c + \lambda^{c^*}\sigma_k\varepsilon)
\end{aligned} \tag{30}$$

$$\begin{aligned}
c\delta\varphi &= -\eta^*\chi - \eta\sigma_2\chi \\
c\delta\chi &= i\mathcal{D}_t\varphi(\eta+\sigma_2\eta^*) + H(\eta-\sigma_2\eta^*) + igc\ L_bA_k^b\sigma_k(\eta+\sigma_2\eta^*) \\
c\delta H &= -i(\eta^*-\eta\sigma_2)\mathcal{D}_t\chi + igc\ L_bA_k^b(\eta^*-\eta\sigma_2)\sigma_k\chi + \\
&\quad igc\ L_b\varphi(\eta^*\lambda^b - \eta\lambda^{b^*} + \eta^*\sigma_2\lambda^{b^*} - \eta\sigma_2\lambda^b)
\end{aligned} \tag{31}$$

The Lagrangians $\mathcal{L}_G^{(o)}$ and $\mathcal{L}_M^{(o)}$ are invariant under the above supertransformations and under the gauge transformations

$$\begin{aligned}
\delta_G(\theta)\lambda^a &= f^a_{bc}\ \lambda^b\theta^c \\
\delta_G(\theta)D^a &= f^a_{bc}\ D^b\theta^c \\
\delta_G(\theta)V^a &= -\frac{1}{gc}(\mathcal{D}_t\theta)^a \\
\delta_G(\theta)A_i^a &= f^a_{bc}\ A_i^b\theta^c
\end{aligned} \tag{32}$$

and analogous for the matter fields.

The commutator of two supersymmetry transformations gives rise to a time translation of parameter $\xi_0 = -(\varepsilon_2^*\varepsilon_1 + \varepsilon_2\varepsilon_1)$ plus a gauge transformation of parameter $\xi_kA_k = -(\varepsilon_2^*\sigma_k\sigma_2\varepsilon_1^* + \varepsilon_2\sigma_2\sigma_k\varepsilon_1)A_k$. We give, as an illustration, the following equations

$$\begin{aligned}
[c\delta_1, c\delta_2]V^a &= -2i\delta_G(\xi_kA_k)V^a \\
[c\delta_1, c\delta_2]\lambda^a &= -2i\xi_0(\mathcal{D}_t\lambda)^a - 2i\frac{1}{gc}\ \delta_G(\xi_kA_k)\lambda^a \\
[c\delta_1, c\delta_2]\varphi &= -2i\xi_0\mathcal{D}_t\varphi - 2i\frac{1}{gc}\ \delta_G(\xi_kA_k)\varphi
\end{aligned} \tag{33}$$

Since the structure of the supertransformations and of the Lagrangian is the same as that of the relativistic models, Fayet's model[7] conserves in the limit its feature concerning the breaking of the gauge symmetry or of supersymmetry. The quantization of the gauge Lagrangian models can be performed by evaluating the anticommutator of two spinorial generators as in the case of the Wess-Zumino model. It happens however a new feature related to new constraints coming from gauge invariance. This requires a detailed discussion which will not be given here.

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