

To be submitted to
Physics Letters B

ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-83/52(P)
1° Agosto 1983

A. Nakamura and F. Palumbo :
VACUUM PROPERTIES OF THE MASSLESS WESS-ZUMINO MODEL
IN THE $c \rightarrow \infty$ LIMIT

VACUUM PROPERTIES OF THE MASSLESS WESS-ZUMINO MODEL IN THE $c \rightarrow \infty$ LIMIT

A. Nakamura and F. Palumbo
INFN - Laboratori Nazionali di Frascati, Frascati, Italy.

ABSTRACT

It is shown that in the $c \rightarrow \infty$ limit of the massless Wess-Zumino model Susy is unbroken but chirality is spontaneously broken. Explicit breaking by boundary conditions is discussed.

1 - The study of symmetry breaking in the tree approximation takes into account only the zero-momentum mode of bosonic fields. A step further in the study of nonperturbative phenomena related to symmetry breaking can be done by means of the $1/c$ expansion⁽¹⁾. To zeroth order massive particles are described by Galilean fields and massless particles by quantum mechanical coordinates which are the zero-momentum components of the relativistic fields. This is consistent with the fact that massless fields, propagating with infinite velocity in the $c \rightarrow \infty$ limit, must be constant over space.

It has been shown⁽²⁾ that the zeroth order reproduces the infrared features of a number of relativistic theories containing both massive and massless fields, including QED.

If all the fields are massless the relativistic theory undergoes a dimensional reduction to zero dimensions becoming ordinary quantum mechanics. The potential parts of the reduced theory and of the relativistic one are the same as far as zero-momentum modes of bosonic fields are concerned, so that they coincide at the tree level. The reduced theory, however, takes into account also zero-momentum modes of fermionic fields and some quantum mechanical effects.

In the case of Susy, moreover, the $1/c$ expansion receives some support from Witten argument⁽³⁾ that Susy breaking, being related to the difference between the number of bosonic and fermionic modes of zero energy, does not depend on the parameters of the theory. Now the velocity of light is

one of the parameters like masses or coupling constants.

We will investigate in this paper both chiral and Susy breaking in the Wess-Zumino model in the $c \rightarrow \infty$ limit. We will also examine Susy breaking by boundary conditions⁽⁴⁾ (b.c.). Susy can in fact be broken spontaneously by imposing different b.c. on fields in different multiplets or explicitly by imposing different b.c. on fields in the same multiplet. In some cases breaking by b.c. gives a positive energy to the vacuum already at the tree level. This is not so for a unique Wess-Zumino multiplet, which makes it necessary to investigate the effect of b.c. at the quantum level.

2 - For periodic b.c. the $c \rightarrow \infty$ limit of the Wess-Zumino Lagrangian density is⁽⁵⁾

$$\begin{aligned} \mathcal{L} = & \left| \partial_t \varphi \right|^2 + \chi^* i \partial_t \chi + \left| h \right|^2 + (h + h^*) x \\ & + g_0 (\varphi^2 h + \varphi^{*2} h^*) - g_0 (\varphi \chi \sigma_2 + \varphi^* \chi^* \sigma_2 \chi^*), \end{aligned} \quad (1)$$

where φ is a complex bosonic coordinate and χ is a Pauli (anticommuting) spinorial coordinate.

This Lagrangian is invariant under the Susy transformations

$$\delta \varphi = -\chi^* \varepsilon \quad ; \quad \delta \chi = -i \partial_t \varphi \sigma_2 \varepsilon^* + h \varepsilon \quad ; \quad \delta h = -i \partial_t \chi^* \sigma_2 \varepsilon^*, \quad (2)$$

which are the $c \rightarrow \infty$ limit of the relativistic ones.

For $x \neq 0$, a mass is generated without Susy breaking. It would be interesting to solve the $c \rightarrow \infty$ problem for any x and investigate how the limit of zero mass is obtained. This would allow us to make contact with previous investigations of the chiral properties of the vacuum in Susy theories⁽⁶⁾. We have been able, however, to solve the problem only for $x=0$. In this case Lagrangian (1) is also invariant under chiral transformations

$$\delta \varphi = -i \alpha \varphi \quad ; \quad \delta \chi = i \frac{\alpha}{2} \chi \quad ; \quad \delta h = 2 i \alpha h. \quad (3)$$

Let us introduce dimensionless variables by rescaling the fields according to

$$\varphi \rightarrow \sqrt{\frac{Z}{L^3 \Lambda}} \varphi \quad ; \quad \chi \rightarrow \frac{1}{L^{3/2}} \chi, \quad (4)$$

where Z is an arbitrary dimensionless parameter, L is an arbitrary length and Λ an arbitrary energy.

Introducing the canonical momenta

$$\pi = \partial_t \varphi^* \quad ; \quad \pi^* = \partial_t \varphi \quad (5)$$

and eliminating the auxiliary fields we get the Hamiltonian density

$$H = \frac{\Lambda}{L^3 Z} \left[\pi^* \pi + g^2 (\varphi^* \varphi)^2 + g \varphi \chi^* \sigma_2 \chi^* + g \varphi^* \chi \sigma_2 \chi \right], \quad (6)$$

where $g = \frac{1}{\sqrt{c}} Z^{3/2} g_0$.

The nonvanishing commutators are

$$[\varphi, \pi] = i, \quad \{\chi^*_{\alpha}, \chi_{\beta}\} = \delta_{\alpha\beta}. \quad (7)$$

Hamiltonian (6) is related to the generator of Susy transformations

$$Q = -\frac{\sqrt{\Lambda}}{\sqrt{Z} L^{3/2}} \frac{1}{\sqrt{2}} \left\{ \left[\pi^* + i g \varphi^{*2} \right] \chi_+ + \left[\pi + i g \varphi^2 \right] \sigma_2 \chi^* \right\} \quad (8)$$

by

$$\{Q_{\alpha}^*, Q_{\beta}\} = \delta_{\alpha\beta} H. \quad (9)$$

Finally the generator of chiral transformations is

$$q = +i (\pi^* \varphi^* - \pi \varphi) + \frac{1}{2} \chi^* \chi. \quad (10)$$

The zero spin states can be written

$$|\Psi\rangle = \left[\Psi_0(\varphi) + \frac{1}{2} \chi^* \sigma_2 \chi^* \Psi_2(\varphi) \right] |0\rangle, \quad (11)$$

where $|0\rangle$ is the fermion vacuum

$$\chi |0\rangle = 0. \quad (12)$$

If Susy is unbroken, a normalizable state must exist satisfying the equations

$$Q |\Psi\rangle = Q^* |\Psi\rangle = 0, \quad (13)$$

which are equivalent to

$$\pi \Psi_0 + i g \varphi^{*2} \Psi_2 = 0 \quad (14)$$

$$\pi^* \Psi_2 + i g \varphi^2 \Psi_0 = 0$$

Introducing polar coordinates

$$\varphi = \frac{1}{\sqrt{2}} \rho e^{i\theta}, \quad (15)$$

and expanding the Ψ_k according to

$$\Psi_k = \sum_n e^{in\theta} f_{k,n}(\rho), \quad (16)$$

Eqs. (14) become

$$\left(-\frac{\partial}{\partial \varrho} + \frac{n}{\varrho}\right) f_{0,n} + \frac{1}{\sqrt{2}} g \varrho^2 f_{2,n-1} = 0 \quad (17)$$

$$\left(-\frac{\partial}{\partial \varrho} - \frac{n-1}{\varrho}\right) f_{2,n-1} + \frac{1}{\sqrt{2}} g \varrho^2 f_{0,n} = 0$$

For small ϱ

$$f_{0,n} \sim \varrho^n \quad ; \quad f_{2,n} \sim \varrho^{-n} \quad (18)$$

so that for the state (11) to be normalizable n must be 0 or 1

$$|\psi\rangle = \begin{cases} \left[f_{0,0} + \frac{1}{2} \chi^* \sigma_2 \chi e^{-i\theta} f_{2,-1} \right] |0\rangle, n=0 \\ \left[f_{2,-1} e^{-i\theta} + \frac{1}{2} \chi^* \sigma_2 \chi f_{0,0} \right] |0\rangle, n=1 \end{cases} \quad (19)$$

where

$$f_{0,0} = \varrho^2 K_{2/3} \left(\frac{1}{3\sqrt{2}} |g| \varrho^3 \right) \quad ; \quad f_{2,-1} = \frac{\sqrt{2}}{|g|} \frac{1}{\varrho^2} \frac{\partial}{\partial \varrho} f_{0,0} \quad (20)$$

$K_{2/3}$ is a modified Bessel function and $f_{0,0}$ and $f_{2,-1}$ have the same asymptotic behaviour for large ϱ

$$f_{0,0} \sim f_{2,-1} \sim \sqrt{\varrho} e^{-\frac{1}{3\sqrt{2}} |g| \varrho^3} \quad (21)$$

We see that Susy is not broken, in agreement with Witten index argument⁽³⁾. Chirality is instead broken, because the two degenerate vacua (19) have chirality 0, 1, respectively.

We have evaluated numerically the lowest eigenvalues of Hamiltonian (6) which are reported in the figure.

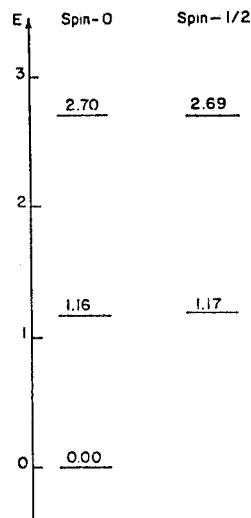


FIG. 1 - Energy levels of the $c \rightarrow \infty$ limit of the Wess Zumino Hamiltonian in units $\Lambda/L^3 Z$ for $g = \sqrt{2}$.

3- Let us now consider the effect of different b.c. on the fields. We will require some of the relativistic fields to satisfy periodic b.c. and others to vanish on the surface of the quantization box. These latter will therefore vanish in the $c \rightarrow \infty$ limit. If the complex scalar field vanishes, Hamiltonian (6) also vanishes. If the spinor field vanishes, the lowest eigenvalue of Hamiltonian (6) is greater than zero, so that breaking by b.c. gives in this case a positive energy to the vacuum.

Suppose now that only the imaginary part of the scalar field vanishes, so that Hamiltonian (6) becomes

$$H = \frac{A}{ZL^3} \frac{1}{2} \left[\pi_1^2 + \frac{1}{2} g^2 \varphi_1^4 + \sqrt{2} g \varphi_1 (\chi^* \sigma_2 \chi + \chi \sigma_2 \chi) \right]. \quad (22)$$

Let us consider states of the form

$$\Psi_{\pm} = (1 \pm \chi^* \sigma_2 \chi) \Phi(\varphi_1) | 0 \rangle. \quad (23)$$

For such states

$$(\chi^* \sigma_2 \chi + \chi \sigma_2 \chi) \Psi_{\pm} = \pm \Psi_{\pm} \quad (24)$$

so that we can replace Eqs. (22) and (23) by

$$H = \frac{A}{ZL^3} \frac{1}{2} \left[\pi_1^2 + \frac{1}{2} g^2 \varphi_1^4 + \sqrt{2} g \varphi_1 \sigma_3 \right] \quad (22')$$

$$\Psi_{\pm} = \Phi(\varphi_1) \lambda_{\pm}, \quad (23')$$

with

$$\sigma_3 \lambda_{\pm} = \pm \lambda_{\pm}.$$

Hamiltonian (22') coincides with that considered by Witten⁽⁷⁾

$$H = \left[\pi_1^2 + W^2(\varphi_1) + \frac{\partial W}{\partial \varphi_1} \sigma_3 \right], \quad (25)$$

with

$$W(\varphi_1) = \frac{1}{\sqrt{2}} g \varphi_1^2. \quad (26)$$

Such Hamiltonian has no vanishing eigenvalue, so that also in this case breaking by b.c. gives a positive energy to the vacuum.

REFERENCES

- (1) F. Palumbo, Lett. Nuovo Cimento, 30, 72 (1981).
- (2) F. Palumbo, Nuclear Phys. 182B, 261 (1981); 197B, 334 (1982).
- (3) E. Witten, Trieste Lectures (Summer 1981); Erice Lectures (1981); S. Cecotti and L. Girardello, Phys. Letters 110B, 39 (1982).
- (4) F. Palumbo, Phys. Letters B, in press.
- (5) S. Ferrara and F. Palumbo, in "Unification of the Fundamental Particle Interactions II, Proc. of the Europhysics Study Conf. October 6-14 (1981), Ed. by J. Ellis and S. Ferrara (Plenum Press 1983) Vol. 15, p. 515.
- (6) See G. Veneziano, Phys. Letters 124B 357 (1983) and references therein.
- (7) E. Witten, Nuclear Physics B188, 513 (1981).