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F. Palumbo: SPIN-ISOSPIN EXCITATIONS IN LIGHT DEFORMED NUCLEI

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SPIN-ISOSPIN EXCITATIONS IN LIGHT DEFORMED NUCLEI

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I will present a collective model of deformed nuclei inspired by σ-τ phases in nuclear matter (Calogero, 1972, 1973, 1975; Migdal 1972, 1978). These phases are characterized by a laminated structure due to one dimensional crystallization (Takatsuka, 1977; Tamagaki, 1979) along the direction of spin quantization (z-axis), which is normal to the planes alternatively occupied by $\sigma_3\tau_3 = 1$ and $\sigma_3\tau_3 = -1$ nucleons. Such a structure has been invented in order to get a nonvanishing contribution from the σ-τ dependent part of the N-N potential

$$\langle V_{\sigma\tau}\rangle\Big|_{\text{direct}} = \frac{1}{2}\sum_{\tau_3(1)\sigma_3(1)}\ \sum_{\tau_3(2)\sigma_3(2)} \int d\vec{r}_1 \int d\vec{r}_2\ \varrho_{\tau_3(1)\sigma_3(1)}(\vec{r}_1)\ \cdot$$
$$\cdot\ \varrho_{\tau_3(2)\sigma_3(2)}(\vec{r}_2)\ \langle\tau_3(1)\sigma_3(1)\tau_3(2)\sigma_3(2)|V_{\sigma\tau}|\tau_3(1)\sigma_3(1)\tau_3(2)\sigma_3(2)\rangle\ . \tag{1}$$

In the above formula $\varrho_{\tau_3\sigma_3}$ is the σ-τ density, and the σ-τ matrix element is

$$\langle\tau_3(1)\sigma_3(1)\tau_3(2)\sigma_3(2)\,|V_{\sigma\tau}|\,\tau_3(1)\sigma_3(1)\tau_3(2)\sigma_3(2)\rangle =$$
$$= \left[V_C + V_T(2z^2 - \vec{r}_T^{\,2})\right]\tau_3(1)\sigma_3(1)\tau_3(2)\sigma_3(2)\ , \tag{2}$$

where $\vec{r}_T$ is the component of the nucleon-nucleon distance in the x-y plane, and V_C, V_T are the central, tensor components of $V_{\sigma\tau}$.
Introducing the σ-τ density operator

$$S_{ik} = \psi^{*}\tau_i\sigma_k\psi\ , \tag{3}$$

eq. (1) can be rewritten in terms of

$$< S_{33}(\vec{r}) > = \sum_{\tau_3 \sigma_3} \varrho_{\tau_3 \sigma_3}(\vec{r}) \, \tau_3 \sigma_3 \, , \qquad (4)$$

$$< V_{\sigma\tau} >_{direct} = \frac{1}{2} \int d\vec{r}_1 \int d\vec{r}_2 < S_{33}(\vec{r}_1) > < S_{33}(\vec{r}_2) > \left[V_C + V_T (2z^2 - \vec{r}_T^{\,2}) \right] . \qquad (5)$$

For $< V_{\sigma\tau} >_{direct}$ not to vanish we need $< S_{33} > \neq 0$, i.e. σ-τ correlations. But if we want in addition to get an attractive contribution from the tensor potential we must require that

$$< 2z^2 - \vec{r}_T^{\,2} > < 0 \, , \qquad (6)$$

a condition which is realized by the laminated structure as illustrated in Fig. 1.

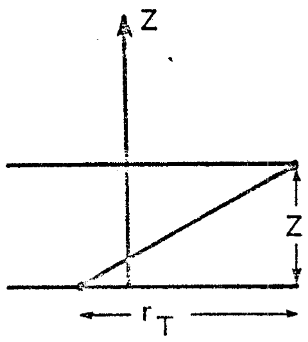

Fig. 1 - The laminated structure in nuclear matter makes $< 2z^2 - \vec{r}_T^{\,2} > < 0$.

Condition (6) can also be fulfilled in an oblate nucleus (Lo Iudice, 1981) by displacing $\sigma_3 \tau_3 = 1$ nucleons w.r. to the $\sigma_3 \tau_3 = -1$ ones, as shown in Fig. 2.

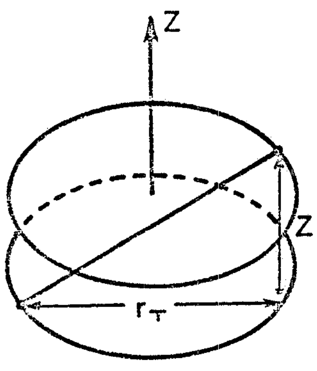

Fig. 2 - The oblate deformation makes $< 2z^2 - \vec{r}_T^{\,2} > < 0$.

Let us consider the potential energy W arising from such a displacement. It will consist of a part W_o which does not depend on spin-isospin, and a spin-isospin dependent one, $W_{\sigma\tau}$. Different possibilities for W are schematically shown in Fig. 3 compared to W_o .

Let us concentrate on small displacements, i.e. zero point σ-τ oscillations. If $W > W_o$ the σ-τ correlation energy is repulsive and, if repulsive enough, it will

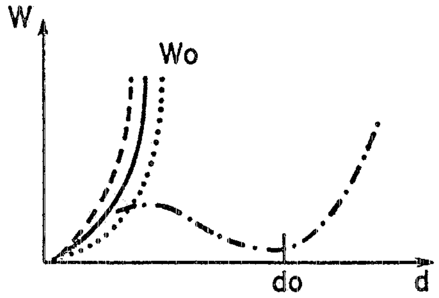

Fig. 3 - The potential displacement energy : the solid line refers to disordered displacement W_o, the dashed line to repulsive $\sigma-\tau$ correlations, the dotted line to attractive $\sigma-\tau$ correlations, the dot-dashed line to the metastable state.

give rise to a hardened collective state. This is related to the zero sound predicted by Ikeda et al. (1963, 1964), and recently confirmed experimentally in ^{208}Pb (Moren, 1980). Little is known experimentally on deformed nuclei however, and the theoretical prediction does not take into account the effect of the tensor force in the presence of nuclear deformation. A preliminary study of this effect has been done by a collective model (Lo Iudice, 1981). This model has been developed for nuclei with axial symmetry and N = Z.

The total nuclear w.f. is assumed to be

$$\Psi_{\nu_3 \nu_T K} = \Phi_{\nu_3 \nu_T K}(\vec{d})\, \Lambda(\vec{d}) \,, \tag{7}$$

where $\Lambda(\vec{d})$ is a Slater determinant of displaced s.p.w.f.

$$\lambda_{n_z n_T m \sigma_3 \tau_3} = \varphi_{n_z n_T m}(\vec{r} - \frac{1}{2}\vec{d}\sigma_3\tau_3)\,\chi_{\sigma_3\tau_3} \,, \tag{8}$$

$\varphi_{n_z n_T m}$ being h.o.w.f. in a cylindrical basis and $\chi_{\sigma_3\tau_3}$ spin-isospin w.f.

The z-axis is taken parallel to the nuclear symmetry axis. If axis-3 of spin space is taken parallel to the symmetry axis, I will talk of longitudinal polarization, if it is taken orthogonal I will talk of transverse polarization. In analogous way I will talk of longitudinal and transverse oscillations.

The collective Hamiltonian is

$$H = \frac{p^2}{2M} + W(\vec{d}) \,, \tag{9}$$

where $M = \frac{1}{4}$ Am. For small oscillations we can take a harmonic approximation to W,

$$W(\vec{d}) \simeq \frac{1}{2}(C_z + K_z)\, d_z^2 + (C_T + K_T)\, \vec{d}_T^2 \,, \tag{10}$$

C_z and C_T being the $\sigma-\tau$ independent restoring constants and K_z and K_T the dependent ones. These latter are determined by the equation

$$\langle \Lambda(d) | V_{\sigma\tau} | \Lambda(d) \rangle \simeq \frac{1}{2} K_z d_z^2 + \frac{1}{2} K_T \vec{d}_T^2 \,. \tag{11}$$

$V_{\sigma\tau}$ is usually approximated by the regularized OPEP plus a contact term which is assumed to account for many effects as discussed in detail in this School

$$V_{\sigma\tau} = -\left(\frac{f}{m_\pi}\right)^2 \varrho(q^2)\vec{\tau}_1 \cdot \vec{\tau}_2 \left[\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}/(q^2+m_\pi^2) - g' \vec{\sigma}_1 \cdot \vec{\sigma}_2\right], \tag{12}$$

where

$$\frac{f^2}{4\pi} = 0.08 \; ; \qquad \varrho(q^2) = \left(\frac{\Lambda^2}{\Lambda^2 + q^2}\right)^2; \qquad \Lambda = 1000 \text{ MeV/c} . \tag{13}$$

This standard form is assumed also in the present model although it is expected to be much less adequate in the context of anisotropic phenomena like longitudinal and transverse modes.

Finally, following the prescription of the unified theory of nuclear vibrations (Bohr, 1975), the σ-τ independent restoring constants are determined by the equations

$$C_z = \frac{A}{4} m \bar{\omega}_z^2 \; , \qquad C_T = \frac{A}{4} m \bar{\omega}_T^2 \; , \tag{14}$$

$\bar{\omega}_z$ and $\bar{\omega}_T$ being s.p.h.o. frequencies.

The numerical evaluation (Lo Iudice, 1981, 1983a) of K_z and K_T confirms, according to the analogy between Figs. 1 and 2, that longitudinal polarization is favored in oblate nuclei for both longitudinal and transverse oscillations.

Specific calculations have been done for ^{12}C and ^{28}Si, assuming $\bar{\omega}_z/\bar{\omega}_T = 1.5$, which corresponds to a deformation parameter $\delta \sim -0.4$.

The constants K_z and K_T have been evaluated for $g' = 0.33$, 0.5 and 0.7. For all these values of g' K_T turns out to be positive, while K_z is negative for $g' = 0.33$, approximately vanishing for $g' = 0.5$ and positive for $g' = 0.7$. The transverse excitation is always collective, while the longitudinal one is not collective for $g' = 0.5$.

The first excited states are characterized by a unique quantum number ($K = 0$, for $n_T = 0$, $n_z = 1$; $K = \pm 1$, for $n_T = 1$, $n_z = 0$) and by the M2 transition strength

$$B(M2, 0 \to K) = \frac{5}{32\pi} \frac{\hbar^2}{m} A(g_p - g_n)^2 \left(\frac{1}{\hbar\omega_z} \delta_{K0} + \frac{3}{2\sqrt{2}} \frac{1}{\hbar\omega_T} \delta_{K,\pm 1}\right)\left(\frac{e\hbar}{2mc}\right)^2 \text{fm}^2 . \tag{15}$$

Longitudinal excitations have magnetic quantum number $K = 0$, while the transverse ones have $|K| = 1$. Since this quantum number is difficult to measure a distinctive characterization of σ-τ excitations requires a comparison of other properties of longitudinal and transverse modes like energy and strength. This is in the same spirit of the comparison between longitudinal and transverse (with respect to momentum transfer) nuclear response (Alberico, 1982) discussed by M. Ericson in this School.

The predicted position and strength of the collective levels are reported in Figs. 4 and 5 with the experimental data (Friebel, 1982 ; Grecksch,1981). They are in closest agreement for $g' \sim 0.5$. The strength in ^{28}Si, however, is fragmented, while in ^{12}C is much larger than predicted.

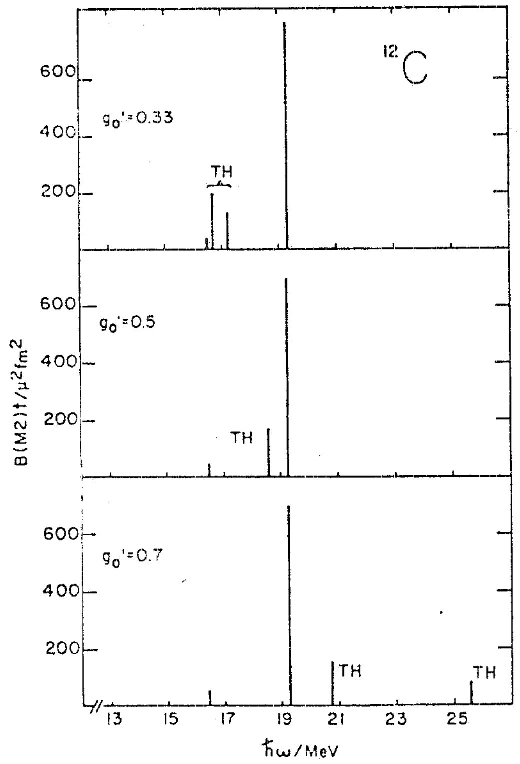

Fig. 4 - Experimental and theoretical (indicated by TH) values of excitation energies and B(M2)↑ strengths for different values of g'_o for ^{12}C.

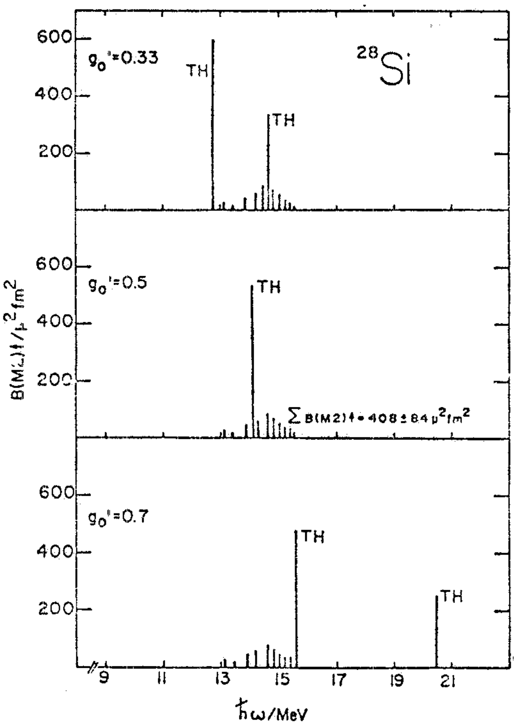

Fig. 5 - The same as Fig. 4 for ^{28}Si.

We see that the OPEP is not strong enough to give attractive correlations in the zero-point motion, but, due to the tensor force, it cancels the repulsion for longitudinal modes. The zero sound propagates only in the transverse directions.

The approximate vanishing of K_z is due to cancellation of a large positive term coming from the short range repulsion and a large negative term coming from the OPEP. It is then conceivable that the nuclear response at high momentum transfer and high energy be enhanced and softened, a precursor to pion condensation (Fayans, 1977 ; Gyulassi, 1977 ; Ericson, 1978). I will come back to this point later.

In nuclei with axial symmetry the longitudinal polarization is the natural one, but it is favored by the OPEP only in oblate nuclei, for prolate nuclei a transverse polarization being preferred (Lo Iudice, 1981). As we have seen, however, the OPEP is not so strong, so that it could be insufficient to establish a transverse polarization which would break, even if very little, axial symmetry. In the absence of a microscopic calculation, which could solve this ambiguity, both longitudinal and transverse polarizations have been considered in prolate nuclei. In spite of this ambiguity, I think that the collective model retains enough predictivity to be worth of experimental tests, but these can be conclusive only if carried out on a number of deformed nuclei.

Calculations for prolate nuclei have not yet been completed. I report preliminary results (Lo Iudice, 1983b) for ^{20}Ne, for which an experiment is in progress (Richter, private communication). In this case with transverse polarization both longitudinal and transverse oscillations are collective for $g' = 0.5$, but less than transverse oscillations in oblate nuclei. For $\delta \sim 0.5$, $\hbar\omega_z = 13.4$ MeV with $B(M2)\uparrow = 205\ \mu^2 fm^2$ and $\hbar\omega_T = 21.7$ MeV with $B(M2)\uparrow = 132\ \mu^2 fm^2$. For longitudinal polarization with the same values of g' and δ, $\hbar\omega_z = 11.7$ MeV with $B(M2)\uparrow = 312\ \mu^2 fm^2$ and $\hbar\omega_T = 21.3$ MeV with $B(M2)\uparrow = 258\ \mu^2 fm^2$. These results do not differ qualitatively from those relative to transverse polarization, there being essentially only an enhancement of collectivity by a factor 1.5. In both cases there should be two more or less collective levels as compared to one in oblate nuclei.

In conclusion the model predicts a reduction of the collectivity of the zero sound (which comes from repulsive σ-τ correlations). Such a reduction is greater in oblate nuclei where it leads to the suppression of the longitudinal zero sound and, possibly, to precursor effects at high momentum transfer and energy.

So far we have discussed small σ-τ oscillations. For large σ-τ displacements W could remain close to W_o, but could also definitely depart from it as shown in the dot-dashed line of Fig. 3, corresponding to a metastable state, in which $\sigma_3\tau_3 = 1$ nucleons would oscillate w.r. to the $\sigma_3\tau_3 = -1$ ones around an average separation d_o. Our study of small oscillations around $d = 0$ also provides information about the nuclear stiffness against metastable separation.

In order to understand the nature of such a hypotetical metastable state, let us consider the equation for the pion field (Palumbo, 1982)

$$(\Box + m_\pi^2)\,\Phi_3 = \frac{f}{m_\pi}\,\partial_k S_{k3}\,. \tag{16}$$

If the oscillation takes place along the third axis with frequency ω

$$\begin{aligned} <S_{k3}(\vec{r}, t)> &= <S_{33}(\vec{r}, 0)>\delta_{k3}\, e^{i\omega t} \\ \Phi_3(\vec{r}, t) &= \Phi_3(\vec{r}, 0)\, e^{i\omega t}\ , \end{aligned} \tag{17}$$

and $\Phi_3(\vec{r}, 0)$ will satisfy the time-independent equation

$$\left[\Delta - (m_\pi^2 - \omega^2)\right]\Phi_3(\vec{r}, 0) = \frac{f}{m_\pi}\,\partial_3 <S_{33}(\vec{r}, 0)>\ , \tag{18}$$

showing that the pion has an effective Compton wave-length $(m_\pi^2 - \omega^2)^{-1/2}$. This should result in a long range interaction which is tempting to relate to the cross section exceeding the geometrical value of anomalous (Friedlander, 1980).

The above is only a sketchy argument. For instance, it does not take into account parity. There are however σ-τ configurations of nuclei with definite parity (Pace, 1978), and to enforce parity one should consider σ-τ oscillations around those equilibrium configurations.

It should be remarked that there has been a number of papers concerning possible σ-τ metastable states with static σ-τ order (Do Dang, 1979; Tripathi, 1980; Giberti, 1982, 1983). They can give rise to coherent effects but can hardly explain a cross-section exceeding the geometrical bound.

Finally I would like to comment on the possibility of precursor effects. These have been originally proposed in the context of pion condensation in nuclear matter. Pion condensation has been proposed as a second order phase transition, and second order phase transitions are announced by precursor phenomena. The σ-τ phases have been instead proposed as a first order phase transition, which is not accompanied by precursor effects. How can the collective model, inspired by σ-τ phases, describe possible precritical effects? The answer is that a phase transition is a well defined notion only for infinite systems. In a finite system there are collective states which can exist only insofar as the system is finite, like surface vibrations. The collective states described by the collective model I have presented are indeed surface effects, as confirmed by the fact that they have been predicted only for light nuclei.

The above remarks about first and second order phase transitions allow me to make a comment on the results presented in this School by Dickoff and F essler showing that pion condensation cannot occur.

Study of a first order phase transition requires the comparison of the energy of the ordered phase to that of the disordered one. The calculation of the energy in the ordered phase is much more difficult, due to the necessary use of complicated s.p.w.f., anisotropic and σ-τ dependent correlation functions and ellipsoidal Fermi surface. Such calculations are presently only at the beginning (Sandler, 1981; Tamiya, 1981a, 1981b; Benhar, 1981, 1983; Pace, 1982).

Study of second order phase transitions, viceversa, can be accomplished by determining the instability of the disordered phase. This is the kind of calculation presented by Dickoff and Faessler. Such a calculation cannot tell anything concerning a first order phase transition, which is not related to a singularity of the response of the system.

Studying by this method the condensation of vapor in the Van der Vaals phase diagram, one would follow the vapor along the soprassaturated phase without realizing the crossing with the liquid phase, a first order phase transition (Palumbo, 1981).

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