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ABSTRACT

We present an improved algorithm to calculate the transverse momentum $(Q_\perp)$ spectrum of $W^\pm$ and Z^0 bosons produced at the Collider using the energy-momentum distribution of soft gluons. We find $\langle Q_\perp^2 \rangle_W = 190$ GeV^2 and $\langle Q_\perp^2 \rangle_{Z^0} = 219$ GeV^2. Also, the calculated transverse distribution gives $\langle Q_\perp \rangle_W \simeq 7$ GeV with cuts. Our results are compared with other calculations and with most recent data from the Collider.

Recent observations of isolated large transverse momentum electrons with associated missing energy at $\sqrt{s}=540$ GeV have signalled the production of weak vector bosons in $\bar{p}p$ collisions[1,2]. It is therefore of interest to obtain quantitative estimates for the transverse spectrum for this process using what is already known about dilepton production at lower energies. In this paper we shall investigate soft QCD bremsstrahlung from the partons producing the weak boson.

In the standard picture with collinear partons, the $Q_\perp$-distribution is that of the QCD radiation emitted by the partons before the interaction which produces the electroweak bosons. We concentrate

on soft gluons because on the theoretical level there exist powerful techniques which allow a complete summation at the leading log level to all orders in α_s[3,4,5] and on the phenomenological level it has been shown that soft effects indeed reproduce almost the entire (measured) transverse distribution[6,7].

Regarding the non-leading terms the theoretical situation is still quite unclear. Our purpose here is to clarify the situation as far as the leading terms are concerned in a parameter free way so that the contribution of the non-leading terms (still under study) may be estimated.

The soft-gluon formula contains two energy scales, one given by Λ which governs the QCD coupling constant and the other is E which is the maximum energy allowed to the gluons. Moreover, in the literature[5,6] one finds another energy scale introduced through the "intrinsic" transverse momentum, which further reduces predictability. In what follows we outline a method which has no intrinsic transverse momentum and in which E is either eliminated or determined without ambiguity. This allows one to study variations introduced through the leftover parameter Λ. Using this approach, an analysis of the $Q_\perp$ spectrum has been performed at lower energies where our results are in agreement with μ-pair production[7], thus confirming the dominance of soft gluons and the formalism used to obtain it. It would be very surprising – and contrary to our experience from QED – to find these effects to become less important as the energy increases.

The framework of our analysis is that of soft QCD radiation treated to all orders and averaged over the hadronic matter coordinates. The cross-section for the process $\bar{p}p \rightarrow \gamma^*, W^{\pm}, Z^0 \rightarrow l_1 + l_2 + X$ can be written as

$$\frac{d\sigma^{had}}{d^4Q} = \int dy_1 \int dy_2 \, \mathcal{G}(y_1,y_2) \, (\frac{d\sigma}{d^4Q})^{QCD}(y_1,y_2,Q) \tag{1}$$

$$= \int dy_1 \int dy_2 \, \mathcal{G}(y_1,y_2) \, \frac{d^4P(K,E)}{d^4K} \, d^4K \, (\frac{d\sigma_E}{d^4q}),$$

where a given hadronic state contributes with probability $\mathcal{G}(y_1,y_2)$ a pair of partons of fractional energies y_1 and y_2 and the parton-parton cross-section has been factored into a product of the soft-gluon 4-momentum distribution $d^4P(K,E)$, and an exclusive cross-section $d\sigma_E$. d^4P is given by[3]

$$\frac{d^4P(K,E)}{d^4K} = \int \frac{d^4x}{(2\pi)^4} \, e^{iK\cdot x - \int_0^E d^3\bar{n}(k)(1-e^{-ik\cdot x})} \tag{2}$$

and $d^3\bar{n}(k)$ denotes the single-gluon spectrum. The remaining exclusive cross-section $d\sigma_E$ is devoid of soft gluons. (Hard gluons may perturbatively be added to it).

The cross-section given by Eq. (1) is more commonly written as

$$\frac{d\sigma}{dx_F \, dQ^2 \, dQ_\perp^2} = (\frac{d\sigma}{dx_F \, dQ_\perp^2}) \cdot (\frac{dP}{dQ_\perp^2}) \tag{3}$$

Such an expression is obtained in the soft limit[8] if $d^4P(K)$ can be factorized as

$$\frac{d^4P(K)}{d^4K} \simeq \mathbb{P}(K_+, K_-) \left(\frac{d^2P(K_\perp)}{d^2K_\perp}\right) , \tag{4}$$

with the light cone variables $K_\pm = \sqrt{s}\,(y_{1,2}-x_{1,2})$. Then, as shown in detail elsewhere[8], $\mathbb{P}(K_+, K_-)$ is absorbed into the running parton densities and $(\frac{dP}{dK_\perp^2})$ is given by

$$\frac{dP}{dK_\perp^2} = \frac{1}{2} \int_0^\infty b\,db\,J_0(K\cdot b) \cdot \exp\left\{-\frac{8}{3\pi}\int_0^E \frac{dk}{k}\,(-\frac{1}{k})\,\alpha_s(k^2/\Lambda^2)\ln(\frac{E+\sqrt{E^2-k^2}}{E-\sqrt{E^2-k^2}})\right\}(1-J_0(bk)) \tag{5}$$

The above factorization which is approximate - annuls the correlation between longitudinal and transverse parts and its use is phenomenologically justified only if one can accurately fix the upper limit E in Eq. (5). Notice that E is crucial in determining the mean value of the distribution and it represents the maximum energy which can be radiated by either of the two colliding partons, since Eq. (5) is obtained through exponentiation of the single soft gluon emission. Thus, $E \lesssim Q/2$ or $M_w/2$. In Ref. (9,10), E has been chosen to be Q.

To eliminate this ambiguity of scale we turn to an independent determination of E based on the following method. We calculate $< Q_\perp^2 >_w$ using a transverse momentum sum rule which does not utilize factorization into longitudinal and transverse variables, and is derived from the complete $d^4P(K)$ and its analyticity in the energy plane. This sum rule reads[8]

$$< Q_\perp^2 (x_1,x_2;s) > = (\frac{4s}{3\pi}) \frac{\int_{x_1}^1 dy_1 \int_{x_2}^1 dy_2\,\alpha_s\left\{\frac{s}{\Lambda^2}(y_1-x_1)(y_2-x_2)\right\}\mathcal{F}^{(Q^2)}(y_1,y_2)}{\mathcal{F}^{(Q^2)}(x_1,x_2)} \tag{6}$$

where the running parton densities $\mathcal{F}^{(Q^2)}(x_1,x_2)$ are given as

$$\frac{d\sigma^{had}}{dx_1\,dx_2} = \sigma_{born}(Q^2) \cdot \mathcal{F}^{(Q^2)}(x_1,x_2) , \tag{7}$$

with $x_1 x_2 = Q^2/s$ and $x_1-x_2 = 2Q_{//}/\sqrt{s}$. As shown in Ref. (7), Eq. (6) reproduces well the low energy πN and $pN \to \mu^+\mu^- + X$ data, if one uses for α_s the singular (but integrable!) expression

$$\alpha_s(k^2/\Lambda^2) = \frac{12\pi}{25}\,\frac{p}{\ln(1+p(\frac{k^2}{\Lambda^2})^p)} \tag{8}$$

with p=5/6 and Λ =100 MeV. Thus, Eq. (6) can be used at higher Q^2 in a completely parameter free manner.

Averaging over x and using the parametrization given by Glück, Hoffmann and Reya[11], we obtain the values

$$< Q_\perp^2 >_W = 190 \text{ GeV}^2 \qquad ; \quad < Q_\perp^2 >_{z^0} = 219 \text{ GeV}^2. \qquad (9)$$

having used $M_W = 81$ GeV and $M_{z^0} = 90$ GeV. (The results do not have a strong dependence upon the exact value for the masses.).

Now we calculate the mean squared value from the simple eikonal formula Eq. (5) and find that if $E = 38$ GeV (for W-production) then

$$< Q_\perp^2 >_{\text{sum rule}} = < Q_\perp^2 >_{\text{eikonal}} \qquad (10)$$

This confirms our earlier remark that in the eikonal approximation $E \simeq M_W/2$. The slight departure of E from $M_W/2$ reflectes the averaging over the rapidity y. In Fig. 1, we show the variation of $< Q_\perp^2(y) >$ as obtained through the sum rule, Eq. (6). A similar calculation for z^0 gives $E = 41.2$ GeV.
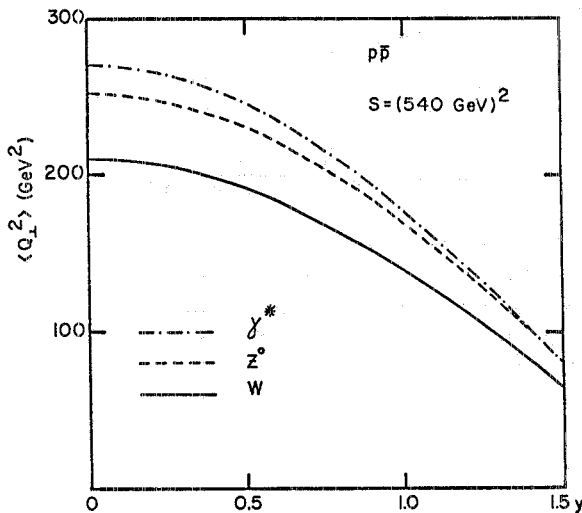

FIG. 1 - $< Q_\perp^2 >$ as a function of rapidity, from Eq. (6). The parton densities are from Ref. (12).

Having fixed the scale E, we may use Eq. (5) to obtain the $Q_\perp$-spectrum for W and z^0 production in $p\bar{p}$ collisions. For their mean values $< Q_\perp >$ we find

$$< Q_\perp >_W = 10.2 \text{ GeV}; \qquad < Q_\perp >_{z^0} = 10.9 \text{ GeV};$$

For comparison with present experiments, we show the effect of imposing a cut at $Q_\perp = 20$ GeV. This reduces $< Q_\perp >$ to become

$$< Q_\perp >_W = 7.4 \text{ GeV} \qquad \text{with cuts.}$$

In Fig. 2 we show the most recent UA1 data[12] on $Q_\perp$-distribution for W-boson and our prediction (curve c) using Eq. (5). For comparison with other estimates we also exhibit (i) a regularized $O(\alpha_s)$ calculation[13] (curve a), (ii) a phenomenological fit (curve b) from Ref. (14) and (iii) curve (d) from Ref. (9). Curves (a) and (b) depend upon a parameter, while ours (curve c) and the result from Ref. (9) (curve d) do not.

Our analysis applies equally well to the transverse momentum of the recoiling jets if we assume
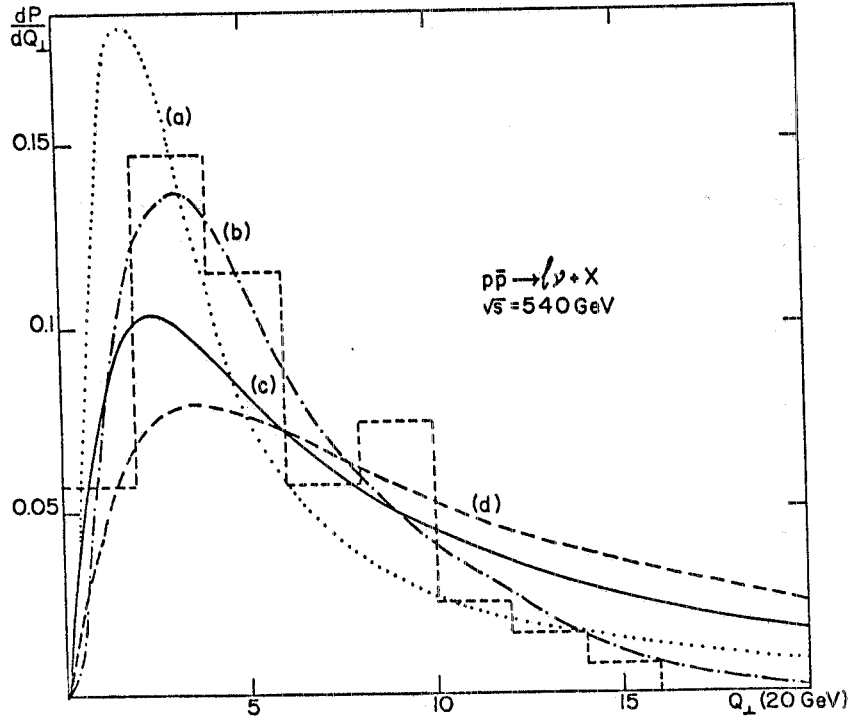
FIG. 2 - Normalized $Q_\perp$-distribution for the W-boson with a $Q_\perp$-cut at 20 GeV. The histogram is obtained using 41 (e) and 20 ($\mu\nu$) events obtained by the UA1 Collaboration[12]. Theoretical curves marked (a), (b) and (d) are from Ref. (13), (14) and (9), and (c) is from our Eq. (5). All curves are normalized to 1 for $0 \leqslant Q \leqslant 20$ GeV.

that around W and Z^o thresholds the dominant jet mechanism is given by the decay of these resonances into quarks which shower into hadrons. Thus, we would predict, for example,

$$< Q_\perp >_{jets} \simeq < Q_\perp >_{Z^o,W} \simeq 10 \text{ GeV,}$$

Finally, we note that Eq. (6) may be rewritten in the more familar form

$$< Q_\perp^2 \cdot (x_1,x_2,s) > = (\frac{4}{3\pi}) s \; \bar{\alpha}_s \; f(x_1,x_2) \tag{11}$$

with

$$\bar{\alpha}_s = \frac{\int_{x_1}^{1} dy_1 \int_{x_2}^{1} dy_2 \; \alpha_s \left\{ \frac{s}{\Lambda^2}(y_1-x_1)(y_2-x_2) \right\} \; \mathscr{F}^{(Q^2)}(y_1,y_2)}{\int_{x_1}^{1} dy_1 \int_{x_2}^{1} dy_2 \; \mathscr{F}^{(Q^2)}(y_1,y_2)} \tag{12}$$

and

$$f(x_1, x_2) = \frac{1}{\bar{\alpha}'^{(Q^2)}(x_1, x_2)} \int_{x_1}^{1} dy_1 \int_{x_2}^{1} dy_2 \, \mathcal{J}^{(Q^2)}(y_1, y_2) \tag{13}$$

Here $\bar{\alpha}_s$ is defined (Eq. (12)) as an average of the running coupling constant over the parton coordinates with the hadronic cross-section as weight function. An accurate knowledge of $< Q_\perp^2 >$ and f at low energies as well as at W and Z^0 would allow us to check whether α really runs and hence determine the scale Λ.

In conclusion, we have presented an improved algorithm for summing soft-gluons to obtain the $Q_\perp$-spectrum for W, Z^0 and γ^* production in hadronic reactions. We were able to eliminate the ambiguity in the energy scale through a sum rule which independently predicts $< Q_\perp^2 >$. Our parameter free $Q_\perp$-spectrum was compared with the present W-boson data and satisfactory agreement was found.

ACKNOWLEDGEMENTS

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