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EXACT FORM OF THE QUARK-QUARK INTERACTION IN NONRELATIVISTIC QCD

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Looking for the features of nonabelian gauge theories which could possibly provide the confinement of color, it is desirable to introduce the most drastic approximation which should respect such features. I will discuss the nonrelativistic approximation, which seems appropriate to a bound state problem.

This approximation is not unique in general, but it is strongly constrained by requiring that Poincaré invariance should contract into Galilei invariance (J.M. Lévy-Leblond, 1967) while other symmetries (gauge invariance, chiral invariance, charge conjugation) should be conserved, and it has been checked (F. Palumbo, 1981; S. Ferrara, 1981) that the low energy behaviour of those relativistic theories for which such behaviour is known is reproduced in the Galilean approximation.

If Galilei invariance is not required, on the other hand, there is no general way to perform unambiguously the nonrelativistic approximation.

The Galilean limit is not known for all field theories. In particular it is not known for the pion-nucleon interaction, whose standard nonrelativistic approximation $\varphi_i \partial_k \psi^* \sigma_k \tau_i \psi$ is not Galilei invariant. No wonder that there will be ambiguities in the relativistic corrections which have been discussed so much in this School.

The Galiean limit is known for gauge theories, and I will show the q-q interaction which results in Galielan QCD. This interaction is non confining in the usual sense, but I will discuss a new possible mechanism for confinement suggested by the comparison with Galilean QED.

I will consider gauge theories in the framework of canonical quantization with two types of boundary conditions. The fields are either required to be periodic (p.b.c.) or to vanish on the surface of the quantization volume. I will refer to this latter boundary condition by the elliptic expression: vanishing boundary conditions (v.b.c.).

Let me first discuss Galilean QCD with p.b.c. The Hamiltonian for two quarks in a singlet color state for the SU(2) color group is (F. Palumbo, 1983)

$$H = -\frac{\hbar^2}{2m_q}(\Delta_1+\Delta_2) + 2\, m_q c^2 + W\,, \qquad (1)$$

where

$$W = -\frac{3}{2}\alpha_s \frac{\hbar c}{|\vec{x}_1 - \vec{x}_2|} + \frac{1}{2}p^2 + \frac{1}{2}\omega^2 q^2 - \frac{3}{2}\hbar\omega$$
$$+ \alpha_s \frac{\hbar c}{|\vec{x}_1 - \vec{x}_2|}\left[1 - \cos \hbar^{-1} g L^{-3/2}\vec{q}\cdot(\vec{x}_1 - \vec{x}_2)\right] . \tag{2}$$

In the last equation L^3 is the quantization volume

$$\alpha_s = \frac{g^2}{4\pi\hbar c}, \qquad \omega^2 = \frac{2g^2}{m_q L^3} \tag{3}$$

and $\vec{q}$ and $\vec{p}$ are the constant terms in the Fourier expansion of the vector potential $\vec{A}$ and its conjugate variable, the electric strength $\vec{E}$. Only these constant terms survive in the Galilean limit, consistently with the fact that in this limit $\vec{A}$ and $\vec{E}$ propagate with infinite velocity.

Cross sections must be evaluated at finite L and the limit $L \to \infty$ must be taken at the end. The importance of following this procedure will be illustrated in the abelian case. It is however obvious that the present interaction is not confining in the usual sense. In this connection it would be interesting to compare the nonrelativistic limit to the lattice approximation, by calculating the q-q potential energy at finite L. In the lattice calculations, however, the q-q potential energy is evaluated for infinite quark mass, and the corresponding calculation in the Galilean limit has not yet been completed. I will therefore report an evaluation of the q-q potential energy which I think instructive, also if valid for light quarks only.

If we take $\vec{x}_1 - \vec{x}_2$ parallel to third axis q_1 and q_2 decouple, so that

$$W = -\frac{3}{2}\alpha_s \frac{\hbar c}{|\vec{x}_1 - \vec{x}_2|} + \frac{1}{2}p_3^2 + \frac{1}{2}\omega^2 q_3^2 - \frac{1}{2}\hbar\omega$$
$$+ \alpha_s \frac{\hbar c}{|\vec{x}_1 - \vec{x}_2|}\left[1 - \cos \hbar^{-1} g L^{-3/2} q_3 |\vec{x}_1 - \vec{x}_2|\right] . \tag{4}$$

Let us introduce the Compton wave length λ_q of the quark and the parameter σ

$$\lambda_q = \frac{\hbar}{m_q c} \quad ; \quad \sigma = \frac{1}{2}\sqrt{\frac{\pi}{2}}\,\alpha_s^{3/2}\frac{\hbar c}{\sqrt{\lambda_q L^3}} . \tag{5}$$

For

$$\hbar^{-2}\frac{g^2}{L} q_3^2 \frac{|\vec{x}_1 - \vec{x}_2|^2}{L^2} \ll 1; \qquad \frac{4\sigma}{\hbar\omega}|\vec{x}_1 - \vec{x}_2| \ll 1, \tag{6}$$

we can approximate (4) by

$$W \simeq -\frac{3}{2}\alpha_s \frac{\hbar c}{|\vec{x}_1 - \vec{x}_2|} + \frac{1}{2}p_3^2 - \frac{1}{2}\hbar\omega + \frac{1}{2}\omega^2\left[1 + 4\frac{\sigma}{\hbar\omega}|\vec{x}_1 - \vec{x}_2|\right] q_3^2 . \tag{7}$$

We can now replace q_3^2 by its expectation value

$$\langle q_3 \rangle^2 = \frac{1}{2}\hbar\omega^{-1}\left[1 + 4\frac{\sigma}{\hbar\omega}|\vec{x}_1 - \vec{x}_2|\right]^{-1/2} \tag{8}$$

both in Eqs. (6) and (7) getting

$$L \ll \frac{2}{\pi\alpha_s}\frac{L^4}{|\vec{x}_1 - \vec{x}_2|^4}\lambda_q \tag{6'}$$

$$W \simeq -\frac{3}{2}\alpha_s \frac{\hbar c}{|\vec{x}_1-\vec{x}_2|} + \frac{1}{2}\hbar\omega\left[1+4\frac{\sigma}{\hbar\omega}|\vec{x}_1-\vec{x}_2|\right]^{1/2} - \frac{1}{2}\hbar\omega$$
$$\simeq -\frac{3}{2}\alpha_s \frac{\hbar c}{|\vec{x}_1-\vec{x}_2|} + \sigma\,|\vec{x}_1-\vec{x}_2|\,. \qquad (7')$$

We thus obtain a linear potential. This is a spurious contribution (vanishing for $L \to \infty$) to the string tension, which must be subtracted from the value obtained in lattice calculations. For $L \sim 1$ fm, $\alpha_s \sim 1$, $\lambda_q \sim 0.7$ fm, $\sigma \sim 150$ MeV fm^{-1}.

The scattering problem with the Hamiltonian (1) has not yet been solved. It is instructive to compare with the abelian case, i.e. Galilean QED, for which the solution is known (F. Palumbo, 1983). The e^+e^- Hamiltonian is

$$H = -\frac{\hbar^2}{2m_e}(\Delta_1+\Delta_2) - \alpha\frac{\hbar c}{|\vec{x}_1-\vec{x}_2|} + \frac{1}{2}p^2 + \frac{1}{2}\omega^2 q^2 - \omega\vec{q}\cdot\vec{I}, \qquad (9)$$

where

$$I_k = -\sqrt{\frac{m_e}{2}}\;\frac{i\hbar}{m_e}\left(\frac{\partial}{\partial x_{1k}} - \frac{\partial}{\partial x_{2k}}\right),\; \omega^2 = \frac{2e^2}{m_e L^3}\,. \qquad (10)$$

It is convenient to introduce creation operators (put $\hbar = c = 1$)

$$a_k^+ = (2\omega)^{-1/2}(p_k + i\omega q_k)$$

and rewrite H as

$$H = -\frac{1}{2m_e}(\Delta_1+\Delta_2) + \frac{3}{2}\omega + \omega a_k^+ a_k + \frac{1}{2}\sqrt{\omega}\; i(a_k^+ - a_k) I_k - \alpha\frac{1}{|\vec{x}_1-\vec{x}_2|} \qquad (11)$$

If we denote the purely electronic states by $|i\rangle$, $|f\rangle$, ... the eigenstates of H can be written

$$|i, n_1, n_2, n_3\rangle = \prod_{k=1}^{3}\frac{1}{\sqrt{n_k!}}\left[a_k^+ + \frac{i}{\sqrt{2\omega}} I_k(i)\right]^{n_k} e^{\frac{i}{\sqrt{2\omega}}(a_k^+ + a_k) I_k(i)}\;|i\rangle, \qquad (12)$$

where $I_k(i)$ is the eigenvalue of the operator I_k in the state $|i\rangle$. Let $|i\rangle$ be the initial state and $|f\rangle$ the final state, $\Delta I_k = I_k(f) - I_k(i)$. Assuming by a proper choice of axes, $\Delta I_k = \delta_{k3}\Delta I$, the S-matrix elements have the expression

$$|\langle i,0,0,0,|S|f,n_1,n_2,n_3\rangle|^2 = \delta_{on_1}\delta_{on_2}|\langle i|S(n_3\omega)|f\rangle|^2 \cdot \frac{1}{n_3!}\left[\frac{(\Delta I)^2}{2}\right]^{n_3} e^{-\frac{(\Delta I)^2}{2\omega}}, \qquad (13)$$

which is the probability for a classical source to radiate n_3 photons of energy $\hbar\omega$.

The limit $L \to \infty$ can now be performed in Eq. (13) showing that

$$\lim_{L\to\infty} \left| \langle i,0,0,0, \left| S \right| f, n_1, n_2, n_3 \rangle \right|^2 = 0 \; . \tag{14}$$

In an actual experiment, however, there will be a finite energy resolution. Let us assume infinite energy resolution in the initial state and energy resolution ΔE in the final state. The theoretical cross section to be compared with experiment is

$$\sigma \sim \lim_{L\to\infty} \sum_{n_3=0}^{\infty} \left| \langle i,0,0,0, \left| S(n_3\,\omega) \right| f, n_1,n_2,n_3 \rangle \right|^2 \theta(\Delta E - n_3\omega)$$
$$= \left| \langle i \left| S\left[\tfrac{1}{2}(\Delta I)^2\right] \right| f \rangle \right|^2 \theta(\Delta E - \tfrac{1}{2}(\Delta I)^2) \, , \tag{15}$$

where $\theta(x)$ is the step function.

The radiative correction of the above equation can also be obtained for $c \to \infty$ from the relativistic formula, if the constant term in the expansion of $\vec{A}$ and $\vec{E}$ is properly taken into account (F. Palumbo, 1983).

This calculation bas been shown for two reasons. First we have seen that it agrees with relativistic QED, and this gives confidence in Galilean gauge theories. The second reason is that it suggests a new mechanism for confinement, as discussed below.

The radiative correction arises from the overlapping of the soft photon state relative to the initial state to the soft photon state relative to the final state, the photon state being a function of the current. This is because $\vec{q}$ is coupled to the current in the electronic Hamiltonian. In Galilean QCD $\vec{q}$ is not coupled to the color current, but from the form of the coupling in the Hamiltonian (1) we see that the gluon state will depend on the structure of the q-q wave-function. Confinement would result if the gluon state relative to a bound state were orthogonal to the gluon state relative to a free state in the $L \to \infty$ limit. In such a case, in fact, the disintegration cross-section would vanish.

To complete the results of Galilean quantum gauge theories with p.b.c. I must mention the consequence of b.c. on the electric potential V. In the first order formulation V appears in the Lagrangian only in the term (common to the relativistic and Galilean theory)

$$\mathcal{L} = \ldots V\,(\mathcal{D}_k E_k + g\,\varrho) + \ldots \tag{16}$$

When we expand in Fourier series, from the variation of the constant term V_o we get

$$(\mathcal{D}_k E_k + g\,\varrho)^a_o = g\left[-f^{abc} A^c_k E^c_k + \varrho^a\right]_o \sim Q^a = 0 \, , \tag{17}$$

i.e. the color charge must vanish. An analogous result holds in the Abelian case.

If we choose v.b.c., the constant terms of $\vec{A}$, $\vec{E}$ and V (b.c. must be the same for $\vec{A}$ and V for Lorentz or Galilean invariance) must vanish, the interaction is purely Coulombic both in the Abelian and nonabelian case and there is no constraint on the charge. The two types of b.c. are perfectly consistent from the theoretical point of view, and give rise to different physical effects which must be compared to phenomenology.

In QED, since charged states exist and the radiative correction of Eq. (15) is in disagreement with experiment, we must choose v.b.c. In QCD, due to the constraint on the color charge, p.b.c. seem to be appropriate.

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