ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-83/32

E. Etim : QUARK MASSES FROM THE VECTOR MESON SPECTRUM

Estratto da : '
"The High-Energy Limit",
ed. by A. Zichichi (Plenum, 1983)

Servizio Documentazione
dei Laboratori Nazionali di Frascati
Cas. Postale 13 - Frascati (Roma)

From: THE HIGH-ENERGY LIMIT
Edited by Antonino Zichichi
(Plenum Publishing Corporation, 1983)

QUARK MASSES FROM THE VECTOR MESON SPECTRUM

E. Etim*

Physics Department

Siegen University, F.R. of Germany

The success of QCD in the deep inelastic region has not been accompanied by progress in the calculation of hadronic properties using the rather promising gauge theory tools. In the absence of such a calculational scheme some attempts are being made, using dispersion relations, to extrapolate QCD predictions to the domain of traditional hadronic physics(1-4). Quark parameters, like charge and mass, get related directly in this way with hadronic parameters. While the meaning of quark charge is rather clear in these treatments, the same is not true of quark mass. Since quarks are postulated to be confined the quark mass which is estimated from the above type of comparison is certainly not the mass which would appear in a renormalised quark propagator. This latter is probably infinite. There are no good reasons to expect that it would even agree with other estimates of quark masses from hadronic mass differences in say an isospin multiplet(5) (Constituent quark masses) or from

*Permanent adress:
Laboratori Nazionale, INFN, Frascati, Italy

current algebra and PCAC[6] (current quark masses). Under these circumstances it is convenient to define quark masses simply as parameters which measure the breaking of flavour symmetries e.g. SU(2), SU(3), SU(3) × SU3) etc. . Depending on the amplitudes from which one extracts these parameters and the assumptions employed in the process there would, quite understandably, be different estimates for the same quark mass. The most one can hope is that they are not incomparably different. We shall consider in this lecture the relationship between quark masses and the masses of their neutral vector meson bound states. We do this by studying the properties of the hadronic contribution to the photon propagator to second order in the electric charge. For low energies the corresponding vacuum polarisation amplitude is dominated by vector mesons while at high energies the quark structure of the current takes over. These two structures are required to match asymptotically. Straight forward extrapolation of the quark model result for the vacuum polarisation amplitude to low energies is known to be inconsistent with data on the vector mesons[3]. On the other hand the high energy tails of the vector meson amplitudes do not reconstruct the asymptotic behaviour characteristic of the quark model. The matching is through an arrangement which is reminiscent of regularisation[1,4]: besides the established vector mesons an infinite number of others, not all of which correspond necessarily to physical resonances, are required in order to approximate the asymptotic quark model behavior of the vacuum polarisation amplitude by a sum of vector meson contributions. The properties of these additional vector mesons are assumed to extrapolate smoothly from those of the known vector mesons. More precisely we assume that to each quark of flavour i corresponds a series of neutral vector mesons with mass spectrum

$$m_{ni}^2 = m_i^2 \ (1+b_i n)^{1+\lambda_i}; \quad n = 0, 1, 2, \ldots\ldots\ldots \tag{1}$$

and coupling to the photon m_{ni}^2/f_{ni} where

$$f_{ni}^2 = f_i^2 \ (1+b_i n); \quad n = 0, 1, 2, \ldots\ldots\ldots \tag{2}$$

The parameters b_i, λ_i are deduced from the masses of the lowest members of the family. There is a slight complication when a vector meson (like the ρ, ω, etc.) is made up of more quark flavours. Eqs(1) and (2) correspond to a definite method of regularising the coefficient of the Schwinger term namely the so-called Zeta function regularisation. This is discussed in the next section. In sect.3 we consider the asymptotic expansions of the vacuum polarisation amplitude and deduce from their comparison the relationship between quark and vector meson masses.

2. THE REGULARISED SCHWINGER COEFFICIENT

In the absence of a complete fundamental theory of hadronic matter in terms of quarks it seems a good first approximation to describe the structure of the hadronic electromagnetic current in terms of its manifested dominant characteristics. At short distances the current tends to manifest a quark structure

$$j_\mu(x) = \sum_i \sum_c Q_i \ \bar{q}_i^c(x) \ \gamma_\mu \ q_i^c(x) \tag{3}$$

The index i stands for flavour and c for colour. The number of colours is taken to be three. Q_i is the charge of the quark i and M_i will stand for its mass. For large distances a certain wealth of neutral vector mesons mediate in the electromagnetic interactions of hadrons. If these vector mesons couple to conserved matter current the following representation of the hadronic electromagnetic current

$$j\mu(x) = \sum_i \sum_n \frac{m_{ni}^2}{f_{ni}} V_{n\mu}^{i}(x) \tag{4}$$

is consistent with gauge invariance[7]. $V_{n\mu}^{i}(x)$ is the field of the n-th vector meson in the flavour family i. The sum over n in eq. (4) should strictly speaking run only over the existing members of the family , if one adopts the simple formulation of field-current identity. We extend it however over all the states in the set whose masses and couplings are given by eqs(1) and (2). In other words although the photon has properties very close to those of vector mesons a finite number of them will represent the photon only incompletely.

Consider the familiar noncovariant T-product

$$T_{\mu\nu}(x) = T(j_\mu(x)\ j_\nu(0)) \tag{5}$$

Its divergence

$$\partial_\mu T_{\mu\nu}(x) = [j_0(x),\ j_\nu(0)]\delta(x_0) = i\,C\,g_{\nu k}\partial_k\delta^4(x) \tag{6}$$

(no sum over k) is not a 4-vector. C is the Schwinger coefficient and $g_{\mu\nu}$ is the Lorentz metric tensor. It is usual to consider besides $T_{\mu\nu}(x)$ the following covariant T-product

$$\hat{T}_{\mu\nu}(x) = T_{\mu\nu}(x) + i\,C\,g_{\mu 0}g_{\nu 0}\delta^4(x) \tag{7}$$

which is obviously such that

$$\partial_\mu \hat{T}_{\mu\nu}(x) = i\, C\, \partial_\nu\, \delta^4(x) \tag{8}$$

is a 4-vector. Taking the vacuum expectation value of (8) (or (6)) one finds the following expression for C

$$C = \frac{1}{\pi}\int_0^\infty ds \;\mathrm{Im}\,\Pi(s) \tag{9}$$

where $\mathrm{Im}\pi(s)$ is the imaginary part of the vacuum polarisation amplitude defined by

$$\begin{aligned}\Pi_{\mu\nu}(q) &= -i\int d^4x e^{iqx} \langle 0|T(j_\mu(x)\, j_\nu(o))|0\rangle \\ &= (q^2\, g_{\mu\nu} - q_\mu\, q_\nu)\,\Pi(q^2) \;+ \\ &+ (g_{\mu\nu} - g_{\mu 0}\, g_{\nu 0})\; C \end{aligned} \tag{10}$$

As is well known if the representation for $j_\mu(x)$ in eq. (3) were used to compute the commutator in eq(6) the result would be zero. This is in conflict with positivity and spectral properties. The conflict is avoided by replacing $\bar{q}(x)\gamma_k\, q(x)$ (k=1,2,3) by

$$\begin{aligned}\bar{j}_k(x;\eta) &= \bar{q}(x_0, \vec{x} + \tfrac{1}{2}\vec{\eta})\;\gamma_k\; q(x_0,\; \vec{x} - \tfrac{1}{2}\vec{\eta}) \;\cdot \\ &\cdot\; \exp\Big(-iQ \int_{\vec{x}\,\frac{1}{2}\vec{\eta}}^{\vec{x}+\frac{1}{2}\vec{\eta}} d\vec{y}\cdot\vec{A}(y)\Big)\end{aligned} \tag{11}$$

and taking the limit $|\vec{\eta}| \to o$ at the end. In this way one discovers immediately that the coefficient C diverges like $1/\eta^2$. In eq.(11) $A_\mu(y)$ is the electromagnetic potential and the exponential there is required to keep $j_\mu(x)$ invariant under the gauge transformations

$$q(x) \longrightarrow q(x)\ \exp\ (iQf(x))$$

$$A_\mu(x) \longrightarrow A_\mu(x) + \partial_\mu f(x) \tag{12}$$

On the other hand if we substitute for $j_\mu(x)$ in (6) from eq(4) and make use of the commutation relation

$$[V^i_{no}(x),\ V^{i'}_{n'\nu}(0)]\ \delta(x_0) = i\delta_{ii'}\ \delta_{nn'}\frac{1}{m^2_{ni}}\ g_{\nu k}\partial_k\delta^4(x) \tag{13}$$

then

$$C = \pi \sum_i \sum_n \frac{m^2_{ni}}{f^2_{ni}} \tag{14}$$

If the sum in eq.(14) were over a finite number of vector mesons C would be finite. The representations (3) and (4) would in this case be incompatible; with an infinite sum C diverges but can be regularised upon using eqs(1) and (2) and then continuing analytically. Substituting in fact from eqs(1) and (2) into (14) one gets

$$C = \pi\sum_i \frac{m_i^2}{f_i^2}\ b_i^{\lambda_i}\sum_{n=0}^{\infty}\ (n + \frac{1}{b_i})^{\lambda_i} \tag{15}$$

Hence

$$\text{Reg } C = \pi\sum_i \frac{m_i^2}{f_i^2}\ b_i^{\lambda_i}\ \zeta(-\lambda_i,\frac{1}{b_i}) \tag{16}$$

$\zeta(\nu,z)$ is the generalised zeta function[8]. Its series representation

$$\zeta(\nu,z) = \sum_{n=0}^{\infty} \frac{1}{(n+z)^{\nu}} \qquad (17)$$

valid for $z \neq 0$ and $\mathrm{Re}\,\nu > 1$ admits the following analytic continuation into the complex ν-plane

$$\zeta(\nu,z) = \frac{1}{2z^{\nu}} + \frac{z^{1-\nu}}{1-\nu} + $$

$$+ 2\int_0^{\infty} \frac{dt}{e^{2\pi t}-1} \frac{\sin[\nu \tan^{-1}(t/z)]}{(t^2+z^2)^{\nu/2}} \qquad (18)$$

The only singularity of $\zeta(\nu,z)$ is the simple pole at $\nu = 1$. Reg C is therefore finite since if all the vector meson masses are not equal $\lambda_i \neq -1$.

If we had a non-zero regularised Schwinger coefficient from the quark model we could have compared it flavour-wise with eq. (16) and deduced therefrom a relationship between the quotient m_i^2/f_i^2 and the product $Q_i^2 M_i^2$. We shall perform the comparison indirectly by examining the asymptotic behaviour of the amplitude $\pi(q^2)$.

3. ASYMPTOTIC EXPANSIONS OF $\pi(q^2)$.

For simplicity consider the contribution of one flavour quark to $\pi(q^2)$. Call it $\pi_i(q^2)$. It is expressible in terms of the integral

$$\pi_i(q^2) = \frac{3Q_i^2}{2\pi^2} \int_0^1 dx\; x(1-x)\,\ell n[1-x(1-x)\frac{q^2}{M_i^2}] \qquad (19)$$

The factor 3 is due to colour. For large q^2, $\pi_i(q^2)$ becomes

$$\pi_i(q^2) \xrightarrow[|q^2| \to \infty]{} - \frac{Q_i}{4\pi^2} (2 + \frac{4M_i^2}{q^2}) [-1 + \frac{1}{2} \ell n (-\frac{q^2}{M_i^2}) - \frac{M_i^2}{q^2} \ell n(-\frac{q^2}{M_i^2}) - \frac{M_i^2}{q^2} - (\frac{M_i^2}{q^2})^2 n(-\frac{q^2}{M_i^2}) + (\frac{M_i^2}{q^2})^2 + \ldots\ldots\ldots\ldots] \quad (20)$$

If we use eq(4) in (10), then $\pi_i(q^2)$ is also given by

$$\pi_i(q^2) = \sum_{n=0}^{\infty} \frac{m_{ni}^2}{f_{ni}^2} \frac{q^2}{m_{ni}^2(m_{ni}^2 - q^2)}$$

$$= \frac{z_i}{b_i f_i^2} \sum_{\ell=0}^{\infty} z_i^{\ell} \zeta(2 + \lambda_i + \ell(1+\lambda_i), \frac{1}{b_i}) \quad (21)$$

where $z_i = q^2/b_i^{1+\lambda_i} m_i^2$. The asymptotic expansion of the series in eq(21) is

$$\pi_i(q^i) \xrightarrow[|q^2| \to \infty]{} - \frac{1}{b_i f_i^2} \left[\frac{\ell n(-z_i)}{1+\lambda_i} + \frac{\zeta(-\lambda_i, \frac{1}{b_i})}{z_i} + \frac{\xi(-1-2\lambda_i, \frac{1}{b_i})}{z_i^2} + \ldots\ldots\ldots\ldots\ldots\ldots \right] \quad (22)$$

Comparing the coefficients of $\ell n(-q^2)$ and $1/q^2$ in eqs (20) and (22) we find

$$\frac{Q_i^2}{4\pi^2} = \frac{1}{b_i f_i^2 (1+\lambda_i)} \tag{23.a}$$

$$-\frac{6Q_i^2 M_i^2}{4\pi^2} = \frac{m_i^2}{f_i^2} b_i^{\lambda_i} \zeta(-\lambda_i, \frac{1}{b_i}) \tag{23.b}$$

Note that the coefficient of $1/q^2$ in eq(23) is exactly the contribution of the i-th vector meson family to the regularised Schwinger coefficient. This is not a general result. We neglected a contribution from the terms determining $\ell n(-z_i)$ which was difficult to estimate accurately. No such difficulty exists in eq(20). The analogous contribution to the coefficient of $1/q^2$ comes from the term $\ell n(1-q^2/4M_i^2)$. It is therefore $-\frac{Q_i^2 M_i^2}{\pi^2}$. If we subtract this from the left hand side of (23.b) we have the result

$$\text{Reg } C = -\frac{Q_i^2 M_i^2}{2\pi^2} = \frac{m_i^2}{f_i^2} b_i^{\lambda_i} \zeta(-\lambda_i, \frac{1}{b_i}) \tag{24}$$

It is easy to see that (24) is just the contribution of the normal fermion (here the coloured quark) to Reg C_i in the Pauli-Villars regularisation of the Schwinger coefficient[9]. In fact regularising $\pi_i(q^2)$ in eq(19) à la Pauli-Villars (PV) one gets

$$\text{Reg } \Pi_i^{(PV)}(q^2) = \sum_{\ell=0} \frac{3Q_{i\ell}^2}{2\pi^2} \int_0^1 dx \; x(1-x)\, \ell n\left[1-x(1-x)\frac{q^2}{M_{i\ell}^2}\right] \tag{25}$$

where $Q_{i\ell=0} = Q_i$, $M_{i\ell=0} = M_i$ and the terms $\ell \geq 1$ are the contributions from regulators. Their masses and charges are required to satisfy[10]

$$\sum_{\ell=0} Q_{i\ell}^2 = 0 \tag{26.a}$$

$$\sum_{\ell=0} Q_{i\ell}^2 \; M_{i\ell}^2 = 0 \tag{26.b}$$

The imaginary part of Reg $\pi_i(q^2)$ is

$$\text{Reg Im } \Pi_i^{(PV)}(S) = \sum_{\ell=0} \theta(S-4M_{i\ell}^2) \frac{Q_{i\ell}^2}{4\pi} \cdot$$

$$\cdot \left(1+2\frac{M_{i\ell}^2}{S}\right) \left(1-4\frac{M_{i\ell}^2}{S}\right)^{1/2} \tag{27}$$

which substituted into eq(9) gives[9]

$$\text{Reg } C_i^{(PV)} = -\sum_{\ell=0} \frac{Q_{i\ell}^2 M_{i\ell}^2}{2\pi^2} = 0 \tag{28}$$

an account of eq(26.b). The $\ell = 0$ term in (28) is the right hand side of (24).

For a linear mass spectrum ($\lambda_i = 0$) we have from eqs(23.a) and (24) the quark masses given in Table (1). Ideal $\omega - \phi$ mixing is assumed ($\sin\theta_V = 1/\sqrt{3}$) so that the quark contents of the light vector mesons are

$$\begin{pmatrix} \rho \\ \omega \\ \phi \end{pmatrix} \quad \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \bar{u}u \\ \bar{d}d \\ \bar{s}s \end{pmatrix} \tag{29}$$

In the u, d, s sector eq(24) becomes

$$- \frac{1}{2\pi} \cdot \frac{1}{2}(M_u + M_d)^2 = \pi \frac{m_\rho^2}{f_\rho^2} (\frac{b_\rho}{2} - 1)$$

$$- \frac{1}{2\pi} \; \frac{1}{18} (M_u + M_d)^2 = \pi \frac{m_\omega^2}{f_\omega^2} (\frac{b_\omega}{2} - 1)$$

$$- \frac{1}{2\pi} \; \frac{1}{9} M_s^2 = \pi \frac{m_\phi^2}{f_\phi^2} (\frac{b_\phi}{2} - 1) \qquad (30)$$

We have made use of the formula

$$\zeta(0,z) = \frac{1}{2} - \frac{1}{z} \qquad (31)$$

The squares of the charge combinations are

$$\left(\frac{Q_u - Q_d}{\sqrt{2}}\right)^2 = \frac{1}{2}$$

$$\left(\frac{Q_u + Q_d}{\sqrt{2}}\right)^2 = \frac{1}{18}$$

$$Q_s^2 = \frac{1}{9} \qquad (32)$$

New data on the excited vector meson states ρ', ω' and ϕ' are now available[11]. Eq(24) is not consistent with these data either with a linear or non-linear mass spectrum. Defining the parameters a_i by

$$m_{1i}^2 = m_i^2 (1 + a_i) \qquad (33)$$

we have from eq(1)

$$(1 + \lambda_i) \ell n(1+b_i) = \ell n(1 + a_i) \qquad (34)$$

Eqs.(23.a), (32) and (34) completely determine b_i and λ_i.

Table (1):

Quark masses computed from eq(24) with a linear vector meson mass spectrum $m_{ni}^2 = m_i^2\ (1 + b_i n)$.

i	m_i (GeV)	m_{1i} (GeV)	b_i	$f_i^2/4\pi$	$M_i = m_i(\frac{1}{2}(1-\frac{b_i}{2}))^{1/2}$ (GeV)
ρ	0.776	1.26	1.64	2.3	0.549
ω	0.783	1.26	1.59	18.3	0.554
φ	1.02	1.50	1.16	13.3	0.727
ψ	3.1	3.686	0.417	11.7	2.189
Υ	9.5	10.01	0.11	31	6.720

Table (2):

Quark masses computed from eq(24) with a non-linear vector meson mass spectrum $m_{ni}^2 = m_i^2(1 + b_i n)^{1+\lambda_i}$

i	m_i (GeV)	m_{1i} (GeV)	b_i	λ_i	$\frac{f_i^2}{4\pi}$	M_i = (in eq(24)) (GeV)
ρ	0.776	1.67	1.34	0.8	2.3	0.027
ω	0.783	1.82	1.15	1.2	18.3	0.051
φ	1.02	1.65	2.13	-0.16	14.3	0.102
ψ	3.1	3.686	1.50	-0.62	12.5	1.04
Υ	9.5	10.02	0.61	-0.78	112.5	3.53

Using them in eq(24) we find the quark masses in Table (2) with the values of m_{1i} there given. Note the rather small values of $1 + \lambda_i$ for the ψ and $\underline{\gamma}$ meson families. This is in agreement with the power growth in n of the S-wave vector meson bound states in potential models[12]. In the latter models λ_i is related to the power growth in r of the potential

$$V_i(r) = A_i + B_i\, r^{\alpha_i} \tag{35}$$

by

$$\lambda_i = \frac{3\alpha_i - 2}{2 + \alpha_i} \tag{36}$$

The quark masses in Table (2) is intermediate between current and constituent quark masses, with a slight tendency towards the latter. The mass parameters used to scale running quark masses in the renormalisation group approach[13] are certainly not the quantities we have computed here. Let me conclude by observing that although we have used the properties of the spectral function of the electromagnetic current to estimate quark masses, other flavour group currents may be used for the same purpose. It would be interesting to carry out the same investigation for the axial vector current and its divergence. There could be important additional information on the Weinberg sum rules and perhaps also on the Adler-Bell Jackiw anomaly.

The quark masses in Table (1) are rather large. If the smaller estimates are correct we would find here another reason, besides the experimental indication from the heavier mesons to exclude the linear mass spectrum.

REFERENCES

1. A.Bramon,E.Etim and M.Greco;Phys.Lett.41B(1972)609
J.J.Sakurai;Phys.Lett.46B(1973)207.
2. R.Shankar;Phys.Rev.D15(1977)755
G.J.Gounaris; Z.Physik C1(1979)399.
3. M.A.Shifman,A.I.Vainstein and V.I.Zakhorov;
Nucl.Phys.B147(1979)385,B147(1979).
4. E.Etim and S.Kitakado,Z.Physik C6(1980)51.
5. Mass differences of the order of $M_s - M_d \sim 150$ MeV,
$M_d - M_u \sim 5$ MeV are sufficient to explain the mass differences of hadrons in these multiplets.
6. M.Gell-Mann,R.J.Oakes and B.Renner,Phys.Rev.175
(1968)2195 H. Leutwyler, Nucl. Phys. B76(1974)413.
7. N.M.Kroll,T.D.Lee and B.Zumino;Phys.Rev.157(1967)
1376.
8. Higher Transcendental Functions,Bateman Manuscript Project,vol1.Ed.A.Erdêlyi(Mc.Graw Hill Book Co., Inc.,New York,1953).
9. G.Källén;Lectures given at the Winter Schools in Karpacz and Schladming,(February and March 1968). published in the Schladming Proceedings (1968) Ed.P.Urban.
10. W.Pauli and F.Villars;Rev.Mod.Phys.21(1949)434.
11. J.Perez-Y-Jorba;Lecture in these proceedings.
12. A.Martin;CERN preprints TH 2843 (April 1980) and TH 2876 (June 1980)
13. C.Becchi,S.Narison,E.de Rafael and F.J.Yndurain, CERN preprint TH.2920 (August 1980)
F.J.Yndurain;CERN preprint TH.2931(August 1980)

D I S C U S S I O N

CHAIRMAN: Prof. E. Etim

Scientific Secretary: R. Kobes

DISCUSSION

- *KREMER:*

In your lecture, you used the formulae

$$Q_i^2 = \frac{4\pi^2}{(1+\lambda_i)b_i f_i^2} \tag{1}$$

$$M_i^2 = \frac{m_i^2}{6}(1+\lambda_i)b_i^{1+\lambda_i}\zeta(-\lambda_i,\frac{1}{b_i}) \tag{2}$$

As I understand it, f_i is the coupling constant of the vector meson. Are λ_i and b_i fit parameters in your model?

- *ETIM:*

These are parameters that characterize the vector meson spectrum. If the spectrum were linear, then λ_i would vanish and b_i would be the only parameter, besides f_i which determines the decay of the lowest vector meson into e^+e^-.

- *KREMER:*

As I understand it, you tried to calculate the current quark masses because you wanted these parameters which describe the breaking of the flavour symmetry. What you got were the constituent quark masses. What went wrong?

- *ETIM:*

From data on the vector meson mass spectrum, I found the parameters λ_i and b_i. I then found that the quark masses coming from (2) are very large. These are the constituent, not current, quark masses. Of course, by going close to the zeros of the zeta function $\zeta(-\lambda_i, \frac{1}{b_i})$, one would get correspondingly smaller masses for the quarks, but these would not give us a picture of the vector meson spectrum as we know it.

- *SCHILDKNECHT:*

You said you use the empirical parameters as input to find the quark masses. On the other hand, you also had a potential,

$$V(r) = A + Br^{\alpha_i}$$

and a relation of α_i to the parameter λ_i.

$$1+\lambda_i = \frac{4\alpha_i}{2+\alpha_i}$$

Could you use the potential model to calculate these parameters, and then find the quark masses?

- *ETIM:*

The relativistic procedure is simpler. I calculated λ_i and b_i from experimental data, and then went back to the non-relativistic potential calculation. Looking at the value of λ_i I find there, I see that it agrees with the analysis of Martin for the heavy quarks. We don't have such calculations for the light quarks.

- *SCHILDKNECHT:*

So for the heavy quarks, you can do everything with the potential. The contact with experiment would be just via the potential, while for the light quarks, you would put in the empirical values.

- *ETIM:*

Yes, that is correct.

- *MEYER:*

Can you predict the top mass?

- *ETIM:*

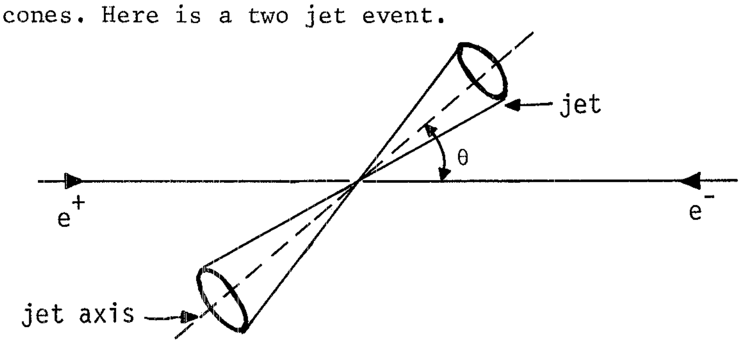
I need the corresponding bound state vector meson mass. I can however predict the ratio of these two masses, assuming everything else to be flavour independent.

- *WIGNER:*

Could you explain the $(1 + \cos^2\theta)$ law?

- *ETIM:*

In e^+e^- collisions, you find the particles come out in jets forming cones. Here is a two jet event.



The centre of the cone defines the jet axis. If the jets originate from quarks or any particles of spin ½, then the probability that the jet axis has a direction θ has a distribution obeying the $(1 + \cos^2\theta)$ law. This law is well confirmed experimentally.

- *SCHILDKNECHT:*

I would like to come to the question of which ρ´ mass to use in determining the quark masses, ρ´(1250) or ρ´(1600). The lower one is still not ruled out; there is some vague evidence perhaps coming from photoproduction. Could you comment on how this would influence your analysis.

- *ETIM:*

If I were to take ρ´(1250), I would just be going very close to the current quark mass. I used ρ´(1600) because it seemed confirmed by several groups.

- *WHITMER:*

The exponent α_i in the potential came out to be very small, suggesting the use of a logarithmic potential. What happens to your results if you do use such a potential?

- *ETIM:*

The logarithm of r is defined in the following way:

$$\ell n\ r = \lim_{\nu \to 0} \frac{r^{\nu}-1}{\nu}$$

When ν is very small, I am really in the logarithmic regime. Therefore, I am approximately using a logarithmic potential.

- *WHITMER:*

You mentioned that there is a relation between the generalized zeta function and regularization. Could you comment on this a little further.

- *ETIM:*

This is not something that has been done very well; there is still a lot of work to be done. It is interesting for those who like to do something different.

It is never said that the Adler anomaly is an effect of regularization. Regularization, by definition, is a mathematical method of subtracting away an infinity. A condition is imposed that this procedure shall not introduce finite terms that contribute to measureable quantities. The only known case that this fails is the Adler anomaly, where there is a $\gamma_5\ \gamma_\mu$ coupling present. This cannot be regularized using dimensional regularization because the generalization of γ_5 to arbitrary dimensions does not exist. You thus need a different way of regularizing it. The first people to look at this were Bell and Jackiw, who used what is known as modified Gupta regularization.

There are two kinds of analytic regularization: dimensional and zeta functional. The zeta functions are defined as

$$\zeta(\nu) = \sum_{n=1}^{\infty} n^{-\nu}$$

$$\zeta(\nu,z) = \sum_{n=0}^{\infty} (n+z)^{-\nu} \quad \ldots z \neq 0$$

The generalized zeta function, $\hat{\zeta}(\nu)$, as used in field theory, is

$$\hat{\zeta}(\nu) = \sum_{n=0}^{\infty} \lambda_n^{-\nu}$$

Now, taking the derivative of $\zeta(\nu)$ with respect to ν and then setting $\nu = 0$ gives

$$\left.\frac{d\zeta(\nu)}{d\nu}\right|_{\nu=0} = \sum_{n=0}^{\infty} \ell n(\lambda_n^{-1}) = \ell n[\prod_n \lambda_n^{-1}]$$

$$= \ell n \text{ Det } A$$

with λ_n^{-1} being the eigenvalues of the operator A. Thus I can write

$$\text{Det } A = \exp\left[\left.\frac{d\hat{\zeta}(\nu)}{d\nu}\right|_{\nu=0}\right]$$

for suitable operators A.

This formula is very useful in path integrals, where you have to compute functional determinants. Most of the time these determinants are divergent, so if you have a way of regularizing the right side of the latter equation, you can do so for det A. That is why people are becoming interested in zeta function regularization.

Another reason for interest in this type of regularization can be seen by considering the function

$$\omega(t) = \sum_{n=1}^{\infty} \exp(-n^2 \pi t)$$

which is a partition function, apart from the n^2 factor. This is related to the elliptic theta function θ_3 by

$$\omega(t) = -\frac{1}{2} + \frac{1}{2}\,\theta_3(o, it)$$

which can be further written as

$$\zeta(2\nu) = \frac{1}{\Gamma(\nu)} \int_0^{\infty} dt\; t^{\nu-1} \omega(\frac{t}{\pi})$$

This is like a propagator; in non-relativistic quantum mechanics, it is the energy operator. To return to dimensional regularization, compute the propagator in D- dimensional Euclidean space.

$$G_\nu(x^2) = \int d^D p \; ip\cdot x \; \frac{1}{p^2+m^2}$$

Carrying out this integration, you find

$$G_\nu(x^2) = \frac{1}{\Gamma(\nu)} \int_0^\infty dt\ t^{\nu-1} [\Gamma(\nu)\pi^{\nu+1}\ \exp(-\frac{x^2}{4}t - \frac{m^2}{t})]$$

$$= \frac{\pi}{2} (\frac{\pi x}{2m})^\nu\ K_\nu(mx)$$

where $\nu = \frac{1}{2}(D-2)$ and $K_\nu(mx)$ is the modified Bessel function. Compare this to

$$\zeta(2\nu) = \frac{1}{\Gamma(\nu)} \int_0^\infty dt\ t^{\nu-1}\ \omega(\frac{t}{\pi})$$

$G_\nu(x)$ is referred to as the generalized zeta function for the theory. Thus, there is a relationship between the generalized zeta function and the propagator.

For these two reasons one is interested in the zeta function regularization; if det A is divergent, then we know how to compute it. The zeta function defined by the series

$$\zeta(\nu, z) = \sum_{n=0}^{\infty} (n+z)^{-\nu}$$

is an analytic function; hence the name analytic regularization. When continued into the complex ν - plane, there is only one pole, at $\nu = 1$.

$$\zeta(\nu, z) = \frac{1}{2z^\nu} + \frac{z^{1-\nu}}{\nu-1} + 2\int_0^\infty \frac{dt}{e^{2\pi t}-1}\ \frac{\sin[\nu\tan^{-1}(\frac{t}{z})]}{(t^2+z^2)^{\nu/2}}$$

In quantum field theory, there is only one type of singularity - the logarithmic. Every other type can be regularized away. To give an example of how this can be done, I compute the vacuum mass in the dual model. As it is well known, this is related to the energy levels of a harmonic oscillator. One in fact has

$$m_o^2 = \frac{1}{\alpha'} \sum_{n=0}^{\infty} (n+\frac{1}{2})$$

which is divergent. In the zeta function regularization scheme, the sum exists, and is given by

$$m_o^2 \overset{reg}{\equiv} \frac{1}{\alpha'}\ \zeta(-1,\frac{1}{2})$$

$$\equiv \frac{1}{\alpha'}\ (\frac{1}{2}-1)\zeta(-1)$$

$$= -\frac{1}{2\alpha'}\zeta(-1)$$

This was first given by Gliozzi. This is how it comes about that the ground state of the dual model is tachyonic. People are now trying to use this scheme to regularize in quantum field theory.

- *GUADAGNINI:*

What do you mean when you say that regularization does not introduce any physical effects?

- *ETIM:*

I will have to qualify that - it does have consequences for non-regularizable theories. The Adler anomaly is an effect of non-regularizability. The regularized Schwinger term I computed was finite, from which I can find a quark mass - that is the effect of the theory not being regularizable. Previously, we studied regularization from the article by Pauli and Villars, where it was stated that there should be no finite effect when you regularize.

- *GUADAGNINI:*

In computing radiative corrections, you have to regularize. However, you have the possibility of different choices of regularization, which give rise to the renormalization group equations. So, regularization does have much physical meaning.

- *ETIM:*

You have different ways of regularizing, each differing by a constant. Because the constants should not appear in the theory, you have the means of setting up an equation, the renormalization group equation, to show this irrelevance. The definition of Pauli and Villars is something different - when you regularize, the finite part should be zero.

- *GUADAGNINI:*

Doesn't that mean that the vacuum polarization shouldn't be a physical effect?

- *ETIM:*

There is a finite part that is not affected by regularization. Let me regularize the vacuum polarization function using the Pauli-Villars regulator method. I then have at the end to send the regulator mass to infinity; nothing finite depends on this mass in this limit.

- *LEVY:*

You said the only true divergence in quantum field theory is of a logarithmic nature. This is obvious if you use dimensional regularization, but not so in other schemes. When you cannot use dimensional regularization, as for example in the Adler anomaly, how do you see that the only divergence is logarithmic?

- *ETIM:*

That is the reason why you had to invent the zeta function regularization. You want to map quantum field theory onto the manifold of this generalized function, which has only one singularity (of logarithmic nature). This scheme of regularization works in all cases, including those where dimensional regularization fails.

- *DALLA TORRE:*

From what you have shown us, it seems current and constituent quark masses are quite similar. What is the physical meaning of this?

- *ETIM:*

First I have to make a correction to the question. The current and constituent quark masses are quite different in magnitude. The values I gave of the constituent quark masses were quite large. The current quark masses are rather small - for example, 10 MeV for the u quark, whereas it was 300 MeV for the constituent mass. The physical significance of the quark mass in a theory with confinement is that it measures the breaking of flavour symmetry. We have used quark masses in this sense for a long time in low energy phenomenology.

- *LEVY:*

How do you disentangle, in computing the masses of the light quarks, the effect of ρ and ω ?

- *ETIM:*

Let us just take the u and d quarks. In the flavour sector corresponding to the ρ, there is an effective quark mass. The masses I gave are not exactly those of the u and d quark, but are mixtures of them.

In general, there would always be a kind of effective mass. If I wanted to get the masses of the individual components, I would diagonalize the mass matrix obtainable from the theory, using always the experimentally determined vector meson mass spectrum.

- *TOSA:*

Apparently, Vector Meson Dominance is a narrow resonance approximation which does not consider any threshold effects. How would this affect your quark masses?

- *ETIM:*

I did not use the narrowness of the resonances. This is a Lagrangian, not a dispersion relation, theory.

- *TOSA:*

Can you give the values of ρ', ρ'', etc.?

- *ETIM:*

ρ' is given, but the other excitations can be calculated. I have not done so.

- *SCHILDKNECHT:*

There is still some confusion here. It has always been a long standing problem that when you have this Vector Meson Dominance picture for e^+e^- annihilation in the low mass region, you need a ρ' (1250); otherwise, you would not get the correct R value. If you use ρ' (1600), don't you obtain too low an R value?

- *ETIM:*

Due to the non-linear vector meson mass spectrum (i.e., the presence of λ_i), the R value comes out right. If you use the linear spectrum, you have to compute the hadronic contribution to charge renormalization. For the ψ and Υ families, you have negative values, which is inconsistent with the positiveness of probability. For the ρ, ω, and ϕ, you have positive values. This is the reason the non-linear spectrum has to be preferred.