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THE INTRINSIC TRANSVERSE MOMENTUM OF DRELL-YAN PAIRS

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ABSTRACT

A soft gluon summation technique is used in conjunction with a singular infrared behaviour of the QCD coupling constant to compute the moments of transverse momentum $\langle Q_\perp^2 \rangle$ and $\langle Q_\perp^4 \rangle$ of lepton pairs produced in hadron-hadron scattering. Remarkably, it is found that an α_s which produces asymptotically linear Regge trajectories, gives the best account of $\langle Q_\perp^2 \rangle$ for lepton pairs produced through valence quarks (and antiquarks), with no need of any intrinsic transverse momentum. Factorization of the cross section into its longitudinal and transverse momentum part is not assumed for the derivation of the mean values.

1. - INTRODUCTION

In this paper we suggest that the intrinsic transverse momentum of Drell-Yan produced lepton pairs is a confinement effect which can be calculated using soft QCD summation techniques averaged over the hadronic matter coordinates.

One of the first QCD predictions /1/ was that the mean square transverse momentum of DY pairs grows linearly with s. On the other hand, a complete description is based on an accurate knowledge of i) the slope of $\langle Q_\perp^2 \rangle$ vs. s and ii) the behaviour of $\langle Q_\perp^2 \rangle$ for very low energy, i.e. in the so called intrinsic transverse momentum region. These two quantities are generally treated as completely different, the first being calculated through perturbative QCD, the second is introduced /2,3,4/ as an ad hoc parameter and describes the fact that the above prediction fails at low energy, where $\langle Q_\perp^2 \rangle$ seems to become energy independent. We believe that this division may be artificial. We suggest a unified treatment of $\langle Q_\perp^2 \rangle$ at high and low energy and present phenomenological evidence to support our assertion. The framework we propose is that of soft QCD radiation averaged over the hadronic matter coordinates. A hadronic cross-section can generally be written as

$$\sigma^{\text{hadronic}} \simeq \sum \int \ldots \int dx_1\, dx_2\, f_i(x_1) f_j(x_2)\, \hat{\sigma}_{ij \to \text{final}}(x_1,x_2;K)\, d^4\mathscr{P}(K)\, D_{\text{obs}}(p_1,\ldots p_n)$$

where the parton-parton cross-section has been factorized into the product of a soft gluon bremsstrahlung distribution $d^4\mathscr{P}(K)$ and an "exclusive" cross-section $\hat{\sigma}_{ij \to \text{final}}$, in which soft gluons do not appear any more. This product is then integrated over the initial parton distributions $f_i(x)$ on the one hand and over the observed final state on the other. Thus for the multiplicity distribution, $D_{\text{obs}}(p_1,p_2,\ldots p_n)$ is related to the n-pion fragmentation function, while for a lepton pair will just be a δ-function in the pair 4-momentum. The distribution $d^4\mathscr{P}(K)$ can be calculated using soft summation techniques.

These techniques have been applied recently /5,6/ to study the mean square transverse momentum $\langle Q_\perp^2 \rangle$ of lepton pairs in the Drell-Yan process. In doing so, one necessarily enters the confinement region and one needs to specify the QCD coupling constant $\alpha_s(k^2/\Lambda^2)$ for small k. Phenomenologically, this is achieved /6,3/ by introducing extra parameters in describing subasymptotic α_s and adding an "intrinsic" transverse momentum for the partons in the hadrons. Most of these approaches employ factorization of the cross-section into its longitudinal and transverse phase space. This introduces further arbitrariness in the energy scale.

In our approach, the sum rule

$$\langle Q_\perp^2(x_1, x_2; s) \rangle = \frac{4s/(3\pi) \int_{x_1}^{1} dy_1 \int_{x_2}^{1} dy_2 \, \alpha_s \left\{ (y_1-x_1)(y_2-x_2)s/\Lambda^2 \right\} \mathscr{F}^{(Q^2)}(y_1,y_2)}{\mathscr{F}^{(Q^2)}(x_1,x_2)}$$

where $\mathscr{F}^{(Q^2)}(x_1, x_2)$ is related to the hadronic cross-section, is obtained without using factorization and is used to predict the mean value of lepton pairs produced in πN as well as pN, from very low s values up to the ISR data points. The constant, "intrinsic", part is obtained using a singular α_s. This can be seen most simply if we take the extreme behaviour

$$\alpha_s(k^2/\Lambda^2) \sim \delta(k^2/\Lambda^2)$$

The sum rule then gives

$$\langle Q_\perp^2 \rangle \sim \Lambda^2$$

i.e. a constant contribution. A physical justification for this procedure is that, if confinement is indeed responsible for the generation of an intrinsic transverse momentum, then it is a reasonable idea to choose a "confining" α_s, i.e. one which would produce, through its catastrophic infrared (IR) behaviour, a physical spectrum consisting of hadrons and not quarks.

In sections 2 and 3 we derive the transverse momentum sum rules using the 4-dimensional soft gluon bremsstrahlung formula /7/ which has already been used successfully to explain mean scaling, KNO function and various transverse and longitudinal distributions in strong interactions. It is important to notice that i) the derivation does not need factorization into longitudinal and transverse phase space, and ii) that the summation technique may be applied even for IR singular (but integrable!) α_s. In section 4, we discuss how to select an α_s so that asymptotically linear Regge trajectories might appear. This expression for α_s is then used in conjunction with the sum rules in section 5 where a comparison with experimental data is presented. For this analysis, available πN data are most suitable since both q and $\bar{q}$ producing the photon are valence and thus, in the spirit of the parton model, ought to be collinear. In fact we find that we obtain very good agreement with the experimental data. No intrinsic transverse momentum is added. The situation for pN scattering is more complicated: $\bar{q}$ in this case is from the "sea" and hence it is unclear whether we can assume it to be truely collinear (also in certain mass ranges Compton graphs might be sizeable /8/). Thus data from the collider would be very interesting to test our ideas. Notice that the few available ISR data already confirm our analysis. In section 6, we compare our present phenomenological analysis with a previous one where an s-dependent and non-singular α_s was used. The results may illustrate why analysis of different processes give different values for Λ.

The paper ends with a conclusion and an appendix where we have developed the cumbersome derivation for $< Q_\perp^4 >$.

2. - THE HADRONIC CROSS-SECTION AND THE SOFT QCD RADIATION FORMULA

As stated in the introduction, we consider the soft QCD radiation formula extended to the so called non-perturbative region. To justify our procedure, we shall briefly state the main points of the derivation of the exponentiated formula in the no-recoil approximation. We will then use it to derive the sum rules in the next section.

We start by noticing that in the short distance scattering of two partons an indefinite number of soft gluons is emitted. In the no-recoil approximation, the cross-section for the process

$$q\bar{q} \longrightarrow l\, l' + X \text{ (gluons)} \tag{1}$$

where l and l' are a lepton pair of total 4-momentum Q, can be factorized

$$d\sigma^{QCD}(Q; K) \simeq d^4\mathscr{P}(K;\epsilon)\, d\sigma_\epsilon\,(Q)$$

where $d^4\mathscr{P}(K;\epsilon)$ is the probability of a 4-momentum loss in d^4K and $d\sigma_\epsilon$ (Q) differs from the lowest order perturbation theory by terms of order α_s, with the double logarithms all included in $d^4\mathscr{P}$. ϵ is an energy cut-off representing the energy up to which the no-recoil approximation is valid. As pointed out by Weinberg for the case of infrared photons and gravitons /9/, this quantity is an ultraviolet cut-off such that a change from ϵ to ϵ' just renormalizes $d\sigma_\epsilon$ (Q) by a factor $(\epsilon / \epsilon')^B$ with B proportional to the coupling constant. Weinberg points out that one can even choose

$$d\sigma_\epsilon \simeq d\sigma_o$$

i.e. to ignore all radiative corrections, but then ϵ should be fixed equal to some typical mass in the reaction. In the following we shall indeed choose to ignore all ultraviolet corrections to $d\sigma_o$. This simplifies the calculation, but mostly it allows to define very clearly what is the full contribution in the no-recoil approximation. It will also be clear from the calculation which follows, how the average over the hadronic matter coordinates produces the appropriate value for ϵ.

The cross-section $d\sigma^{had}$ which is measured in a hadronic experiment can be expressed in terms of $d\sigma^{QCD}$ as follows:

$$d\sigma^{had} = \int \dots \int dy_1\, dy_2\, G(y_1,y_2)\, d\sigma^{QCD}\ (Q;K)$$

where $G(y_1,y_2)$ is related to the probability that a given hadronic state contribute a $q\bar{q}$ pair of fractional energies y_1 and y_2. We thus have

$$\frac{d\sigma^{had}}{d^4Q} = \int dy_1 \int dy_2\, G(y_1,y_2)\, \frac{d^4\mathscr{P}(K)}{d^4K}\, d^4K\, \frac{d\sigma_o}{d^4Q}$$

where we have suppressed the ϵ-dependence in $d^4\mathscr{P}(K)$ to simplify the notation. To zeroeth order in α_s, we can write

$$\frac{d\sigma_o}{d^4Q} = \sigma_{born}\, \delta^4(P - K - Q)$$

with $P^\mu \equiv \sqrt{s}/2(y_1 + y_2, \vec{0}, y_1 - y_2)$ and $Q^\mu \equiv (Q_o, \vec{Q}_\perp, Q_3)$, and where σ_{born} is the cross-section for

$$q\bar{q} \rightarrow l\, l'$$

i.e. for process (1) without any gluon in the final state. We thus have

$$\frac{d\sigma^{had}}{d^4Q} = \int dy_1 \int dy_2\, G(y_1,y_2) \int d^4\mathscr{P}(K)\, \delta^4(P-K-Q)\, \sigma_{born}(Q)$$

$$= \sigma_{born}(Q) \int dy_1 \int dy_2 G(y_1,y_2)\, F(P-Q) \qquad (2)$$

with

$$F(K) = d^4\mathscr{P}(K)/d^4K \qquad (3)$$

The probability $d^4\mathscr{P}(K)$ is determined by applying the methods of statistical mechanics to a Bloch-Nordsieck type model for the emission of soft gluons. This model states that as long as the recoil effect of the emitted radiation on the emitting particles can be neglected, the distribution of emitted massless quanta can be taken to be Poissonian, i.e.

$$P(\{n_k\}) = \prod_k \frac{\bar{n}_k^{\,n_k}}{n_k!} \exp(-\bar{n}_k)$$

where n_k is the number of gluons which are emitted with momentum $\vec{k}$, $\bar{n}_k$ is the average value of the number of massless quanta. $P(\{n_k\})$ is the probability of process (1) ending up with n_{k_1} gluons of momentum k_1, n_{k_2} gluons with momentum k_2, etc. In the soft limit, we of course do not observe individual distributions, but only a 4-momentum loss K. The probability of observing this loss is given by

$$d^4\mathscr{P}(K) = \Sigma\, P(\{n_{k'}\})\, \delta^4(K-\Sigma k'\, n_{k'})\, d^4K$$

where the sum is carried over all the values of the $n_{k'}$. The four dimensional δ-function selects the distributions $\{n_{k'}\}$ with the right energy momentum loss K. The sum over the distributions $\{n_k\}$ can be carried out introducing a four-vector selector variable, so that δ can be replaced by

$$\delta^4(K-\Sigma k'\, n_{k'}) = (2\pi)^{-4} \int d^4x\ \exp\left(i(K-\Sigma k'\, n_{k'})\cdot x\right)$$

One can in this way invert the order between forming the product in $P(\{n_k\})$ and the summation over all the distributions. The sum over the n_k can then be easily carried out and one obtains /10/

$$d^4\mathscr{P}(K) = (2\pi)^{-4}\, d^4K \int d^4x\, \exp\left(iK\cdot x - h(x)\right) \qquad (4)$$

with

$$h(x) = \sum_k \bar{n}_k ((1 - \exp(-ik\cdot x)) \qquad (5)$$

We have gone through this rather well known derivation to point out that Eq. (4) is not a perturbative result and that the exponentiation of the function h(x), which is related to the single gluon spectrum, follows only from the hypothesis of individual soft gluon distributions which are Poissonian and which obey an overall energy-momentum conservation constraint.

In continuous notation, Eq. (5) now becomes

$$h(x;\epsilon) = \int_0^{\epsilon} d^3\bar{n}(k)\ (1-\exp(-i\,k\cdot x))$$

where $d^3\bar{n}(k)$ is the average number of gluons of momentum $\vec{k}$. To first order in α_s, one has

$$d^3\bar{n}(k) \simeq \left(\frac{C_F}{\pi}\right)\frac{dk_\perp^2}{k_\perp^2}\frac{dk_{/\!/}}{k_{/\!/}}\alpha_s\left(\frac{k^2}{\Lambda^2}\right) \simeq \frac{C_F}{\pi}\frac{dk_+\,dk_-}{k_+k_-}\alpha_s\left(\frac{k_+k_-}{\Lambda^2}\right) \qquad (6)$$

where $k_\pm = k_o \pm k_3$ and $C_F=4/3$. We consider α_s as a function of $k_\perp$ /11/ with massless gluons for which $k_\perp^2 = k_+k_-$.

Eq. (6) exibits the usual infrared singularity in the number of massless quanta, in addition to a possible singularity coming from α_s in the $k_\perp \rightarrow 0$ region. However it should be noted that, as long as α_s is not more singular than $k_\perp^{-2p}$ with $p<1$, the function h(x) is still finite. Thus there is a priori no reason to limit the validity of Eqs. (4) and (5) to the perturbative region where α_s is small. In fact, the (IR) small $k_\perp$ region is particularly emphasized since here the probability for gluon emission is higher as α_s is larger.

Now we proceed to define the cross-section by integrating Eq. (2) in the transverse momentum

$$\frac{d\sigma^{had}}{dq_o\,dq_3} = \sigma_{born}(Q)\int dy_1 \int dy_2\, G(y_1, y_2)\,\mathbb{P}(P-Q) \qquad (7)$$

with

$$\mathbb{P}(K) = \int \frac{d^4\mathscr{P}(K)}{d^4K}\, d^2\vec{K}_\perp = (2\pi)^{-2}\int dt\, dx_3 \exp\left[itK_o - i\,x_3K_3 - h(t, x_3; \epsilon)\right]$$

$$= \int F(K)\, d^2\vec{K} \qquad (8)$$

and

$$h(t, x_3; \epsilon) = \int_0^\epsilon d^3\bar{n}(k)\,(1 - \exp(-it\,k + i\,x_3\,k_3))$$

We now compare Eq. (7) with the usual parametrization for the cross-section /12/ in terms of Q^2-dependent parton densities, i.e.

$$\frac{d\sigma^{had}}{dx_1\, dx_2} = \sigma_{born}(Q)\, \mathscr{F}^{(Q^2)}(x_1, x_2) \qquad (9)$$

with $x_1\, x_2 = Q^2/s$ and $x_1 - x_2 = 2Q_3/\sqrt{s}$. We identify

$$\mathscr{F}^{(Q^2)}(x_1, x_2) = (s/2)\int_{x_1}^1 dy_1 \int_{x_2}^1 dy_2\, G(y_1, y_2)\,\mathbb{P}(P-Q) \qquad (10)$$

An inspection of Eq. (8) shows that if we send $\alpha_s \to 0$, i.e. $h(x) \to 0$, one gets

$$(s/2)\ \mathbb{P}(P-Q) \xrightarrow[\alpha_s \to 0]{} (s/2)\,\delta(P_o - Q_o)\,\delta(P_3 - Q_3)$$

$$\longrightarrow \delta(y_1 - x_1)\,\delta(y_2 - x_2)$$

thus giving

$$\mathscr{F}^{(Q^2)}(x_1, x_2) \xrightarrow[\alpha_s \to 0]{} G(y_1, y_2) \qquad (11)$$

This checks the normalization of the function $G(y_1, y_2)$. For the Drell-Yan process, we shall have

$$G(y_1, y_2) = \sum_i f_i(y_1)\,\bar{f}_i(y_2)\,e_i^2 \qquad (12)$$

and

$$\sigma_{born} = 4\pi\alpha^2/(3Q^2)$$

3. - THE TRANSVERSE MOMENTUM SUM RULES

Now we calculate $\langle Q_\perp^{2n} \rangle$ from Eq. (2), for n=1,2,3. We start from the basic definition:

$$\langle Q_\perp^{2n} \rangle (x_1, x_2; s) \int \frac{d\sigma^{had}}{d^4Q} d^2\vec{Q} = \int Q_\perp^{2n} \frac{d\sigma^{had}}{d^4Q} d^2\vec{Q}_\perp$$
$$= \sigma_{born}(Q^2) \int dy_1 \int dy_2\, G(y_1, y_2) \int Q_\perp^{2n} d^2\vec{Q}_\perp\, F(P-Q) \qquad (13)$$

It is rather straightforward to show that

$$\int K_\perp^2 d^2\vec{K}_\perp\, F(K) = \int k_\perp^2 d^3\bar{n}(k)\, \mathbb{P}(K-k) \equiv \Pi_2(K) \qquad (14)$$

$$\int K_\perp^4 d^2\vec{K}_\perp\, F(K) = 2\int k_\perp^2 d^3\bar{n}(k) \int k_\perp'^2 d^3\bar{n}(k')\, \mathbb{P}(K-k-k')$$
$$+ \int k_\perp^4 d^3\bar{n}(k)\, \mathbb{P}(K-k) \qquad (15)$$

The above relations follow from repeated applications of the partial integration rule.

Below we show in some detail the steps leading to the sum rule for $\langle Q_\perp^2 \rangle$. Those for $\langle Q_\perp^4 \rangle$ are more cumbersome and hence are relegated to the Appendix. Factorization of the QCD radiation into its light cone and transverse momentum part is not assumed. Previously, in Ref. /5/, we had derived the same sum rule for $\langle Q_\perp^2 \rangle$ using a power-law for $\mathbb{P}(K)$. We now find this not to be necessary.

Using the analyticity of $\mathbb{P}(K)$, we see from Eq. (8) that

$$\mathbb{P}(K) = 0 \qquad \text{for} \qquad K_\pm \simeq \sqrt{s}\,(y_{1,2} - x_{1,2}) \leqslant 0$$

Hence, Eqs. (6) and (14) lead to

$$\Pi_2(K) = \frac{C_F}{\pi} \int_0^{K_+} dk_+ \int_0^{K_-} dk_-\, \alpha_s\left(\frac{k_+ k_-}{\Lambda^2}\right) \mathbb{P}(K_+ - k_+, K_- - k_-; \epsilon) \qquad (16)$$

where we have made explicit the dependence upon the light cone variables. To compute $\langle Q_\perp^2 \rangle$ from Eq. (13), we need

$$R_2 = \int_{x_1}^{1} dy_1 \int_{x_2}^{1} dy_2 \; G(y_1, y_2) \; \Pi_2(K)$$

which, upon using Eq. (16), may be rewritten as

$$R_2 = \frac{C_F s}{\pi} \int_{x_1}^{1} dy_1 \int_{x_2}^{1} dy_2 \; G(y_1, y_2) \int_{o}^{y_1 - x_1} dz_1 \int_{o}^{y_2 - x_2} dz_2 \; \alpha_s(z_1 z_2 s/\Lambda^2) \, .$$

$$\mathbb{P}(\sqrt{s}(y_1 - x_1 - z_1), \sqrt{s}(y_2 - y_2 - z_2) \, ; \, \sqrt{s \; y_1 \; y_2}/2)$$

Here, for definiteness, we have set $\epsilon = \sqrt{s \, y_1 \, y_2}/2$, i.e. we define ϵ to be the CM energy of the $(q\bar{q})$ system. Now a simple change of variable and order of integration gives

$$R_2 = (C_F s/\pi) \int_{x_1}^{1} dt_1 \int_{x_2}^{1} dt_2 \; \alpha_s((t_1 - x_1)(t_2 - x_2) \, s/\Lambda^2).$$

$$\cdot \int_{t_1}^{1} dy_1 \int_{t_2}^{1} dy_2 \; G(y_1, y_2) \, \mathbb{P}(\sqrt{s}(y_1 - t_1), \sqrt{s}(y_2 - t_2) \, ; \sqrt{s \, y_1 y_2}/2)$$

This last expression can be considerably simplified in terms of the Q^2 dependent parton densities, Eq. (10):

$$R_2 = (2C_F/\pi) \int_{x_1}^{1} dt_1 \int_{x_2}^{1} dt_2 \; \alpha_s((t_1 - x_1)(t_2 - x_2) \, s/\Lambda^2) \, \mathscr{F}^{(Q^2)}(t_1, t_2) \tag{17}$$

Returning to Eq. (13) and substituting for the cross-section on the left hand-side, using Eqs. (9) and (10), we obtain from Eq. (17)

$$\langle Q_\perp^2 (x_1, x_2; s) \rangle = \frac{(C_F s/\pi) \int_{x_1}^{1} dy_1 \int_{x_2}^{1} dy_2 \; \alpha_s \; ((y_1 - x_1)(y_2 - x_2) s/\Lambda^2) \; \mathscr{F}^{(Q^2)}(y_1, y_2)}{\mathscr{F}^{(Q^2)}(x_1, x_2)} \tag{18}$$

This is our sum rule for the mean square transverse momentum of lepton pairs as a function of x_1, x_2 and s. If one requires the same at fixed τ, a (partial) integration over $x_{1,2}$ may be performed to obtain

$$\langle Q_\perp^2(\tau, s)\rangle = \frac{\int_\tau^1 \frac{dx}{x}\langle Q_\perp^2(x, \tau/x\,; s)\rangle \mathscr{F}^{(Q^2)}(x, \tau/x)}{\int_\tau^1 \frac{dx}{x}\mathscr{F}^{(Q^2)}(x, \tau/x)} \tag{19}$$

As shown in the Appendix, the fourth moment sum rule reads

$$\langle Q_\perp^4(x_1,x_2;s)\rangle = s^2\,(C_F/\pi)\left[\mathscr{F}^{(Q^2)}(x_1, x_2)\right]^{-1}\cdot$$

$$\left[\int_{x_1}^1 dt_1\int_{x_2}^1 dt_2 (t_1-x_1)(t_2-x_2)\ \ \alpha_s((t_1-x_1)(t_2-x_2)\, s/\Lambda^2)\mathscr{F}^{(Q^2)}(t_1,t_2) +\right.$$

$$+ (2C_F/\pi)\int_{x_1}^1 dt_1\int_{x_2}^1 dt_2\ \ \alpha_s((t_1-x_1)(t_2-x_2)s/\mathscr{F}^2)\,.$$

$$\left.\cdot\int_{t_1}^1 dw_1\int_{t_2}^1 dw_2\ \ \alpha_s((w_1-t_1)(w_2-t_2)s/\Lambda^2)\,\mathscr{F}^{(Q^2)}(w_1, w_2)\right] \tag{20}$$

The general pattern of the sum rules for higher moments ($\langle Q_\perp^6\rangle$, etc.) is quite clear from Eq. (20). They are given by successive foldings of α_s with the parton densities. The expressions are somewhat complicated and so we do not reproduce them here.

4. - CHOOSING α_s

As mentioned in the introduction, we seek a phenomenological α_s which reflects confinement. Let us choose

$$\alpha_s(k^2/\Lambda^2) = p\left[b_o \ln\ (1+p(\frac{k^2}{\Lambda^2})^p)\right]^{-1} \tag{21}$$

where b_o=25/(12π) (for 4 flavours) and p is a constant to be fixed shortly. For large k^2, we recover the asymptotic freedom limit. If p=1, we obtain Richardson's expression /13/, if p=0 we get a constant α_s. Following Ref. /13/ we can compute the growth of energy E(r) as a function of the distance r between the $q\bar{q}$ pair. For large r, $E(r)\sim r^{-1+2p}$. The parameter p can now be related to the asymptotic form of the Regge spectrum through an argument due to Polyakov /14/ used in a somewhat related context. For large angular momentum J, and large r, we have

$$E_J(r) \sim C_1\, r^{-1+2p} + C_2\, J^2/r^2 \qquad (22)$$

This leads to the mass spectrum

$$M^2(J) \sim J^{4(2p-1)/(2p+1)} \qquad (23)$$

Asymptotically linearly rising Regge trajectories can therefore be obtained provided p=5/6. We shall use Eq. (21) for all of our analysis to follow. In Fig. 1 we show a comparison of α_s as given by Eq. (21) with the asymptotic freedom expression

$$\alpha_s^{AF}(k^2/\Lambda^2) = \left[b_o \ln(k^2/\Lambda^2) \right]^{-1}$$

for a value of Λ=0.1 GeV. The justification for such a value for Λ can be found in section 5, where we do phenomenological analysis of DY pair transverse momentum.

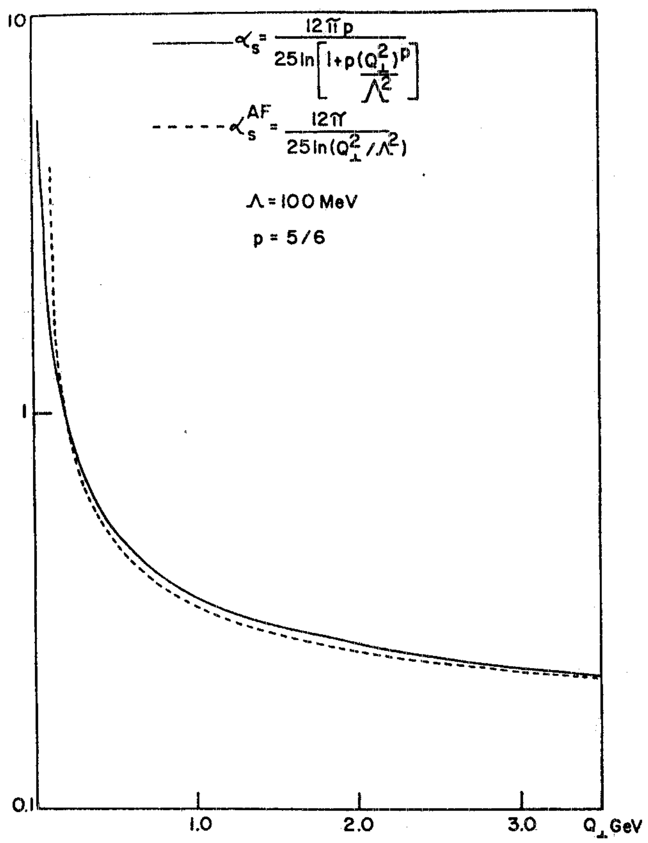

FIG. 1 - Comparison of our α_s with that given by the asymptotic freedom formula.

Notice that

$$\alpha_s(k^2/\Lambda^2) \simeq \alpha_s^{AF}(k^2/\Lambda^2) \qquad \text{for } k \gtrsim 2\Lambda$$

i.e. that our expression differs from the asymptotic freedom one only for distances $r \gtrsim 1$ Fermi.

It is pleasing that such a simple functional form for α_s like the one we propose is able to incorporate the stringent requirements of asymptotic freedom and approximately linear mass spectrum.

5. - COMPARISON WITH EXPERIMENT

Our model contains two parameters, p and Λ, and needs the parton distributions. The parameter p has already been fixed to be 5/6 in section 4. Since we do not introduce ad hoc intrinsic transverse momenta, we are left with only one parameter Λ.

Fig. 2a shows how the parameter Λ controls the mean square transverse momentum, $\langle Q_\perp^2 \rangle$, of di-muon for π^- beam /15,16,17/. We will adopt the value 0.1 GeV for Λ in the following analysis. It is very interesting that this value is very near to the values determinated by the analyses of e^+e^- /18 / and deep inelastic scattering /19/. This suggests strongly that our extrapolation of the form for $\alpha_s(Q^2)$ into the soft region is reasonable.

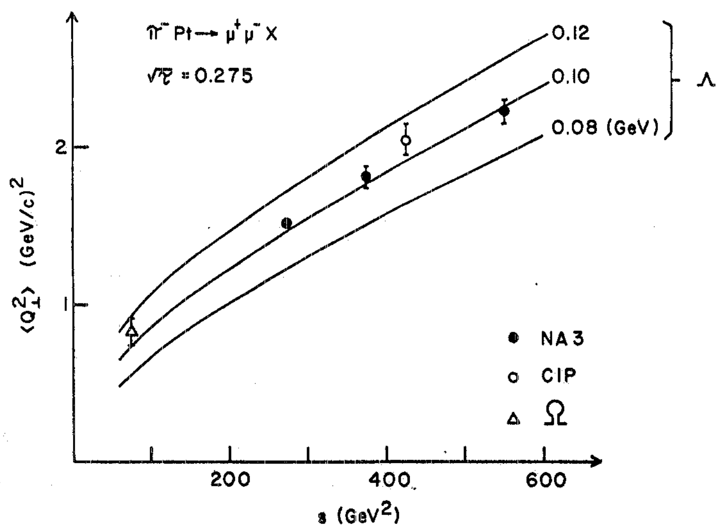

FIG. 2a - $\langle Q_\perp^2 \rangle$ vs s for the muon-pair in π^-Pt reaction for three different Λ(0.08, 0.10 and 0.12 GeV). p = 5/6. Parton distributions are taken from Ref. /12/. The experimental data are taken from Refs. /15,16,17/ and /21/.

In Fig. 2b, we show the energy dependence of $\langle Q_\perp^2 \rangle$ for p beam /20,21/. It is more complicated to compare our model with these data because i) in pN reaction a Compton-type diagram, which we have not included, may play an important role, ii) the results are sensitive to the sea quark distributions which are not so reliable as those of valence quarks and iii) the data are less extensive than for πN. Considering these facts, we see that the theoretical values are quite reasonable. The values of $\langle Q_\perp^2 \rangle$ of p beam are less than those of π-beam because mainly slow partons contribute to the production of transverse momenta in the former case.

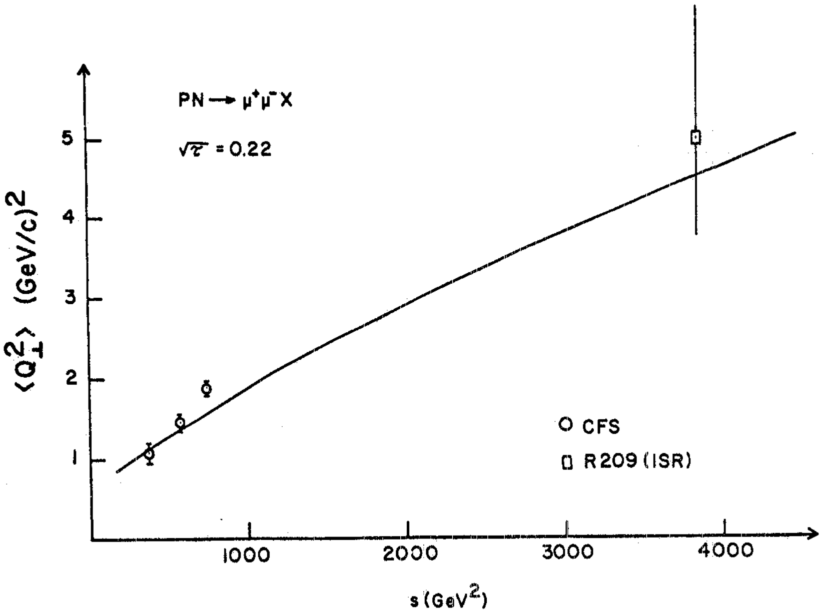

Fig. 2b - $\langle Q_\perp^2 \rangle$ vs s for pN reaction. $\Lambda = 0.1$ GeV, p = 5/6. The evoluted parton densities from Ref. /22/ are employed. The experimental data are taken from Refs. /20/ and /21/.

In Table I, we present calculated values of $\langle Q_\perp^2 \rangle$ for $\pi^\pm$, p and $\bar{p}$ beams along with the experimental data. τ distributions of $\langle Q_\perp^2 \rangle$ for lepton pairs for π-beam are plotted in Fig. 3a-d for several energies. We have averaged over rapidity y according to the experiments.

In the above analyses we neglected, for simplicity, the Q^2 dependence of the parton densities and used the NA3 parametrization /12/ except in Fig. 2b. We have calculated the curve in Fig. 2b using evoluted parton density à la Glück, Hoffman and Reya /22/.

Next let us go to discuss the $Q_\perp$-distribution. For this purpose, we employ an

TABLE I - $\langle Q_\perp^2 \rangle$ for different reactions with Λ = 0.1 GeV and p = 5/6. Parton distributions are from Ref. /12/. The results shown are obtained upon averaging over the rapidity interval: $0 \leq y \leq 0.4$ at M = 4.4 GeV. The experimental data are taken from Ref. /16/.

P_{lab}		π^-Pt	π^+Pt	pPt	$\bar{p}$Pt	π^-p
150 GeV/c	Exp.	1.44±0.03	1.25±0.21	0.89±0.25	1.26±0.11	1.55±0.13
	Th	1.49	1.25	0.80	1.17	1.60
200 GeV/c	Exp	1.74±0.04	1.76±0.08	1.38±0.07	1.43±0.28	1.73±0.13
	Th	1.69	1.39	0.92	1.36	1.83

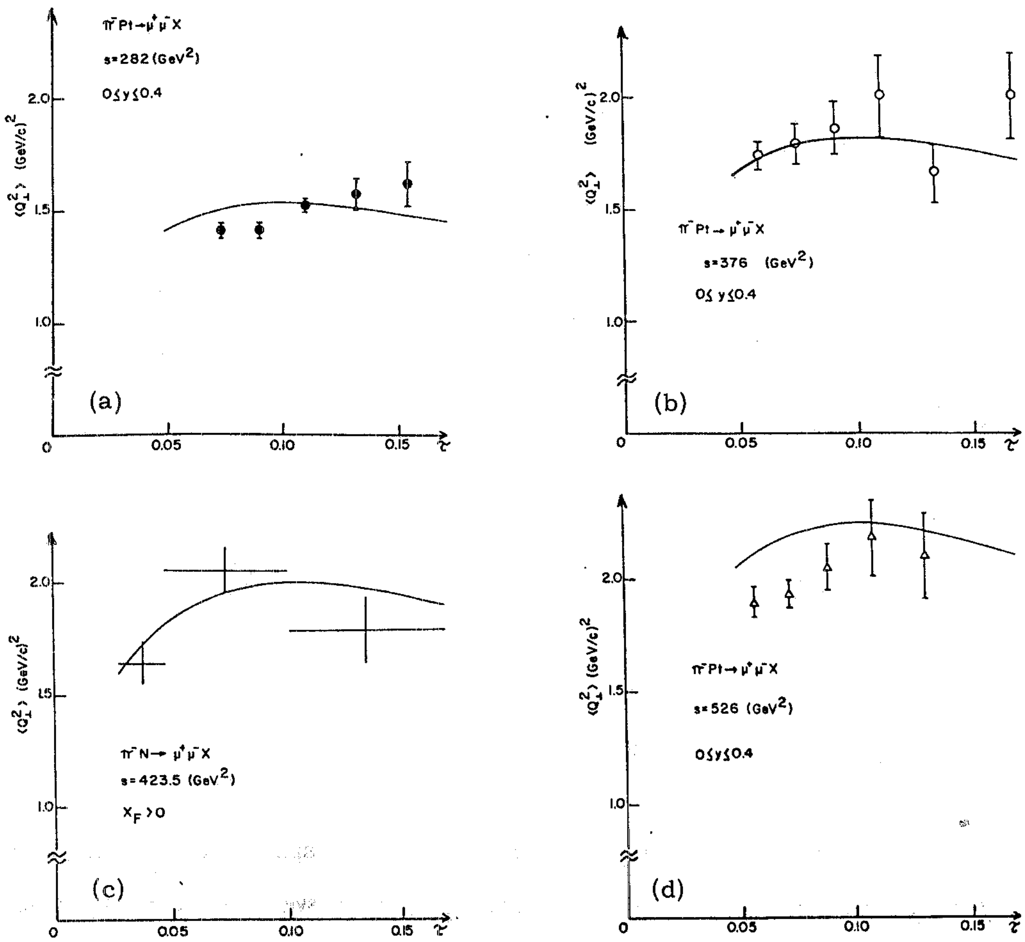

FIG. 3 - $\langle Q_\perp^2 \rangle$ vs τ for π-beam at s = 282, 376, 424 and 526 GeV². Λ = 0.1 GeV ; p = 5/6. Parton distributions are from Ref. /12/. The experimental data are taken from Ref. /16/ for Figs. (a), (b) and (d) end from Ref. /17/ for Fig. (c).

approximation, i.e. the factorization:

$$\frac{d\sigma}{dy\, dM\, dQ_\perp^2} = \frac{d\sigma}{dy\, dM} \cdot \frac{d\mathscr{P}}{dQ_\perp^2}$$

Under this assumption, the $Q_\perp$ -distribution is given by

$$\frac{d\mathscr{P}}{dQ_\perp^2} = \frac{1}{2}\int_o^\infty b\, db\, J_o(bQ_\perp)\exp(-\Delta(b,E)) \tag{25a}$$

where

$$\Delta(b,E) = (4/3\pi)\int_o^{E^2} (dq^2/q^2)\,\alpha_s(q^2/\Lambda^2)\ln\left[(E+\sqrt{E^2-q^2})/(E-\sqrt{E^2-q^2})\right]\cdot\left[1-J_o(bq)\right] \tag{25b}$$

Up to 3rd moment of $Q_\perp^2$, the distribution can be written as

$$\begin{aligned}\frac{d\mathscr{P}}{dQ_\perp^2} = (4C_2)^{-1}\exp\left[-Q_\perp^2/4C_2\right]\cdot\Big[&1+(C_4/C_2^2)(2-Q_\perp^2/C_2+Q_\perp^4/16\,C_2^2)\\ &-(C_6/C_2^3)(6-9Q_\perp^2/2C_2+9Q_\perp^4/16C_2^2-Q_\perp^6/64C_2^3)\\ &+\left[(C_8+C_4^2/2)/C_2^4\right](24-24Q_\perp^2/C_2+9Q_\perp^4/2C_2^2-Q_\perp^6/4C_2^3+Q_\perp^8/256C_2^4)\Big]\end{aligned} \tag{26}$$

where C_n are given by

$$\Delta = C_2b^2 - C_4b^2 + C_6b^6 - C_8b^8 + \ldots\ldots$$

and they are related to the moments as

$$4\,C_2 = \langle Q_\perp^2 \rangle$$

$$64\,C_4 = \langle Q_\perp^4 \rangle - 2\langle Q_\perp^2 \rangle^2$$

In Eq. (25b), E is a parameter because, due to the approximation of factorization, there is no correlation between the longitudinal and transverse momenta. We have fixed E so that the formula (25) reproduces the same $\langle Q_\perp^2 \rangle$ as that given by the sum rule, i.e. Eq. (18). Once we fix E, we can calculate the higher moments without ambiguity.

In Fig. 4a-b, we show the transverse momentum distribution of πN data from NA3 collaboration /16/ on arbitrary scale. Though we neglect terms which depend on the

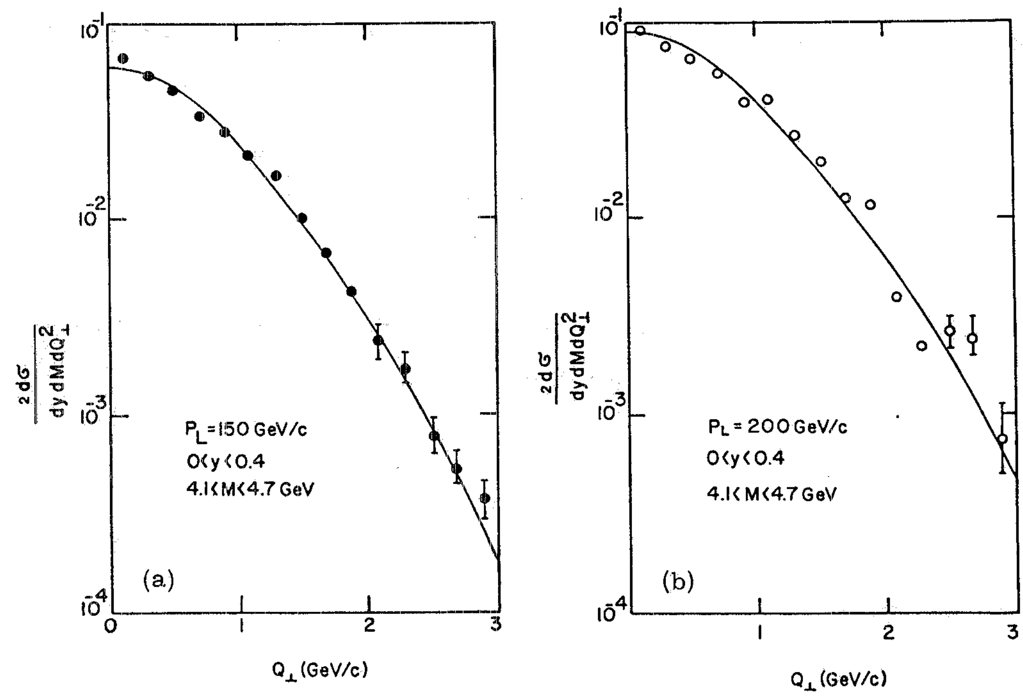

FIG. 4 - The transverse momentum distribution of muon-pair in the reaction π^-Pt at s = 282 and 376 GeV^2, Λ = 0.1 ; p = 5/6. Eq. (26) is used for the theoretical curves. The experimental data are taken from Ref. /16/.

higher moments, the formula (26) gives quite reasonable results. Note that usually /2,3,4,6/ the intrinsic momentum is introduced as an additional term in Eq. (25):

$$\Lambda \longrightarrow C_{int}\, b^2 + \Lambda$$

In contrast, with infrared divergent α_s, whose form is determined in section 4, we are able to produce satisfactory results without such additional insertions.

6. - HADRONIC AVERAGES AND THE Λ SCALE IN QCD

In the previous section we have obtained the values of the mean square transverse momentum of Drell-Yan pairs produced in a variety of hadronic processes using only one phenomenological parameter, the QCD scale Λ . p was not a parameter in the sense that it was fixed from Polyakov' argument on lineraly rising Regge trajectories /14/.

The value of Λ which best fits the data is

$$\Lambda \simeq 100 - 120 \text{ MeV}$$

We would like to compare this value with the one which describes the data if Eq. (18) is written as

$$\langle Q_\perp^2 (x_1, x_2; s) \rangle = (C_F s/\pi)\, \bar{\alpha}_s \quad \frac{\int_{x_1}^{1} dy_1 \int_{x_2}^{1} dy_2 \mathcal{F}^{(Q^2)}(y_1, y_2)}{\mathcal{F}^{(Q^2)}(x_1, x_2)} \tag{27}$$

This type of analysis was followed in Ref. /5/, its main justification being our lack of knowledge of α_s in the infrared region. Using for $\bar{\alpha}_s$

$$\bar{\alpha}_s = \left[(b_o \ln(s/\Lambda^2_{had}) \right]^{-1}$$

and NA3 parametrization for the parton densities, it was found that the NA3 data points for the quantity

$$\frac{\langle Q_\perp^2 \rangle - \langle Q_\perp^2 \rangle_{intrinsic}}{s}$$

were fitted by

$$\Lambda_{had} \simeq 300 \text{ MeV}$$

This discrepancy, like similar discrepancies in other determinations of Λ, can be understood if one notices that $\bar{\alpha}_s$ used in Eq. (27) is the hadronic average of α_s as used in Eq. (18). Comparing these two equations, in fact, we obtain

$$\bar{\alpha}_s = \frac{\int_{x_1}^{1} dy_1 \int_{x_2}^{1} dy_2\, \alpha_s \left[(y_1 - x_1)(y_2 - x_2)\, s/\Lambda^2 \right] \mathcal{F}^{(Q^2)}(y_1, y_2)}{\int_{x_1}^{1} dy_1 \int_{x_2}^{1} dy_2\, \mathcal{F}^{(Q^2)}(y_1, y_2)}\ .$$

i.e. we find that $\bar{\alpha}_s$ is the value of α_s averaged over the cross-section characterizing a given process. Thus lambda depends not only from the process, but at low energy from the infrared behaviour, whose characteristics we have incorporated in the parameter p. At large energies, on the other hand we can neglect the contribution from the IR region and simbolically write

$$\ln (s/\Lambda^2_{had}) \simeq \langle \ln (s(y_1 - x_1)(y_2 - x_2)/\Lambda^2) \rangle_{had}$$

i.e.

$$\Lambda^2_{had} \simeq \Lambda^2 / < (y_1 - x_1)(y_2 - x_2) >_{had}$$

We conjecture that in general one can write

$$\Lambda_{had} = \Lambda / \langle y - x \rangle_{had}$$

where $\langle 0 \rangle_{had}$ represents average over initial and (eventually) final hadronic states, with the microscopic cross-section as weight function.

7. - CONCLUSIONS

We conclude by symmarizing the two central results of this paper. The first is a derivation of sum rules for the means ($< Q_\perp^2 >$, $< Q_\perp^4 >$, etc.) of transverse momentum of dileptons (with minor modification it also applies to diphotons) reproduced in hadronic collisions. This was done through a folding of the appropriate parton densities with the soft gluon distribution. The other important point concerns the assertion - and partial confirmation through phenomenological analysis - that an IR singular coupling constant can be employed to recover also the so-called "intrinsic" piece of transverse momentum of partons. This approach allows one to construct accurate transverse momentum distributions given only the (x-distribution of the) parton densities. The success of this program in predicting the entire $\langle Q_\perp^2 \rangle$ distribution has led us to investigate how good is the "potential" constructed - for heavy quarks at least - through our α_s. The results, while interesting and not in contradiction with experimental facts, are outside the scope of this paper and will be reported elsewhere.

Acknowledgments

We would like to thank M. Greco and G. Parisi for useful discussions.

APPENDIX

Derivation of the Sum Rule for $< Q_\perp^4 (x_1, x_2; s) >$

From Eqs. (13) and (15) of the text, we have

$$< Q_\perp^4 (x_1, x_2; s) > = (s/2) \quad R_4 / \mathscr{F}^{(Q^2)}(x_1,x_2) \tag{A1}$$

where

$$R_4 = \int_{x_1}^{1} dy_1 \int_{x_2}^{1} dy_2 \; G(y_1, y_2) \left\{ \int k_\perp^4 \, d^3 \bar{n}(k) \, \mathbb{P}(K-k) + + 2 \int k_\perp^2 \, d^3 \bar{n}(k) \int k'^2_\perp \, d^3 \bar{n}(k') \, \mathbb{P}(K-k-k') \right\} \equiv R_{41} + R_{42} \tag{A2}$$

Calculation of R_4 proceeds almost identically to that of R_2 and one readily obtains

$$R_{41} = s^2 (C_F/\pi) \int_{x_1}^{1} dt_1 \int_{x_2}^{1} dt_2 (t_1-x_1)(t_2-x_2) \, \alpha_s((t_1-x_1)(t_2-x_2)s/\Lambda^2)(2/s) \mathscr{F}^{(Q^2)}(t_1,t_2) \tag{A3}$$

The remaining term R_{42} reads

$$R_{42} = 2(C_F/\pi)^2 \int_{x_1}^{1} dy_1 \int_{x_2}^{1} dy_2 \, G(y_1, y_2) \cdot \int_{o}^{K_+} dk_+ \int_{o}^{K_-} dk_- \, \alpha_s(k_+ k_- / \Lambda^2) \int_{o}^{K_+ - k_+} dk'_+ \int_{o}^{K_- - k_-} dk'_- \; \alpha_s(\frac{k'_+ k'_-}{\Lambda^2}) \, \mathbb{P}(K-k-k') \tag{A4}$$

Once again changing variables and order of integration we obtain

$$R_{42} = 2s^2 (C_F/\pi)^2 \int_{x_1}^{1} dt_1 \int_{x_2}^{1} dt_2 \, \alpha_s((t_1-x_1)(t_2-x_2)s/\Lambda^2) \int_{t_1}^{1} dw_1 \int_{t_2}^{1} dw_2 \, \alpha_s((w_1-t_1)(w_2-t_2)s/\Lambda^2) . \cdot \int_{w_1}^{1} dy_1 \int_{w_2}^{1} dy_2 \, G(y_1, y_2) \, \mathbb{P}(\sqrt{s}(y_1-w_1), \; \sqrt{s}(y_2-w_2) \, ; \, \sqrt{s \, y_1 \, y_2}/2)$$

which may be converted in terms of running parton densities, Eq. (10), to become

$$R_{42} = 2s^2(C_F/\pi)^2 \int_{x_1}^{1} dt_1 \int_{x_2}^{1} dt_2 \ \alpha_s((t_1-x_1)(t_2-x_2)s/\Lambda^2) \ . \tag{A5}$$
$$\cdot \int_{t_1}^{1} dw_1 \int_{t_2}^{1} dw_2 \ \alpha_s((w_1-t_1)(w_2-t_2)\, s/\Lambda^2)\, (2/s)\, \mathscr{F}^{(Q^2)}(w_1, w_2) \ .$$

Substituting Eqs. (A3) and (A5) into Eq. (A1) we have our final result:

$$\langle Q_\perp^4(x_1, x_2; s)\rangle = s^2(C_F/\pi) \left[\mathscr{F}^{(Q^2)}(x_1, x_2)\right]^{-1} \cdot$$
$$\cdot \left[\int_{x_1}^{1} dt_1 \int_{x_2}^{1} dt_2 (t_1-x_1)(t_2-x_2)\, \alpha_s((t_1-x_1)(t_1-y_2)s/\Lambda^2)\ \mathscr{F}^{(Q^2)}(t_1, t_2) + \right.$$
$$+ \ 2(C_F/\pi) \int_{x_1}^{1} dt_1 \int_{x_2}^{1} dt_2 \ \alpha_s((t_1-x_1)(t_2-x_2)s/\Lambda^2) \ .$$
$$\left. \cdot \int_{t_1}^{1} dw_1 \int_{t_2}^{1} dw_2 \ \alpha_s((w_1-t_1)(w_2-t_2)s/\Lambda^2)\, \mathscr{F}^{(Q^2)}(w_1, w_2)\right] .$$

This proves Eq. (20) of the text.

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