

Talk given at the  
XVIII Rencontre de Moriond,  
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THE KNO FUNCTION AND OTHER SOFT GLUON EFFECTS AT THE COLLIDER

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ABSTRACT

We calculate the effect of soft QCD radiation on the energy and transverse momentum distribution at the collider. In particular using the energy distribution we predict the shape of the KNO function and the  $s$ -dependence of KNO scaling violations. Through the transverse momentum distribution, we calculate the mean square transverse momentum of the weak bosons,  $W^\pm$  and  $Z_0$ . We find that the  $\langle Q_{\perp}^2 \rangle$  for the  $W$  is larger than that for the  $Z_0$ .

We present two applications of the soft gluon bremsstrahlung formalism which describes the soft QCD radiation emitted in high energy parton scattering. The effects of this radiation have already been studied in a variety of processes, like  $e^+e^-$  (1), Drell-Yan (2), DIS (3). It can be expected, however, that these effects are most evident at the collider (4,5) where extremely high energy particles are created and destroyed, a process which leads to the liberation of a considerable portion of the energy in form of soft QCD radiation.

To study the effect of this radiation, we consider the 4-momentum distribution of soft gluon emission, summed to all orders in  $\alpha_s$ . When integrated over the unobserved 3-momentum variables, this distribution gives the probability of observing a certain energy loss. We propose that the multiplicity distribution in  $pp$  or  $p\bar{p}$  collisions or  $e^+e^-$  annihilation directly reflect this energy distribution. This follows from the suggestion that pions in an inclusive process are always emitted softly so that the total energy they carry is mostly that of the soft gluons. Thus the shape of the KNO function can be predicted.

For a rather different application, we then turn to the 4-dimensional distribution and predict the mean square transverse momentum of the intermediate vector boson. In the quark-parton model, in fact, where the partons are all collinear, the transverse momentum distribution of a directly produced virtual  $\gamma$ ,  $W$  or  $Z_0$  is that of the soft gluon radiation emitted by the quarks before the interaction. When the soft gluon bremsstrahlung formula is integrated over the parton densities, we obtain a numerical value for  $\langle Q_{\perp}^2 \rangle \simeq 190 \text{ GeV}^2$  for  $W$  and a slightly larger value for  $Z_0$ .

In the following, Sect. 1. briefly describes the relation between mean scaling and KNO scaling. In Sect. 2. we present our results for the KNO function. In Sect. 3. we discuss the transverse momentum of the  $W$ . Due to space limitations, we present here only the results of the calculation, details can be found in Ref. (6).

## 1. - KNO SCALING AND MEAN SCALING

The original derivation of KNO scaling follows from the definition of topological cross-sections and Feynman scaling (7). KNO scaling, however, is an example of a more general behaviour known as mean scaling or scaling-in-the-mean. This was first observed in 1974 by F.T. Dao et al. (8), who put forward the hypothesis that the shape of single particle distributions in the transverse and longitudinal momentum variables are independent of multiplicity and incident energy, if the distributions are plotted against the mean variables  $x/\langle x \rangle$ . These authors also noted that, when the variables are the charged and neutral multiplicities, one had the familiar KNO scaling. We suggest that mean scaling (and hence KNO scaling) can be understood in QCD as follows. A hadronic cross-section can generally be written as

$$\sigma^{\text{hadronic}} \simeq \sum \iint f_i(x_1) f_j(x_2) \hat{\sigma}_{ij \rightarrow \text{final}}(x_1, x_2; K) d^4P(K) \mathcal{D}_{\text{observed}}(p_1, \dots, p_n) \quad (1)$$

where the parton parton cross section has been factorized into the product of a soft gluon bremsstrahlung distribution  $d^4P(K)$  and an "exclusive" cross-section  $\hat{\sigma}_{ij \rightarrow \text{final}}$  in which soft gluons do not appear any more. This product is then integrated over the initial parton distributions  $f_i(x)$  on the one hand and over the observed final state on the other. Thus, for the multiplicity distribution,  $\mathcal{D}_{\text{obs}}(p_1, \dots, p_n)$  is related to the n-pion fragmentation function, while for a lepton pair it will just be a  $\delta$ -function in the pair 4-momentum<sup>(2)</sup>.

Eq. (1) now clearly illustrates how mean scaling appears. Because of soft gluon bremsstrahlung, we have a distribution function  $d^4P(K)$  which is integrated over the hadronic matter coordinates. This distribution function is given by

$$d^4P(K) = d^4K \int \frac{d^4x}{(2\pi)^4} e^{iK \cdot x} \cdot e^{-\int d^3\vec{n}(k)(1-e^{-ik \cdot x})} \quad (2)$$

where  $K$  is the 4-momentum of the emitted radiation and  $d^3\vec{n}(k)$  is the single soft gluon probability which has been exponentiated. In Eq. (2), the first term in the exponent represents soft virtual gluons (vertex corrections) while the second represents real emission. The presence of both terms ensures the cancellation of the infrared divergence, energy-momentum conservation being maintained through the term  $\exp(-ik \cdot x)$ . If we write

$$d^3\vec{n}(k) \simeq \frac{4}{3\pi} \frac{d^3\vec{k}}{k} \frac{\alpha_s(k^2)}{k^2} \quad (3)$$

we see that  $d^3\vec{n}(k)$  has an approximate scaling behavior which reflects in  $d^4P(K)$ . When we average over the hadronic matter coordinates, the microscopic scales characterizing the quark and gluon system, are turned into their mean values. This happens for both the momentum variables as well as for the energy scales. Hence mean scaling and KNO scaling follow.

## 2. - THE KNO FUNCTION

Integrating Eq. (2) in all the 3-momentum variables, one obtains the energy distribution

$$dP(K_0, \Omega) = dK_0 \int \frac{dt}{2\pi} e^{iK_0 \cdot t} \int_0^{\Omega} d^3\vec{n}(k)(1-e^{-ikt})$$

In terms of this function, the charged n-particle cross-section can be written as

$$\sigma_n^{\text{ch}(s)} = \sum_{i,j,\text{final}} \int dx_1 f_i(x_1) \int dx_2 f_j(x_2) \int d\Omega \sigma_{i,j \rightarrow \text{final}}(x_1, x_2; \Omega) \cdot \int d\Omega_{\text{vis}} \frac{dP(\Omega_{\text{vis}}, \Omega)}{d\Omega_{\text{vis}}} \delta(\Omega_{\text{vis}} - \sum_1^n \omega_i) \int d\omega_1 \dots \int d\omega_n \mathcal{D}_{\pi}^{(n)}(\Omega; \omega_1 \dots \omega_n) \quad (4)$$

with  $\Omega_{\text{vis}}$  representing the energy of the soft QCD radiation which converts into pions. The

n-particle probability distribution defined by

$$P(n,s) = \frac{\sigma_n^{\text{ch}}(s)}{\sum \sigma_n^{\text{ch}}(s)}$$

can then be approximately obtained by putting

$$P(n,s) \approx \int d\Omega_{\text{vis}} \frac{dP(\Omega_{\text{vis}}, \langle \Omega \rangle)}{d\Omega_{\text{vis}}} \delta(\Omega_{\text{vis}} - n \langle \omega \rangle)$$

i.e. by assuming that the averaging process on the hadronic coordinates corresponds to substitute all the QCD energy scales with their mean values. To do so, we take the following two steps:

i) for the total pion energy  $\sum \omega_i$  in Eq. (4), we write

$$\sum_1^n \omega_i \rightarrow n \langle \omega(s) \rangle$$

ii) for the energy scale  $\Omega$ , we let

$$\Omega \rightarrow \langle \Omega(s) \rangle$$

and then

$$\langle n(s) \rangle = \beta(s) \frac{\langle \Omega(s) \rangle}{\langle \omega(s) \rangle}$$

where  $\beta(s)$  is the soft gluon energy spectrum which can be calculated from Eq. (3) and is the maximum energy allowed by the kinematics to a single gluon. After some straightforward manipulations, one obtains

$$P(n,s) = \beta(s) \int \frac{dt}{2\pi} e^{i\beta \frac{n}{\langle n \rangle} t - \beta \int_0^1 \frac{dk}{k} (1 - e^{-ikt})} \quad (5)$$

We propose that the KNO function be given by

$$\psi\left(\frac{n}{\langle n \rangle}\right) = \langle n \rangle P(n,s).$$

This function obeys the two normalization conditions

$$\int_0^{\infty} \psi(z) dz = 1 \quad \text{and} \quad \int z dz \psi(z) = 1.$$

We can calculate the moments of this distribution. Using Eq. (5) and the definition of Refs. (4,5) we find

$$\gamma_2 = \frac{1}{2\beta}; \quad \gamma_3 = \frac{1}{3\beta^2}; \quad \gamma_4 = \frac{1}{4\beta^3}$$

i.e.

$$\gamma_2 \approx \frac{25}{32} \frac{1}{\ln \ln s/\Lambda^2} .$$

The above equation shows how KNO scaling is violated. In the asymptotic freedom region, the cumulants  $\gamma_i$  should start decreasing with increasing energy. The function  $\beta(s)$  is the same function from which scaling violations in DIS are predicted<sup>(3)</sup>.

Using  $\Lambda=100$  MeV, we calculate  $\beta$  to be approximately 1.82. In Fig. 1 we show the fit to the UA1 data<sup>(4)</sup>.

In Fig. 2 the UA1 data for  $|\eta| < 1.5$  have been fitted using  $\beta=1.1$ . The value for  $\beta$  is here strictly empirical. Its main justification lies in the fact that the selection of events in the range  $|\eta| < 1.5$  means a reduced phase space and hence a smaller  $\beta$ .

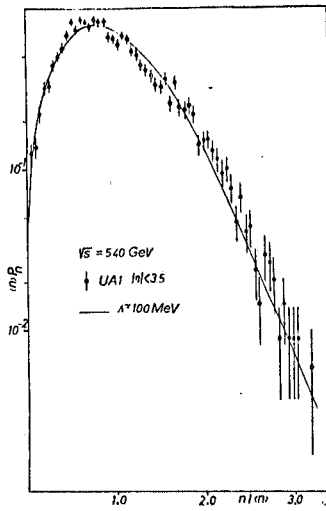


FIG. 1

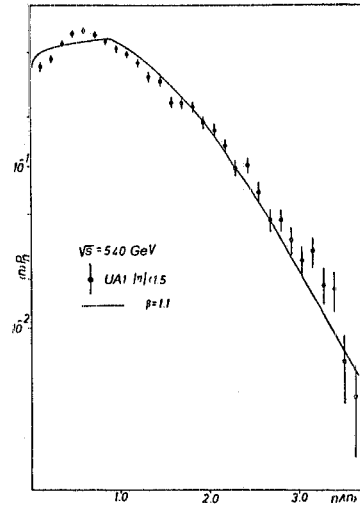


FIG. 2

### 3. - THE WEAK BOSON TRANSVERSE MOMENTUM

The work described in this section has been done in collaboration with A. Nakamura. For current processes in hadronic collisions, the following sum rule can be derived<sup>(6)</sup>:

$$\langle Q_{\perp}^2 \rangle \cdot \int_{\tau}^1 \frac{dx}{x} g(x, \tau/x) = \frac{sC_F}{\pi} \bar{\alpha}_s \int_{\tau}^1 \frac{dx}{x} G(x, \tau/x)$$

with

$$\bar{\alpha}_s G(x_1, x_2) = \int_{x_1}^1 dt_1 \int_{x_2}^1 dt_2 \alpha_s \left\{ \frac{s}{\Lambda^2} (t_1 - x_1)(t_2 - x_2) \right\} g(x_1, x_2)$$

and  $g(x_1, x_2)$  defined<sup>(9)</sup> as

$$\frac{d\sigma}{dx_F} = G\pi\sqrt{2} \frac{x_1 x_2}{\sqrt{x_F^2 + 4\tau}} \cdot W(x_1, x_2) \quad \text{for W-production}$$

and

$$\frac{d\sigma}{dx_F} = 2 G\pi\sqrt{2} \frac{x_1 x_2}{\sqrt{x_F^2 + 4\tau}} \cdot Z(x_1, x_2) \quad \text{for } Z^0\text{-production}$$

We used a phenomenological  $\alpha_s$  given by

$$\alpha_s(Q^2) = \frac{12\pi}{25} \frac{p}{\ln \left[ 1 + p \left( \frac{Q^2}{\Lambda^2} \right)^p \right]}$$

with  $\Lambda = 100$  MeV and  $p = 5/6$ , and evolved parton densities à la Glück, Hoffmann and Reya<sup>(10)</sup>.

In Table I we show the values obtained for  $\langle Q_{\perp}^2 \rangle$  for the three cases, W,  $Z^0$  and  $\gamma^*$  at  $Q^2 = M_{Z^0}^2 = (92.8 \text{ GeV})^2$  for comparison. An inspection of this table shows that the  $Z^0$  should have a  $Q_{\perp}^2$  larger than W by about 10%.

TABLE I

$\gamma^* \rightarrow \mu^+ \mu^-$	$W^+ \rightarrow \ell \nu$	$Z^0 \rightarrow \mu^+ \mu^-$
217 GeV <sup>2</sup>	180 GeV <sup>2</sup>	206 GeV <sup>2</sup>

In conclusion, we have presented the results of two calculations of soft gluon effects at the collider: one showing that the shape of the KNO function is correctly reproduced by the (non-trivial) energy distribution of soft QCD radiation and the other predicting the value of the mean square transverse momentum of the intermediate vector boson.

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