To be submitted to
Lett. Nuovo Cimento

ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-83/12(P)
3 Marzo 1983

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HAS PROTON HAD TIME ENOUGH TO DECAY?

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ABSTRACT

Following the spectral analysis of decay times recently used by Khalfin, the decay rate of a system with N protons is considered. For N sufficiently large (macroscopic), it would seem that cosmological time scales are not directly involved in experimental detectors of the proton decay events.

In a recent letter, Khalfin(1) has used a rigorous theorem relating the survival amplitude of a quantum state to the spectral resolution of its energy distribution to discuss the hypothesis of proton decay implicit in certain gauge field theories(2,3). Our purpose is to discuss some of the interesting physical points raised in Ref. (1) in terms of the laboratory proton decay detecton devices.

It is a well known result of quantum mechanics that the amplitude A(t), for a quantum state to survive a time interval t, is the characteristic function (in the sense of probability theory) of the energy distribution P(E) of the quantum state

$$A(t) = \int e^{-iEt/\hbar} dP(E) \tag{1}$$

As a consequence of Eq. (1), the survival probability of a quantum state

$$Z(t) = |A(t)|^2 = \iint e^{-i(E-E')t/\hbar} dP(E)\, dP(E') \tag{2}$$

is a (real) characteristic function of the probability law for Bohr transition frequencies

$$Z(t) = \int \cos(\omega t)\, d\varrho(\omega) \tag{3}$$

where

$$\hbar\omega = (E-E'),$$

and $d\varrho(\omega)$ is the probability that a Bohr transition frequency is in the interval $d\omega$.

Khalfin points out that for the present estimates[3] of proton decay lifetimes, the Bohr transitions contributing to the survival probability of a single proton wave function yield time scales much longer than cosmological times. Hence, experimental efforts to detect proton decay may have some unforseen difficulties. Our purpose is to discuss this view.

The experimental methods, of course, involve a macroscopic detector, containing a large number N of protons, hence Eqs. (1-3) will refer (in what followes) to a general N proton state - rather than the one proton state considered by Khalfin. Macroscopic systems always have closely spaced levels, and thus very small Bohr transition frequencis. A macroscopic level spacing based on the entropy S typically yields a number

$$\exp(S/k_B), \qquad \text{say } (10^{10^{20}}),$$

which makes cosmological time scales look infinitely small - in any units!

For macroscopic Bohr transition frequencies built up from infinitely small units, the limit law of probability theory serves as a useful tool. If such a frequency distribution - for the time scales observable in laboratories - is built up from independent infinitesimal distributions then the limit law of Kolmogorof (generalized from the Gaussian Central Limit Theorem)[4], implies a survival probability of the form

$$Z(t) = \exp\left\{-\int_{-\infty}^{\infty} \frac{dG(\omega)}{\omega^2}\,[1-\cos(\omega t)]\right\}, \tag{4}$$

given only the mild hypothesis of a finite energy (second moment) uncertainty for the quantum state of interest

$$(\Delta E)^2 = \int E^2\, dP(E) - \left(\int E\, dP(E)\right)^2 = \hbar^2 \int_{-\infty}^{\infty} dG(\omega). \tag{5}$$

Fermi's Golden rule relates the Kolmogorov spectral function $dG(\omega)$ to the transition rate $\Gamma(\omega)$,

$$dG(\omega) = \frac{1}{\pi}\, \Gamma(\omega)\, d\omega. \tag{6}$$

For example, let H_W be the effective four-fermion interaction Hamiltonian - connecting a three quark state to a lepton - computed from some grand unified theory. Then, to lowest order in perturbation theory,

$$\Gamma(\omega) = \left(\frac{2\pi}{\hbar^2}\right) \bar{\sum_{I}} \sum_{F} p_I \, |\langle F | H_w | I \rangle|^2 \, \delta(\omega - \omega_{FI}) \tag{7}$$

The Bohr transition frequencies of all sizes receive their just weights in the Fermi-Van Hove correlation version of Eq. (7)

$$\Gamma(\omega) = \left(\frac{2}{\hbar^2}\right) \int_0^\infty dt \, \mathrm{Cos}\,(\omega t) \, \mathrm{Re} \langle H_w(t) \, H_w(0) \rangle \tag{8}$$

One notes that time variations of the weak interaction $H_w(t)$ are determined by the strong interaction part of the Hamiltonian H_s,

$$H_w(t) = e^{it H_s/\hbar} \, H_w(0) \, e^{-it H_s/\hbar} \tag{9}$$

Thus, there is both a long, weak interaction time $\Gamma(0)^{-1}$ and a short, strong interaction time τ_s defined as

$$2 \int_0^\infty d\omega \, \Gamma(\omega) = \Gamma(0)/\tau_s = \left(\frac{\Delta E}{\hbar}\right)^2 \tag{10}$$

Provided that the condition

$$\Gamma(0) \, \tau_s \ll 1 \tag{11}$$

holds true, from Eqs. (4) and (6) written as

$$Z(t) = \exp \left\{ -t \int_{-\infty}^{\infty} \left(\frac{du}{u^2}\right) \Gamma\left(\frac{u}{t}\right) (1 - \cos u) \right\} , \tag{12}$$

one deduces the following: for an (unobservably) short strong interaction time, the decay is approximately gaussian, i.e.,

$$Z(t) \simeq \exp \left\{ -\frac{(\Delta E)^2 \, t^2}{2 \hbar^2} \right\} , \qquad t \lesssim \tau_s \tag{13}$$

while for times long on the time scale of strong interactions, one obtains an exponential decay

$$Z(t) \simeq \exp \left\{ -\Gamma(0) \, t \right\} , \qquad t \gg \tau_s \tag{14}$$

In this manner, $\Gamma(0)$ is observed if $\Gamma(0)^{-1}$ represents a laboratory time scale for the macroscopic apparatus.

For N protons, it is a reasonable working hypothesis, that

$$H_w = \sum_{j=1}^{N} h_j \tag{15}$$

where h_j is the Fermi matrix element for the decay of the j-th proton, and that different protons decay independently

$$< h_i(t)\, h_j(t') > \simeq \delta_{ij}\, K(t-t') \tag{16}$$

This leads to

$$\Gamma(0) \simeq N\gamma, \tag{17}$$

via Eqs. (8), (15) and (16), where γ is the usual single proton decay width.

It does not appear to us that cosmological time scales enter into the correlation times of $< H_w(t)\, H_w(t') >$ and so the observation of $\Gamma(0)$ would indeed yield proper single proton decay width.

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