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QUANTIZATION OF GALILEAN GAUGE THEORIES

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QUANTIZATION OF GALILEAN GAUGE THEORIES

F. Palumbo
INFN, Laboratori Nazionali di Frascati, Frascati (Italy).

ABSTRACT: Galilean gauge theories are quantized using Dirac's theory of canonical quantization of constrained systems. The infrared sector for the Abelian case is exactly solved and shown to be analogous to that of the relativistic case.

Looking for the features of nonabelian gauge theories which could possibly provide the confinement of color, it is desirable to introduce the most drastic approximations which could preserve such features. We have considered the limit of infinite light velocity, which gives rise to Galilean gauge theories(1-4).

We have already checked in previous papers that the low energy behaviour of relativistic theories (for which such behaviour is known) is reproduced by the corresponding Galilean theories. This has been shown for the Goldstone and Higgs models, the Schwinger mechanisms for spontaneous mass generation(3), the Wess-Zumino and the Fayet models of supersymmetry(4) and for QED(1). Concerning the latter, which is most relevant to QCD, we have shown by performing directly the $c \to \infty$ limit on the quantum theory, that Galilean QED has infrared divergences giving rise to radiative corrections which factorize as in the relativistic case.

The above results give some confidence that also the infrared behaviour of QCD should not qualitatively change in the Galilean approximation. Relativistic effects can be in any case accounted for by means of a $1/c$ expansion(2).

The classical gauge-fields Lagrangian density is

$$\mathscr{L}_G = E_i \partial_t A_i - \frac{1}{2} E_i^2 - V \mathscr{D}_i E_i - \Lambda_k \Phi_k \,, \tag{1}$$

where $\mathscr{D}_i$ are the spatial covariant derivatives

$$\mathscr{D}_i = \partial_i - g t_a A_i^a \tag{2}$$

in the adjoint representation and

$$\Phi_k^a = \frac{1}{2} \varepsilon_{kij} (\partial_i A_j^a - \partial_j A_i^a + g f^{abc} A_i^b A_j^c) \,. \tag{3}$$

The Lagrangian (1) contains the primary constraints

$$\Phi_k^a = 0 \,, \tag{4}$$

$$\mathscr{D}_i^{ab} E_i^b = 0 \,, \quad \text{for the non abelian case}$$

$$\partial_i E_i = 0 \,, \quad \text{for the abelian case} \,. \tag{5}$$

The constraints (5) are common to the relativistic case.

By applying Dirac's theory[5] of constrained systems we find that the only secondary constraints are

$$\chi_k^a = \varepsilon_{kij} \mathscr{D}_i^{ab} E_j^b \,. \tag{6}$$

We must now choose the boundary conditions and the gauge fixing. We choose periodic boundary conditions in a cubic box of volume Ω.

In the Abelian case we choose the Coulomb gauge

$$\partial_i A_i = 0 \,. \tag{7}$$

The solution to all the constraints with periodic boundary conditions is

$$A_i = \frac{1}{\sqrt{\Omega}} q_i \,, \qquad E_i = \frac{1}{\sqrt{\Omega}} p_i \,, \tag{8}$$

where q_i and p_i are x-independent and can be shown to satisfy canonical Poisson brackets.

For the non-abelian case we restrict ourselves to SU(2), although we will show separately[6] that the present results generalize in a straightforward way to any SU(n), and choose the gauge fixing

$$A_3^a - \delta^{a3} Q = 0 \, , \tag{9}$$

Q being an arbitrary function. The solution to the constraints is

$$A_k^a = \delta^{a3} \left(\frac{1}{\sqrt{\Omega}} q_k + \partial_k A \right) , \qquad E_k^a = \delta^{a3} \frac{1}{\sqrt{\Omega}} p_k \, , \tag{10}$$

where q_k and p_k are x-independent and A is an arbitrary function which satisfies periodic boundary conditions. This function can be put equal to zero and q_k, p_k, can be shown to satisfy canonical Poisson brackets. We see that the quantum gauge fields do not carry any color index.

In the presence of matter nothing changes of the above with the exception of the constraint (5) which becomes

$$\begin{aligned} &\mathcal{D}_i E_i + g\varrho = 0 \, , \quad \text{for the non abelian case} \\ &\partial_i E_i + g\varrho = 0 \, , \quad \text{for the abelian case} \end{aligned} \tag{11}$$

with

$$\begin{aligned} &\varrho^a = \varphi^* t^a \varphi - \bar{\varphi}^* t^a \bar{\varphi} \, , \quad \text{for the non abelian case} \\ &\varrho^a = \varphi^* \varphi - \bar{\varphi}^* \bar{\varphi} \, , \quad \text{for the abelian case .} \end{aligned} \tag{12}$$

φ and $\bar{\varphi}$ being the matter, antimatter fields respectively. The solutions to the constraints remain the same for the gauge fields A_i, while

$$\begin{aligned} &E_i = \frac{1}{\sqrt{\Omega}} p_i - \Delta^{-1} \cdot \partial_i \varrho \, , \qquad \text{for the Abelian case ,} \\ &E_i^a = \delta^{a3} \frac{1}{\sqrt{\Omega}} p_i - (\mathcal{D}^{-2} \mathcal{D}_i \varrho)^a \, , \qquad \text{for SU(2) .} \end{aligned} \tag{13}$$

When expanded in Fourier series, eq. (11) contains or does not contain the constant term according to the value of the constant term V_o of V. If $V_o = 0$, ϱ_o is arbitrary while if $V_o \neq 0$, $\varrho_o = 0$. The constant term of ϱ,

$$\varrho_o = \frac{1}{\sqrt{\Omega}} \int_\Omega d^3x \, \varrho(x) \tag{14}$$

is the charge. So in the first case, which is appropriate to QED, the electric charge can be arbitrary, while in the second case, which is appropriate to QCD, the color charge must be zero.

The quantization of the classical matter field Lagrangian, whose expression is given in refs. (1, 3) requires no discussion. Finally we arrive to the Hamiltonian

$$H = H_o + V , \tag{15}$$

where V is the Coulomb interaction both in the Abelian and non-abelian case. In the Abelian case

$$H_o = \frac{1}{2} p^2 + \frac{1}{2} \omega^2 q^2 - \omega q_k I_k - \frac{1}{2m} \int d^3x (\varphi^* \Delta \varphi + \bar{\varphi}^* \Delta \bar{\varphi}) , \tag{16}$$

with

$$I_k = \sqrt{\frac{m}{N}} \int d^3x \frac{1}{mi} (\varphi^* \partial_k \varphi - \bar{\varphi}^* \partial_k \bar{\varphi}) , \qquad \omega^2 = \frac{g^2 N}{m \Omega} , \tag{17}$$

N being the number of particles plus antiparticles.

In the non-abelian case, restricted to singlet color states due to the vanishing of the color charge

$$\begin{aligned} H_o = & \frac{1}{2} p^2 + \frac{1}{2} \omega^2 q^2 - \frac{1}{2m} \int d^3x (\varphi^* \Delta \varphi + \bar{\varphi}^* \Delta \bar{\varphi}) - \\ & - \frac{1}{2} g^2 \int_\Omega d^3x \, d^3y \, \varrho^a(x) \left\{ \left[\mathscr{D}^{-2} - \Delta^{-1} \right] (x-y) \right\}^{ab} \varrho^b(y) . \end{aligned} \tag{18}$$

The reason why some terms containing the gauge fields have been inserted in H_o is that they affect the properties of the physical in- and out- states rather than to give rise to a real interaction.

Let us see this in the Abelian case, whose infrared sector is exactely solved. The cross section for a process accompanied by emission of n photons is

$$\sigma_n = \sigma \exp\left[- \frac{(\Delta I)^2}{2\omega} \right] \frac{1}{n!} \left[\frac{(\Delta I)^2}{2\omega} \right]^n \tag{19}$$

where σ is the cross section evaluated in the absence of coupling of the matter to the radiation and

$$(\Delta I)^2 = (\langle f | I_k | f \rangle - \langle i | I_k | i \rangle)^2 , \tag{20}$$

$|f\rangle$ and $|i\rangle$ being the final and initial states respectively. For $\Omega \to \infty$, $\omega \to 0$ and $\sigma_n \to 0$, as in the relativistic case. As in the relativistic case one observes that in an actual experiment, due to the finite energy resolution, there is a probability $P(E, \Delta E)$ to assign energy E to particles having energy $E - \Delta E$. We must therefore evaluate the quantity[7]

$$\lim_{\Omega\to\infty} \sum_{n=0}^{\infty} \bar{\sigma}_n P(E, n\omega) = \sigma P(E, \frac{1}{2}(\Delta I)^2) . \qquad (21)$$

This correction, which is the only survivor in the Galilean limit, must of course be present also in relativistic QED with periodic boundary conditions. It would on the other hand be killed by a different choice of boundary conditions implying the vanishing of the fields on the surface of the quantization volume. We can therefore test the boundary conditions by checking the above formula in experiments at very low energy by varying the energy resolution. This point will be discussed in detail somewhere.

The above should explain our definition of H_0. The photons do not really have interactions in Galilean QED, but charged particles are surrounded by soft photons and the overlapping of these clouds which are different for initial and final states gives rise to the radiative correction of eq. (21).

The infrared sector has not yet been solved for non-abelian gauge theories. We already see, however, that the possibility of emission of soft gluons would not cause any problem from the phenomenological point of view because they do not carry color.

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