To be submitted to
Physics Letters B

ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-83/8(P)
21 Febbraio 1983

G. Martinelli and Zhang Yi-Cheng: ONE LOOP CORRECTIONS TO EXTENDED OPERATORS ON THE LATTICE

Servizio Documentazione
dei Laboratori Nazionali di Frascati
Cas. Postale 13 - Frascati (Roma)

INFN - Laboratori Nazionali di Frascati
Servizio Documentazione

LNF-83/8(P)
21 Febbraio 1983

ONE LOOP CORRECTIONS TO EXTENDED OPFRATORS ON THE LATTICE

G. Martinelli
INFN, Laboratori Nazionali di Frascati

and

Zhang Yi-Cheng
SISSA, Trieste, and Istituto di Fisica 'G. Marconi', Università di Roma

ABSTRACT

We have computed at first order in perturbation theory the general expressions to establish the relation between lattice one site extended fermion operators (used in recent Montecarlo simulations) and the corresponding operators in the continuum.

Recent Montecarlo simulations studied the correlation functions of spatially extended fermion operators in lattice QCD[1,2]: extended operators are expected to be more appropriate than local operators in investigating the properties of P-wave mesons[1] and they could be used to compute on the lattice the matrix elements relevant for deep inelastic scattering. In this paper, extending previous calculations[3,4,5], we compute at first order in the coupling constant the relations between certain extended operators on the lattice and the corresponding operators in the continuum. The extended operators we consider here are defined by splitting the quark fields by one lattice distance:

$$O^{+}_{i,\delta}(x) = \frac{1}{2}\left[\bar{\psi}_1(x)\,\Gamma_i U_\delta(x)\,\psi_2(x+\hat{\delta}) + \bar{\psi}_1(x+\hat{\delta})\,\Gamma_i U^{+}_\delta(x)\,\psi_2(x)\right] \longrightarrow \bar{\psi}_1(x)\,\Gamma_i\,\psi_2(x) \tag{1}$$

$$O^{-}_{i,\delta}(x) = \frac{1}{2\underline{a}}\left[\bar{\psi}_1(x)\,\Gamma_i U_\delta(x)\,\psi_2(x+\hat{\delta}) - \bar{\psi}_1(x+\hat{\delta})\,\Gamma_i U^{+}_\delta(x)\,\psi_2(x)\right] \longrightarrow \frac{1}{2}\,\bar{\psi}_1(x)\,\Gamma_i \overleftrightarrow{D}_\delta\,\psi_2(x)$$

where $\psi_{1,2}$ and $U_\delta(x)$ are the quark and gluon fields respectively, Γ_i is one of the 16 dirac matrices, $\underline{a}$ is the lattice spacing and D_δ is the covariant derivative.

At first order in perturbation theory these operators depend on the regularization procedure; sandwiching them between arbitrary external quark states of momentum p and p' we can write:

$$O^{\pm\,\overline{MS},\,LATT}_{i,\delta}(p,p') = O^{\pm\,0}_{i,\delta}(p,p') + \frac{\alpha_s}{4\pi}\,C_F\,\tilde{O}^{\pm\,\overline{MS},\,LATT}_{i,\delta}(p,p') \tag{2}$$

$O^{\pm\,0}_{i,\delta}(p,p')$ is the bare matrix element of the operator, $\tilde{O}^{\pm\,\overline{MS},\,LATT}_{i,\delta}(p,p')$ is obtained by computing the diagrams shown in Figg. 1 and 2 for the vertex function and the quark self energy[3] in some continuum renormalization scheme ($\overline{MS}$ in our case) and

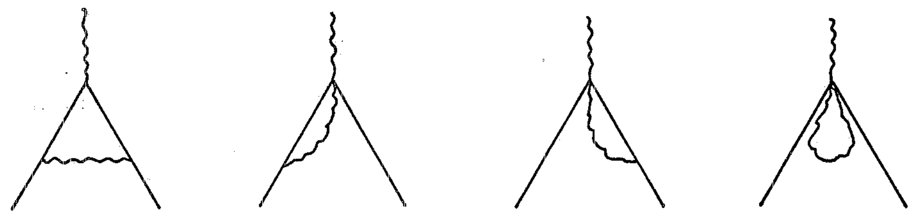

FIG. 1 - Feynman diagrams for the vertex corrections at order α_s, only the first diagram is present in the continuum and local operator.

FIG. 2 - Order $O(\alpha_s)$ diagrams for the quark self energy; only the first diagram exists in the continuum.

on the lattice. In the limit $a \to 0$ the general form for the quantities $\Delta^{\pm}_{i,\delta} = \tilde{O}^{\pm\,\overline{MS}}_{i,\delta}(p,p') - \tilde{O}^{\pm\,LATT}_{i,\delta}(p,p')$ is:

$$\Delta^+_{i,\delta} = (\gamma^\nu\gamma^\varrho \Gamma_i \gamma^\varrho\gamma^\nu)(\eta_i + I_1) + (\gamma^\nu\gamma^\delta \Gamma_i \gamma^\delta\gamma^\nu) I_2 + (\gamma^\nu \Gamma_i \gamma^\nu) I_3 + \\ + (\gamma^\delta \Gamma_i \gamma^\delta) I_4 + \Gamma_i I_5 \tag{3a}$$

$$\Delta^-_{i,\delta} = i\,\frac{(p+p')\delta}{2}\Big[(\gamma^\nu\gamma^\varrho\Gamma_i\gamma^\varrho\gamma^\nu)(\eta'_i + I'_1) + (\gamma^\nu\gamma^\delta\Gamma_i\gamma^\delta\gamma^\nu) I'_2 + (\gamma^\nu\Gamma_i\gamma^\nu) I'_3 + \\ + (\gamma^\delta\Gamma_i\gamma^\delta) I'_4 + \Gamma_i I'_5\Big] + i\,\frac{(p+p')\varrho}{2}\Big[(\gamma^\nu\gamma^\varrho\Gamma_i\gamma^\delta\gamma^\nu + \gamma^\nu\gamma^\delta\Gamma_i\gamma^\varrho\gamma^\nu)(\theta_i + I'_6) + \\ + (\Gamma_i\gamma^\delta\gamma^\varrho + \gamma^\varrho\gamma^\delta\Gamma_i) I'_7 + (\gamma^\delta\Gamma_i\gamma^\varrho + \gamma^\varrho\Gamma_i\gamma^\delta) I'_8 + \frac{r}{a} T_{i,\delta}\Big] \tag{3b}$$

$I_1, \ldots\ldots, I'_8$ are independent of the external momenta.

Because $O^-_{i,\delta}$ have not a definite spin there not multiplicatively renormalized.

$\eta_i = -1/4$ if $\Gamma_i = 1$ or γ_5; $\eta_i = -1/2$ for γ_μ or $\gamma_\mu\gamma_5$;

$\eta'_i = -1/12$ if $\Gamma_i = 1$ or γ_5; $\eta'_i = -1/6$ for γ_μ or $\gamma_\mu\gamma_5$;

$\theta_i = -1/24$ if $\Gamma_i = 1$ or γ_5; $\theta_i = -1/12$ for γ_μ or $\gamma_\mu\gamma_5$.

All the repeated indices but δ are summed. The linear divergent term (proportional to r/a) is independent on the external momenta and it can be eliminated by subtracting to the matrix element of $O^-_{i,\delta}$ its zero momentum matrix element in analogy with what is usually done for the linear divergent term of the quark self energy:

$$O^-_{i,\delta}(p, p') \Rightarrow O^-_{i,\delta}(p,p') - O^-_{i,\delta}(0,0)\,. \tag{4}$$

However it is precisely this term that renders the vector current, defined on the lattice as

$$V_\mu(x) = \frac{1}{2}\Big[\overline{\psi}(x)(\gamma^\mu - r)U_\mu(x)\overline{\psi}(x+\hat{\mu}) + \overline{\psi}(x+\hat{\mu})(\gamma^\mu + r)U^+_\mu(x)\psi(x)\Big] = \\ = O^+_{\gamma_\mu,\mu}(x) - \underline{a}\cdot r\, O^-_{1,\mu}(x)\,, \tag{5}$$

conserved. In eq. (5) r is the usual Wilson parameter which defines the fermionic action on the lattice[(6)(x)].

The explicit expressions for $I_1 \ldots\ldots I_5$ and $I'_1 \ldots\ldots I'_8$ are very complicated and are too tedious to present[(o)].

(x) As a test of our computation for $O^+_{i,\delta}(x)$ we computed separately $O^+_{\gamma_\mu,\mu}(x)$ and the linear divergent piece in $O^-_{\mu,\mu}(x)$ summed them and checked that they cancel for any value of r.

(o) Available on request.

We report in Table I and II their numerical values at six different values of r ranging from 1 (Wilson fermions) and 0 (naive fermions). Using eqs. (3a) and (3b) and the tables it is very easy to extract the $O(\alpha_s)$ corrections to any one site extended operator.

TABLE I - We give the values of $I_1 \dots I_5$ defined in eq. (3a) at several values of r. In I_5 the terms coming from the quark self energy corrections, which has been computed elsewhere[3] are not included. The numerical precision on the integrals is $\sim 5\%$.

I_1	I_2	I_3	I_4	I_5	r
- 0. 13	- 0. 84	0	0	6. 9	0
- 0. 18	- 0. 25	- 0. 033	0. 74	6. 6	0. 2
- 0. 078	0. 038	0. 16	0. 85	6. 0	0. 4
0. 058	0. 091	0. 32	0. 70	5. 5	0. 6
0. 18	0. 088	0. 43	0. 55	5. 0	0. 8
0. 28	0. 074	0. 49	0. 43	4. 7	1. 0

TABLE II - The values of $I'_1 \dots I'_8$ of eq. (3b) are given. The terms coming from the quark self energy are not included in I'_5 and $I'_1 = I'_6$. The largest part of corrections is given by the second and third diagrams in Fig. 1.

I'_1	I'_2	I'_3	I'_4	I'_5	I'_7	I'_8	r
- 0. 19	1. 0	0	0	9. 0	- 0. 32	0	0
- 0. 13	0. 73	0. 14	- 0. 41	9. 8	- 0. 37	- 0. 021	0. 2
- 0. 039	0. 44	0. 28	- 0. 56	10. 8	- 0. 31	- 0. 072	0. 4
0. 028	0. 28	0. 36	- 0. 52	11. 4	- 0. 18	- 0. 11	0. 6
0. 076	0. 19	0. 39	- 0. 44	11. 7	- 0. 042	- 0. 14	0. 8
0. 11	0. 14	0. 42	- 0. 36	12. 0	0. 092	- 0. 16	1. 0

The extended operators coupled to π, ϱ, δ, A^1 and B mesons are[1]:

$$\pi(x) = \sum_\mu O^+_{\gamma_5,\mu}(x) \; ; \qquad A^1_i(x) = \varepsilon_{ijk} O^-_{\gamma_j,k}(x) \; ;$$

$$\varrho_i(x) = \varepsilon_{ijk} O^+_{\gamma_j,k}(x) \; ; \qquad B_i(x) = O^-_{\gamma_5,i}(x) ; \tag{6}$$

$$\delta(x) = \sum_\mu O^-_{\gamma_\mu,\mu}(x) ;$$

the expression for $\pi(x)$ in eq. (6) can be used to have a rough idea of the $O(\underline{a}^2)$ corrections in Montecarlo simulations:

$$\pi(x)^{BARE}_{ext} \simeq \pi(x)^{BARE}_{local} + O(\underline{a}^2) . \tag{7}$$

For example from eqs. (3a), (3b) and (6) and the tables one finds for $r = 1$:

$$\langle \pi(x) \rangle^{\overline{MS}} = \left[1 + \frac{\alpha_s}{4\pi} C_F (16 I_1 + 4 I_2 - 4 I_3 - I_4 + I_5 - 4) \langle \pi(x) \rangle \right]^{LATT} .$$

From which, using the results of ref. (3) it follows at $r = 1$:

$$\left[m_q \langle \pi(x) \rangle \right]^{\overline{MS}} = \left[1 + 0.05\, g^2 \right] \left[m_q \langle \pi(x) \rangle \right]^{LATT}$$

and

$$\langle \varrho_i(x) \rangle^{\overline{MS}} = \left[1 - 0.09\, g^2 \right] \langle \varrho_i(x) \rangle^{LATT} ;$$

$$\langle \delta(x) \rangle^{\overline{MS}} = \left[1 - 0.03\, g^2 \right] \langle \delta(x) \rangle^{LATT} ; \tag{8}$$

$$\langle A^1_i(x) \rangle^{\overline{MS}} = \left[1 + 0.00..\, g^2 \right] \langle A^1_i(x) \rangle^{LATT} ;$$

$$\langle B_i(x) \rangle^{\overline{MS}} = \left[1 + 0.00..\, g^2 \right] \langle B_i(x) \rangle^{LATT} ;$$

m_q is the quark mass: $\langle \ldots \rangle$ stands for any matrix element. The correction to A^1 and B mesons turn out to be numerically equal for any fixed r: for $r = 1$ it is completely negligible.

Given the pion and rho propagators $G_{\varrho,\pi}$ one predicts at $r = 1$:

$$\frac{(G_\pi)_{LOCAL}}{(G_\pi)_{EXTENDED}} \simeq 1 - 0.22\, g^2 ; \qquad \frac{(G_\varrho)_{LOCAL}}{(G_{\varrho'})_{EXTENDED}} \simeq 1 - 0.18\, g^2 , \tag{9}$$

where $\varrho'_i(x)^{EXTENDED} = O^+_{\gamma_i, i}(x) \sim \overline{\psi}_1(x)\, \gamma_i\, \psi_2(x)$ when $\underline{a} \to 0$. For this current as well as for the axial current we agree within our numerical accuracy with the results of ref. (5).

This computation allows us to compare the energy momentum tensor on the lattice and in the continuum:

$$O^{\mu_1 \mu_2}(x) = \overline{\psi}_1(x) (\gamma_{\mu_1} D_{\mu_2} + \gamma_{\mu_2} D_{\mu_1}) \psi_2(x) - \text{trace terms} . \qquad (10)$$

The matrix elements of this operator on the lattice can be used to calculate the fraction of longitudinal momentum carried by a quark inside the parent hadron. One finds :

$$\langle O^{\mu,\mu} \rangle^{\overline{MS}} = (1 - 0.02\, g^2) \langle O^{\mu,\mu} \rangle^{LATT} . \qquad (11)$$

In this case the correction is very small.

We computed the general formulae to relate at order α_s one site extended operators on the lattice to the corresponding operators in the continuum and found as was already observed in ref. (5) that these corrections for $r = 1$ are larger for operators which are local on the lattice as expected.

REFERENCES

(1) - A. Haselfratz, P. Hasenfratz, Z. Kunszt and C. B. Lang, Phys. Letters 117B, 81 (1982).

(2) - G. Martinelli, G. Parisi and R. Petronzio, Unpublished.

(3) - A. Gonzales Arroyo, F. J. Yndurain and G. Martinelli, Phys. Letters 117B, 437 (1982).

(4) - G. Martinelli and Zhang Yi-Cheng, Frascati preprint LNF-82/69 (1982); to appear in Phys. Letters B.

(5) - B. Meyer and C. Smith, Edinburgh preprint No. 82/325 (1982).

(6) - L. H. Karsten and J. Smit, Nuclear Phys. B183, 103 (1981).