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DISTRIBUTION AT THE COLLIDER

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QCD RADIATION AND THE MULTIPLICITY DISTRIBUTION
AT THE COLLIDER

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ABSTRACT

The multiplicity distribution in hadron-hadron collisions is discussed as arising from the soft gluon bremsstrahlung accompanying parton-parton scattering. KNO scaling is mean scaling in the energy variable and is seen to arise when the microscopic system of quarks and gluons is averaged over the hadronic matter coordinates. The shape of the multiplicity distribution for the UA1 data at the collider energy is well fitted by a soft QCD radiation formula, using a QCD parameter $\Lambda \approx 100$ MeV. The reduced cumulants γ_i are expected to decrease with energy.

The work I will discuss has been done in collaboration with Y. Srivastava. In this work we study the contribution of soft gluon bremsstrahlung to multiparticle production at very high energy^(1, 2, 3). This mechanism has already been seen at work in a score of high energy processes like DIS⁽⁴⁾, Drell-Yan^(5, 6), e^+e^- ⁽⁷⁾. Although one cannot say that all or even most high energy hadron physics is explained in terms of soft gluon bremsstrahlung, we believe the latter to be an important mechanism often competitive with hard QCD processes and sometimes dominant.

The material I will present can be summarized in the following three points :

1. KNO scaling is scaling-in-the-mean;
2. The shape of the multiplicity distribution can be obtained from the ener-

gy distribution of the soft QCD radiation emitted in parton-parton scattering;

3. KNO scaling violations have the same origin as scaling violations in Deep Inelastic Scattering and they depend on Λ_{QCD} . A typical value for Λ is 100 MeV.

In the following, I will discuss in detail the above points in separate sections.

1. - KNO Scaling and Mean Scaling

In 1974, F. T. Dao et al.⁽⁸⁾ put forward the hypothesis that the shape of single particle distributions in the transverse and longitudinal momentum variables p_t and p_L are independent of multiplicity and incident energy, if the distributions are plotted against the "mean" variable $x/\langle x \rangle$. According to their hypothesis, the energy, multiplicity and initial state dependence lie in the average value $\langle x \rangle$. To test their ansatz, they analyzed single inclusive pion cross-sections at various energies and for different values of the charged multiplicity and found that when normalized all the distributions (in a given variable) fell on top of each other and that this was true for the transverse as well as the longitudinal momentum variable. These authors note that when the variables are the charged and neutral multiplicity, one has the familiar KNO scaling⁽⁹⁾. In Figs. 1 and 2 we show their data compilation together with the QCD radiation curve which will be discussed in the next section. What is quite remarkable is that mean scaling can be observed also for current processes. In fact a similar analysis for SPEAR data⁽¹⁰⁾ in the energy range $3 \leq W \leq 10$ GeV again shows that the data for p_t arrange themselves on a single curve⁽⁵⁾. Fig. 3 illustrates mean scaling for e^+e^- . A completely analogous scaling takes place in μ -pair production⁽¹¹⁾, as can be seen from Fig. 4. It should be noted that the two mass ranges we have analyzed⁽¹²⁾ correspond to rather different values of $\langle p_t \rangle$, one set having $\langle p_t \rangle = 1.2$ GeV and the other $\langle p_t \rangle = 1.35$

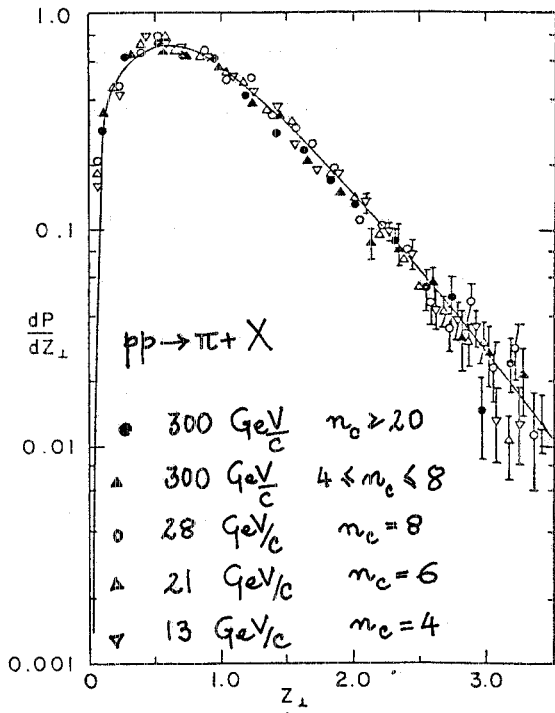


FIG. 1

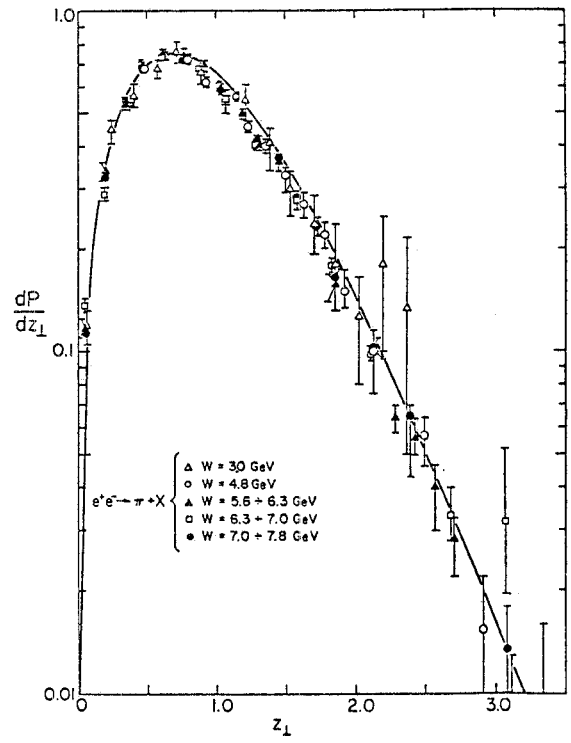


FIG. 2

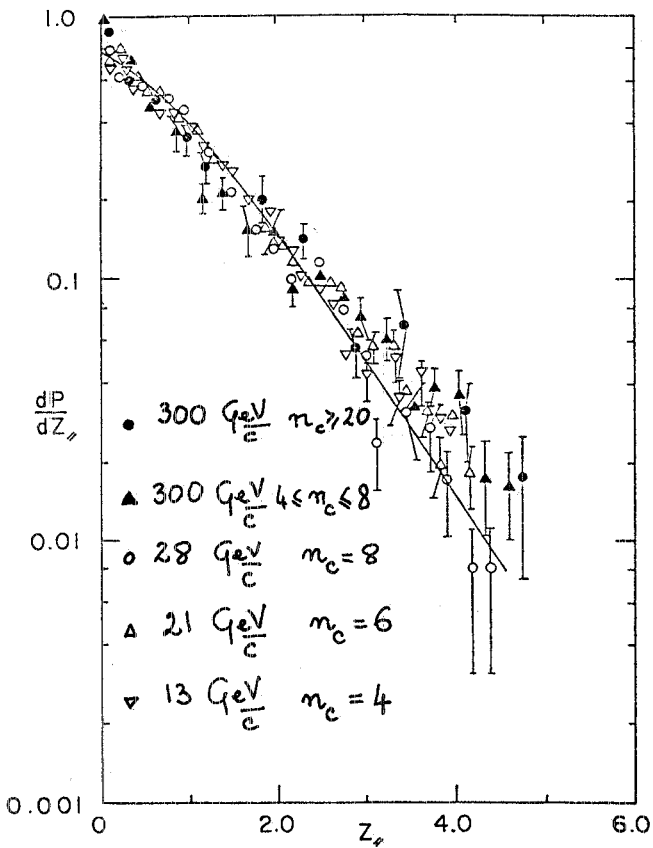


FIG. 3

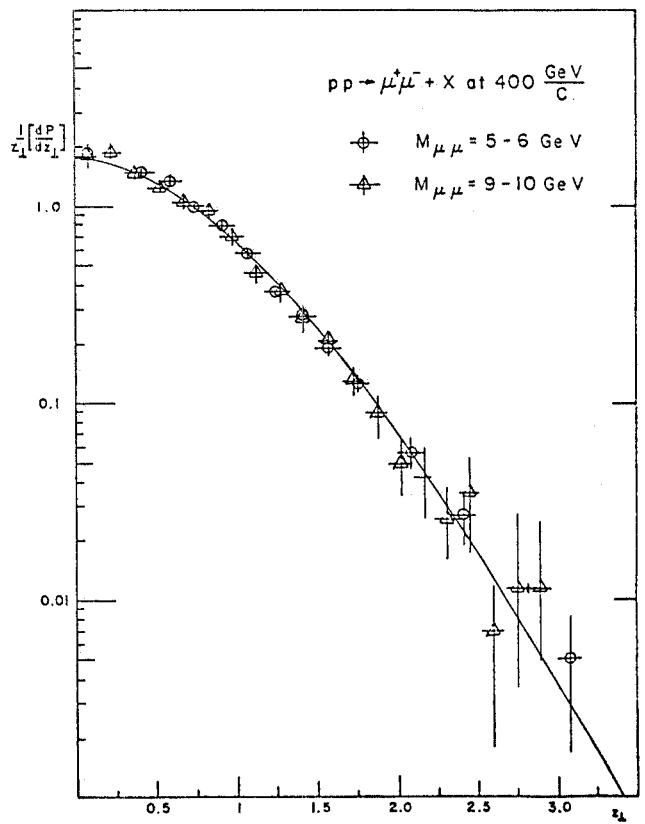


FIG. 4

GeV (in the \mathcal{Y} -region). Finally, if the variable under scrutiny is $n/\langle n \rangle$, one will observe KNO scaling and interpret it as one more instance of scaling in the mean.

Before turning to discuss the shape of all these distributions, we would like to comment upon "mean" scaling in general. A scaling behaviour reflects the presence of a distribution function for which scaling in certain parameters occur. The question one may ask is why these parameters are precisely the mean values of the variables under scrutiny. A possible answer is that we are observing a system which has been averaged over the hadronic matter coordinates. As a result of such average, the microscopic scales characterizing the quark and gluon system are turned into their mean values. An illustration of this procedure can be seen in sect. 3 and ref. (13).

2. - The shape of the Scaling Function

As discussed in the previous section, high energy inclusive distributions show scaling in the variables $p_t/\langle p_t \rangle$, $p_L/\langle p_L \rangle$ and $n/\langle n \rangle$. These three variables can be associated to the momentum and energy characterizing a universal distribution which we propose to be due to soft gluon bremsstrahlung. Why soft? In QED the no-recoil approximation can be used only in a limited fashion, i. e. to describe photons carrying at most 10 or 20% of the emitting electron's energy. Beyond this value, in fact, one has to start using the hard bremsstrahlung formulae, since the soft approximation breaks down. Why then in QCD, the main mechanism of particle production in the central region seems to be the no-recoil one? A tentative answer is the following: it is a general physical law that when a particle which is coupled to a massless field changes its momentum through scattering or annihilation or pair production, then massless quanta are emitted. In QCD, once two quarks are produced like in e^+e^- or annihilate like in DY or scatter off a photon like in DIS or are somehow extracted from their hadronic shells through a high energy process, they will start the soft bremsstrah-

lung process. Now, because α_s is Q^2 -dependent, with $\alpha_s(Q^2) \xrightarrow[Q^2 \rightarrow \infty]{\longrightarrow} 0$, a hard emission process is non-leading relative to a soft one. It is "cheaper" for a quark to lose most of its energy in a sequence of non-recoil emission events than otherwise: the loss of energy is preferentially an adiabatic process. A check of this hypothesis can be found in the fact that the shape of the multiplicity distribution in pp and $p\bar{p}$ collisions, when plotted in the KNO variable, is obtained from the soft gluon energy distribution. Before discussing the latter in detail, we shall comment upon the distribution which underlies the mean scaling curves appearing in Figs. 1, 2, 3, 4. The characteristics of this distribution can be summarized as follows:

- a) the shape of the distribution is not the same (albeit is similar) for all three variables p_t , p_L and E or n;
- b) for a given variable the shape depends on the single soft gluon distribution for the process under examination, a function which we shall call β ;
- c) for a given variable and a given process, the shape depends on energy through $\beta(s)$.

Concerning points (b) and (c), it should be noticed that the p_t distribution for semihadronic processes like $e^+e^- \rightarrow \pi + X$ and $pp \rightarrow \mu^+\mu^- + X$ seems to have exactly the same shape at comparable energies, while for purely hadronic processes the normalized curves are "fatter". A comparison between the normalized SPEAR distributions and the power-like fit $p_t/(1+(p_t/p_0)^2)^6$ is shown in Fig. 5, where the dotted curve is the empirical fit and the full curve is the same QCD radiation curve fitting the SPEAR

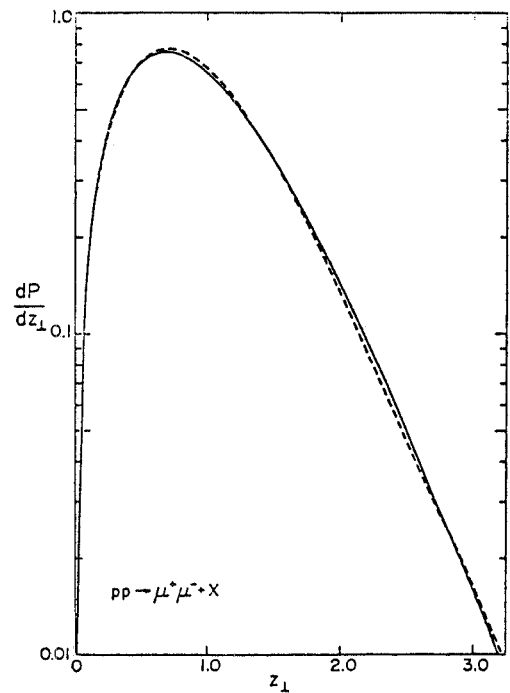


FIG. 5

data in Fig. 3. The empirical fit had been used by the CFS collaboration⁽¹¹⁾ for their μ -pair data. All the QCD curves were obtained from the function

$$d^4P(K) = d^4K \int \frac{d^4x}{(2\pi)^4} e^{iK \cdot x} \cdot e^{-\int d^3\vec{n}(k)(1-e^{-ik \cdot x})}$$

where $d^3\vec{n}(k)$ represents the (exponentiated) single gluon distribution function. When integrated in all variables but K_L , the above function was used to give the longitudinal momentum distribution of Fig. 2. Likewise for the transverse momentum variable and the curves shown in Figs. 1, 3, 4. To obtain the energy distribution, one will integrate in all the three momentum variables and obtain

$$dP(K_0, \Omega) = dK_0 \int \frac{dt}{2\pi} e^{iK_0 t} e^{-\int_0^\Omega d\vec{n}(k)(1-e^{-ikt})} \quad (1)$$

This function and its relation to the multiplicity distribution in hadronic reactions will be discussed in detail in the next section.

3. - The multiplicity Distribution in pp or p \bar{p} Collisions

Our starting hypothesis is that the total energy carried by the pions, $\sum_1^n \omega_i$, is mostly that of the soft QCD radiation, Ω_{vis} , emitted by the interacting partons, i. e. we put

$$\sum_1^n \omega_i = \Omega_{vis} \cdot$$

The charged n-particle cross-section can then be written as

$$\begin{aligned} \sigma_n^{ch}(s) = & \sum_{i,j,final} \int dx_1 f_i(x_1) \int dx_2 f_j(x_2) \int d\Omega \hat{\sigma}_{i,j \rightarrow final}(x_1, x_2; \Omega) \cdot \\ & \cdot \int d\Omega_{vis} \frac{dP(\Omega_{vis}, \Omega)}{d\Omega_{vis}} \delta(\Omega_{vis} - \sum_1^n \omega_i) \int d\omega_1 \dots \int d\omega_n \mathcal{D}_\pi^{(n)}(\Omega; \omega_1 \dots \omega_n) \end{aligned} \quad (2)$$

where $dP(\Omega_{\text{vis}}, \Omega)/d\Omega_{\text{vis}}$ is the energy distribution of soft gluons as in eq. (1) and $\mathcal{D}_{\pi}^{(n)}(\Omega; \omega_1, \omega_2, \dots, \omega_n)$ is the inclusive probability for production of n pions out of a total available energy Ω . In eq. (2), $f(x)$ represents the initial parton densities, $\hat{\sigma}$ is the parton-parton cross-section, summed over all unobserved final states. To study the multiplicity distribution, we now take the following two steps:

- i) $\sum_1^n \omega_i \rightarrow n \langle \omega(s) \rangle$ as a result of averaging over the pion energies;
and
- ii) $\Omega \rightarrow \langle \Omega(s) \rangle$ as a result of averaging over the quark coordinates.

Lets us briefly comment upon the above two steps.

- i) We start by making the substitution

$$\sum_1^n \omega_i \longrightarrow n \langle \omega(n, s) \rangle$$

i. e. by considering that the effect of integrating over the individual pion energies with the fragmentation function as weight function is that of substituting, everywhere it appers, $\sum_1^n \omega_i$ with $n \langle \omega(n, s) \rangle$. We then use the experimental fact that the mean energy per charged track is approximately a constant in n and set

$$\langle \omega(n, s) \rangle \sim \langle \omega(s) \rangle + O\left(\frac{1}{n}\right).$$

We can now scale out $\langle \omega \rangle$ in $dP(n \langle \omega \rangle, \Omega)$ and write

$$\begin{aligned} \sigma_n^{\text{ch}}(s) \simeq & \sum_{\substack{i, j \\ \text{final}}} \int f_i(x_1) dx_1 \int f_j(x_2) dx_2 \int \hat{\sigma}(x_1, x_2; \Omega) \frac{d\Omega}{\langle \omega \rangle} \cdot \\ & \cdot \int \frac{dt}{2\pi} e^{int} e^{-\int_0^{\Omega/\langle \omega \rangle} d\bar{n}(k)(1-e^{-ikt})} \end{aligned}$$

The above equation is valid if $d\bar{n}(k)$ scales. This is certainly not true in QCD. However, if we write

$$d\bar{n}(k) \simeq \frac{4}{3\pi} \frac{dk}{k} \int \frac{dk_{\perp}^2}{k_{\perp}^2} \alpha_s(k_{\perp}^2)$$

and notice that in the very large s region we can consider the k_{\perp} integration to be independent of k , then the scaling takes place and we can write

$$d\bar{n}(k) \simeq \frac{dk}{k} \frac{16}{25} \ln \ln \left(\frac{s}{\Lambda^2} \right) . \quad (3)$$

ii) To take the second step and use mean scaling, we substitute $\Omega/\langle\omega\rangle$ with its mean value. We calculate, using eq. (2),

$$\langle n(s) \rangle = \frac{\int n \sigma(n, s) dn}{\int \sigma(n, s) dn}$$

and, from the definition

$$\frac{\langle \Omega \rangle}{\langle \omega \rangle} = \frac{\sum \int f_i(x_1) dx_1 \int f_j(x_2) dx_2 \int \hat{\sigma}_{ij \rightarrow \text{final}}(x_1, x_2; \Omega) \frac{d\Omega}{\langle \omega \rangle} \frac{\Omega}{\langle \omega \rangle}}{\sum \int f_i(x_1) dx_1 \int f_j(x_2) dx_2 \int \hat{\sigma}_{ij \rightarrow \text{final}}(x_1, x_2; \Omega) \frac{d\Omega}{\langle \omega \rangle}}$$

we find

$$\langle n(s) \rangle = \beta(s) \frac{\langle \Omega(s) \rangle}{\langle \omega(s) \rangle} .$$

This equation has a very simple semi-classical interpretation. We notice that the quantity

$$\int_0^{\Omega} k d\bar{n}(k) = \int_0^{\Omega} dW(k) \simeq \beta(s) \Omega(s)$$

represents the energy radiated during parton scattering in the frequency range $0 - \Omega$. But then we can write for the radiated energy

$$\beta(s) \langle \Omega(s) \rangle = \langle n(s) \rangle \langle \omega(s) \rangle$$

i. e. the mean energy radiated equals the mean energy per track times the average multiplicity.

From eq. (1), one can see that the energy distribution function $dP(\Omega_{\text{vis}}, \Omega)$ scales in $\Omega_{\text{vis}}/\Omega$, so that the function $P(n, s)$ defined as

$$P(n, s) = \frac{\sigma_n^{\text{ch}}(s)}{\sum \sigma_n^{\text{ch}}(s)} \simeq \int d\Omega_{\text{vis}} \frac{dP(\Omega_{\text{vis}}, \langle \Omega \rangle)}{d\Omega_{\text{vis}}} \delta(\Omega_{\text{vis}} - n \langle \omega \rangle)$$

clearly will scale in $n \langle \omega \rangle / \langle \Omega \rangle$, i. e. in $\beta n / \langle n \rangle$. After some straightforward manipulation, one obtains

$$P(n, s) = \beta(s) \int \frac{dt}{2\pi} e^{i\beta \frac{n}{\langle n \rangle} t - \beta \int_0^1 \frac{dk}{k} (1 - e^{-ikt})}$$

We propose that the KNO function be given by

$$\Psi\left(\frac{n}{\langle n \rangle}\right) = \langle n \rangle P(n, s) .$$

This function obeys the two normalization conditions

$$\int_0^{\infty} \Psi(z) dz = 1 \quad \text{and} \quad \int z dz \Psi(z) = 1 .$$

We can calculate the moments of this distribution. Using eq. (1) and the definition of refs. (1, 2, 3), we find

$$\gamma_2 = \frac{1}{2\beta} ; \quad \gamma_3 = \frac{1}{3\beta^2} ; \quad \gamma_4 = \frac{1}{4\beta^3}$$

i. e.

$$\gamma_2 \simeq \frac{25}{32} \frac{1}{\ln \ln s / \Lambda^2} .$$

The above equation shows how KNO scaling is violated. As soon as eq. (3) is satisfied, i. e. in the asymptotic freedom region, the cumulants γ_i should start decreasing with increasing energy. The function $\beta(s)$ is the same function from which scaling violations in DIS are predicted⁽⁴⁾.

Using $\Lambda = 100$ MeV, we calculate β to be approximately 1.82. In Fig. 6 we show the fit to the UA1 data⁽²⁾. We notice that for very small values of the multiplicity the following behaviour obtains :

$$\Psi\left(\frac{n}{\langle n \rangle}\right) \underset{\beta \frac{n}{\langle n \rangle} \ll 1}{\sim} \left(\beta \frac{n}{\langle n \rangle}\right)^{\beta-1}.$$

This shows that the closer β is to 1, the flatter the curve will be. This can be seen in Fig. 7 where the UA1 data for $|\eta| < 1.5$ have been fitted using $\beta = 1.1$. The value for β is here strictly empirical. Its main justification lies in the fact that the selection of events in the range $|\eta| < 1.5$ means a reduced phase space and hence a smaller β .

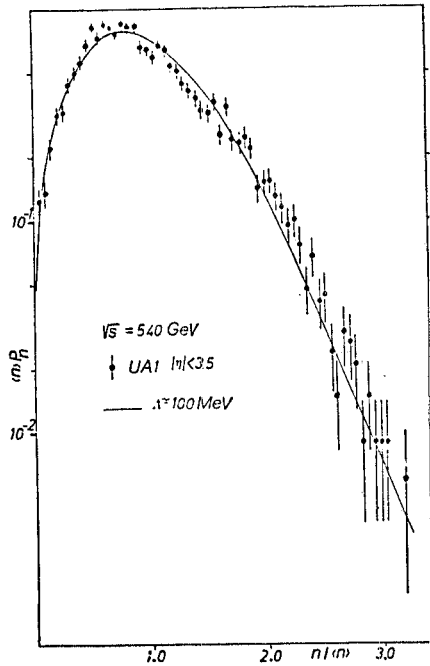


FIG. 6

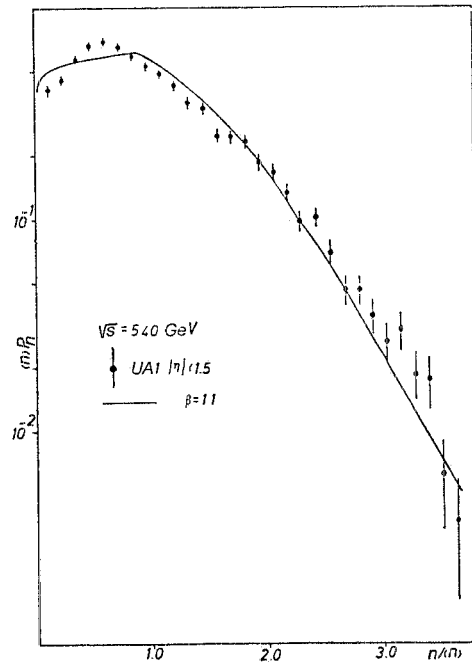


FIG. 7

4. - Conclusion

Finally, we would like to give a tentative answer to the question as to whether there is a connection between mean scaling, KNO scaling and geometric scalings^(14,15,16). There certainly is a connection insofar they

all reflect the same averaging process done on a common universal phenomenon, like soft gluon bremsstrahlung. However, geometric scaling and KNO scaling do seem to reflect different components of the same distribution: geometric scaling, being related to the spatial properties of the cross section, is probably connected to the transverse momentum distribution, while KNO scaling is related to the energy distribution.

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