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CRITICAL VOLTAGE LAW FOR CHIRAL PARTICLE PRODUCTION NEAR A MONOPOLE

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ABSTRACT

Recent progress in understanding the implications of winding number transitions and topological quantized magnetic flux in terms of a Sine-Gordon theory in charge space, is here discussed in terms of a previously derived critical voltage law for such processes in superconducting rings. In particular, chiral particle production in the neighborhood of a monopole will be of interest.

The close analogy between the theory of superconductors and the theory of non-Abelian gauge theories of fundamental interactions is well known[(1)]. Of interest here is the notion of topological magnetic flux quantization which appears in non-Abelian gauge theories in the Dirac unit

$$g = \left(\frac{2\pi\hbar c}{q}\right) \tag{1}$$

by virtue of the 't Hooft-Polyakov monopole[(2)] (q is the fundamental charge of the theory), and appears in superconducting rings in the unit

$$\Phi_o = \left(\frac{2\pi\hbar c}{2e}\right) \tag{2}$$

by virtue of the single-valued nature of the electron pair wave function.

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Recent progress in the theory of the Dirac-'t Hooft-Polyakov magnetic flux quantum by both Rubakov[3] and Callan[4] has produced a Sine-Gordon field theoretic description in charge space. Recent progress in both theoretical[5] and experimental[6] understanding of winding number transitions in superconducting rings has produced the notion of a critical voltage law in charge space[7].

Our purpose is to explain certain theoretical features of the monopole in terms of a critical voltage law. This relates advances made in recent years in understanding the nature of magnetic flux quantization due to topology in two different (but closely analogous) fields of research.

To understand how a critical voltage arises for the chiral production of particles near a monopole, one notes that the monopole field

$$\underset{\sim}{B} = (g/r^2)\hat{\underset{\sim}{r}} , \tag{3}$$

and the (solid) angular average of the electrostatic potential gradient

$$-\left(\frac{dV}{dr}\right) = \int \frac{(d\Omega)}{(4\pi)} \hat{\underset{\sim}{r}}\cdot\underset{\sim}{E} , \tag{4}$$

enter into the integral

$$\int(\underset{\sim}{E}\cdot\underset{\sim}{B})(d^3\underset{\sim}{r}) = -\int_0^\infty \left(\frac{dV}{dr}\right)\left(\frac{g}{r^2}\right)(4\pi r^2\, dr) , \tag{5}$$

in such a mannar that the monopole core voltage

$$V_g = -\int_0^\infty dV(r) , \tag{6}$$

directly determines the chiral production rate N_5, through the anomaly equation[8], to be

$$\left(\frac{1}{2}N_5\right) = (\text{constant})\cdot V_g . \tag{7}$$

Clearly, this is an "Ohm's law" for chiral current.

On the other hand, the topological winding numbers $n = 0, \pm 1, \pm 2, \ldots$ appear as eigen values of the equation

$$\left(\frac{1}{2}N_5\right)|n\rangle = n|n\rangle , \tag{8}$$

which in the language of θ-state, $\langle\theta|n\rangle = e^{in\theta}$,

$$\langle\theta|\left(\frac{1}{2}N_5\right)|n\rangle = n\langle\theta|n\rangle = -i\frac{\partial}{\partial\theta}\langle\theta|n\rangle , \tag{9}$$

yields the global operator representation for chiral number, i. e.

$$\left(\frac{1}{2}N_5\right)\Psi(\theta) = -i\frac{\partial}{\partial\theta}\Psi(\theta) , \tag{10}$$

where $\Psi(\theta)$ is the wave function for the topological monopole configuration. With $\Omega(\nu)$ as the direct tunneling frequency for $\Delta N_5 = 2\nu$ transitions, and neglecting the effects of the Witten angle[9] - for simplicity only - one associates with θ-states the energy

$$W(\theta) = \sum_{r=1}^{\infty} \hbar\,\Omega(\nu)\left[1 - \cos(\nu\theta)\right] , \tag{11}$$

and thus the chiral production rate

$$(\dot{N}_{5/2}) = (\frac{i}{\hbar})\left[W(\theta),\ -i(\frac{\partial}{\partial\theta})\right] = -\frac{\partial W(\theta)}{\partial\theta} = -\sum_{r=1}^{\infty} \nu\,\Omega(\nu)\sin(\nu\theta) . \tag{12}$$

Eqs. (7) and (12) yield the critical voltage law for the monopole potential provided that the θ-representation is expressed in charge units

$$V_g = (\text{constant}) \sum_{r=1}^{\infty} \nu\,\Omega(\nu)\sin(\frac{2\pi\nu Q}{q}) . \tag{13}$$

In such units, it is evident that the energy in Eq. (11), i. e. $W(\frac{2\pi Q}{q}) = \hbar\Omega\left[1 - \cos(\frac{2\pi Q}{q})\right]$, if only direct $\Delta N_5 = \pm 2$ transitions were allowed, minimizes at $Q = 0,\ \pm q,\ \pm 2q, \ldots$ which would be slightly phase shifted if the Witten angle were included.

As in the Josephson effect, whereby a more detailed quantum electrodynamic Sine--Gordon model[10] can be represented for qualitative purposes by a simple pendulum, such a model for the problem of chiral particle production is worthy of note.

A simple analog Josephson-Sine-Gordon pendulum Hamiltonian can be written as

$$H = -(\frac{qg}{2\Lambda})(\frac{\partial}{\partial\theta})^2 + \hbar\Omega\left[1 - \cos\theta\right] , \tag{14}$$

where both Λ and $(\frac{c}{\Omega})$ are length scales set by the fermion mass μ, i. e. Λ and $(\frac{c}{\Omega})$ are of order $(\hbar/\mu c^2)$. Thus, the wave function $\Psi(\theta, t)$ describing the state of the Vacuum in the region of the monopole is that of a fairly freely swinging pendulum

$$i\hbar\,\frac{\partial\Psi(\theta,t)}{\partial t} = H\,\Psi(\theta, t) , \tag{15}$$

in the sense that the WKJB barrier factor for a 2π quantum tunneling rotation of the pendulum (corresponding to a charge q creation) is of order unity. This verifies the ease with which both charge and chiral particle production accompanies the physical existence of a monopole.

A perhaps more sophisticated pendulum equation (with a similar physical content) has been used[11], previous to this work, to describe possible cosmological baryon production and destruction. The 't Hooft-Polyakov-Dirac monopole provides a mechanism for causing such baryon number non-conservation.

FOOTNOTES AND REFERENCES

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