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QUANTUM CHROMODYNAMICS RADIATION AND KNO SCALING

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ABSTRACT

We obtain the KNO scaling function from the parton model and QCD, after averaging the microscopic system of quarks and gluons over the hadronic matter coordinates. Energy conservation and soft-gluon emission distribution functions are used to derive an expression which exhibits scaling violations and fits the charged multiplicity very well.

Charged particle multiplicities in hadronic reactions have recently received a great deal of attention(1,2,3). In particular, various authors have tried to relate the observed KNO scaling behaviour(4) to geometrical models and hence to geometric scaling(5). In this paper we try to understand KN0 scaling within the framework of the parton model and its QCD corrections as a behaviour which obtains after averaging the microscopic system, which consists of quarks and gluons, over hadronic matter coordinates. In this picture the scaling laws clearly reflect a mean behaviour of the microscopic system.

We start by considering hadron-hadron scattering at high energy, i.e. a situation where two point-like constituents can interact at short distance. In such a scattering process, gluons are always emitted. We assume that the mesons produced in a process like

$$pp \to n\pi + pp$$

or

$$p\bar{p} \to n\pi + \text{leading baryons}$$

come from the QCD radiation emitted by the quarks participating in a hard scattering process. Let Ω be the energy which is lost to radiation by each quark leg. In hadron-hadron scattering this quantity is unobserved and has to be integrated between a minimum and a maximum value determined by the kinematics. Let Ω_{vis} be the energy (always a finite fraction of Ω) which is converted into pions. Notice that Ω_{vis} is the energy carried by a colour singlet state, and that the number of gluons which carry this energy cannot be determined. In other words, while we can count the pions, we cannot count the gluons. Thus we must sum over all possible gluon configurations. Let $P(\{n_g\})$ represent a distribution of $n_1(k_1)$ gluons of momentum k_1, $n_2(k_2)$ gluons of momentum k_2, etc. and total energy $\Sigma k n(k) = \Omega_{vis}$. Each individual distribution can be taken to be Poissonian, if we assume that the gluons are emitted independently of one another. This corresponds to the no-recoil approximation. The no-recoil approximation can be justified in QCD since the running coupling constant makes a large recoil event (characterized by a large momentum transfer Q^2) non-leading relative to a no-recoil one for which α_s is presumably large. Then the probability that out of an energy Ω, an amount Ω_{vis} is emitted via independent gluon production is given by

$$\frac{dP(\Omega_{vis},\Omega)}{d\Omega_{vis}} = \sum_{n_g} P(\{n_g\})\, \delta(\Sigma k\, n(k) - \Omega_{vis}) = \int \frac{dt}{2\pi}\, e^{i\Omega_{vis} t}\, e^{-\int_0^{\Omega} d\bar{n}(k)\,(1-e^{-ikt})} \quad (1)$$

The above energy distribution function must next be folded with the cross section for parton-parton scattering $\hat{\sigma}_{ij \to final}(x_1,x_2;\Omega)$, where the available energy for the final state has been reduced by the amount Ω, and x_1,x_2 are the momentum fractions of the partons produced with densities f(x). For an experimental situation in which the leading proton momenta are not observed and the π^o's are escaping detection, we have

$$\frac{d\sigma_n^{ch}}{d\omega_1 \ldots d\omega_n} = \sum_{i,j,final} \int f_i(x_1)dx_1 \int f_j(x_2)dx_2 \int d\Omega\, \hat{\sigma}_{ij \to final}(x_1, x_2;\Omega) \cdot \int d\Omega_{vis} \frac{dP(\Omega_{vis},\Omega)}{d\Omega_{vis}}\, \delta(\Omega_{vis} - \sum_1^n \omega_i)\, D_\pi^{(n)}(\Omega;\omega_1 \ldots \omega_n) \quad (2)$$

where $D_{\pi}^{(n)}(\Omega; \omega_1 \dots \omega_n)$ is the inclusive probability for production of n pions out of an available energy Ω. The charged n particle cross-section is then obtained as follows:

$$\sigma_n^{ch}(s) = \sum_{\substack{i,j \\ \text{final}}} \int f_i(x_1)dx_1 \int f_j(x_2)dx_2 \int d\Omega \ \hat{\sigma}_{ij \to \text{final}}(x_1, x_2, \Omega) \quad Q(n,\Omega) \tag{3}$$

where

$$Q(n,\Omega) = \int d\omega_1 \dots \int d\omega_n \, D_{\pi}^{(n)}(\Omega; \omega_1 \dots \omega_n) \int \frac{dt}{2\pi} e^{it\sum_i^n \omega_i} e^{-\int_0^\Omega d\bar{n}(k)(1-e^{-ikt})} \tag{4}$$

The effect of integrating the energy distribution function over the individual pion energies can be taken into account by using the mean value theorem and making the substitution

$$\int d\omega_1 \dots \int d\omega_n D_{\pi}^{(n)}(\Omega; \omega_1 \dots \omega_n) \, e^{it\sum_1^n \omega_i} \longrightarrow e^{it\, n\, \langle \omega(n,s) \rangle} \int d\omega_1 \dots \int d\omega_n D_{\pi}^{(n)}(\Omega; \omega_1 \dots \omega_n) \tag{5}$$

where each ω_i in the exponential has been replaced by its mean value $\langle \omega(n,s) \rangle$. We now have

$$Q(n,\Omega) = \int d\omega_1 \dots \int d\omega_n D_{\pi}^{(n)}(\Omega; \omega_1 \dots \omega_n) \int \frac{dt}{2\pi} e^{in\, t \langle \omega(n,s) \rangle} e^{-\int_0^\Omega d\bar{n}(k)(1-e^{-ikt})} \tag{6}$$

To proceed further, we must make some assumptions about the function $D_{\pi}^{(n)}$. This function represents the probability that n pions of energies $\omega_1, \omega_2, \dots \omega_n$ are formed out of an available energy Ω. Thus it is similar to a fragmentation function of a quark into n pions. In an independent emission model, we can write

$$D_{\pi}^{(n)}(\Omega; \omega_1 \dots \omega_n) \underset{\text{large } n}{\sim} d_{\pi}^{(1)}(\Omega; \omega_1) \dots d_{\pi}^{(1)}(\Omega; \omega_n) \tag{7}$$

and normalize each $d_{\pi}^{(1)}$, i.e.

$$\int_0^\Omega d\omega \; d_{\pi}^{(1)}(\Omega;\omega) = 1 \tag{8}$$

assuming that each pion could in principle be obtained from the entire available energy. Notice that energy conservation is being maintained by the energy distribution function. Within the above model, the mean energy per charged track $\langle\omega(n,s)\rangle$ can be taken to be a costant in n. Clearly this assumption, which follows from the independent emission model, is only valid up to corrections of order 1/n. (This assertion, which is supported by the experimental data, will be commented upon later). Then, if we write

$$\langle\omega(n,s)\rangle \underset{\text{large } n}{\sim} \langle\omega(s)\rangle \tag{9}$$

the charged particle cross-section becomes

$$\sigma^{ch}(n,s) = \sum_{i,j,\text{final}} \int f_i(x_1) \int f_j(x_2) dx_1\, dx_2 \int \hat{\sigma}_{ij \to \text{final}}(x_1, x_2; \Omega)\, d\Omega \quad . \\ \cdot \int \frac{dt}{2\pi}\, e^{in\langle\omega\rangle t}\, e^{-\int_0^{\Omega} d\bar{n}(k)(1-e^{-ikt})} \tag{10}$$

with n a continuous variable, in the large n region.

The details of the parton subprocess can now be averaged over, by substituting for Ω its mean value $\langle\Omega\rangle$ as follows. We calculate, in the large n limit,

$$\langle n(s)\rangle_{ch} = \frac{\int n\, dn\, \sigma^{ch}(n,s)}{\int dn\, \sigma^{ch}(n,s)} \tag{11}$$

Using the normalization property

$$\langle\omega\rangle \int dn \int \frac{dt}{2\pi}\, e^{in\langle\omega\rangle t - \int_0^{\Omega} d\bar{n}(k)(1-e^{-ikt})} = 1 \tag{12}$$

and after performing an integration by parts of the numerator, we obtain

$$\langle n(s)\rangle_{ch} = \frac{\sum \int f_i(x_1)dx_1 \int f_j(x_2)\, dx_2 \int \hat{\sigma}(x_1,x_2;\Omega)d\Omega \,/\langle\omega\rangle^2 \int_0^{\Omega} k d\bar{n}(k)}{\sum \int f_i(x_1)dx_1 \int f_j(x_2)dx_2 \int \hat{\sigma}(x_1,x_2;\Omega)\, d\Omega/\langle\omega\rangle} \tag{13}$$

The integral $\int_0^{\Omega} k d\bar{n}(k)$ represents the total energy dissipated by the emitted quarks in the frequency band from 0 to Ω. This quantity reflects the detailed dynamics, as it is different for different parton subprocesses. We parametrize it as

$$\int_0^{\Omega} k\, d\bar{n}(k) = \beta(s)\Omega \tag{14}$$

with β approximated by a constant in Ω. This gives

$$\langle n(s)\rangle_{ch} \sim \beta(s) \frac{\sum \int f_i(x_1)dx_1 \int f_j(x_2)dx_2 \int \hat{\sigma}(x_1,x_2;\Omega)\,\Omega/\langle\omega\rangle \; d\Omega/\langle\omega\rangle}{\sum \int f_i(x_1)dx_1 \int f_j(x_2)dx_2 \int \sigma(x_1,x_2;\Omega)\, d\Omega/\langle\omega\rangle} \tag{15}$$

Thus the mean value for $\Omega/\langle\omega\rangle$ is $\langle n\rangle/\beta$. This allows us to use the mean value theorem once more and finally obtain

$$\mathbb{P}^{ch}(n,s) = \frac{\sigma^{ch}(n,s)}{\sigma_{in}(s)} =$$

$$= \frac{\sum \int f_i(x_1)dx_1 \int f_j(x_2)dx_2 \int \hat{\sigma}(x_1,x_2;\Omega)d\Omega/\langle\omega\rangle \int \frac{dt}{2\pi} e^{int - \int_0^{\Omega/\langle\omega\rangle} d\bar{n}(k)(1-e^{-ikt})}}{\sum \int f_i(x_1)dx_1 \int f_j(x_2)\,dx_2 \int \hat{\sigma}(x_1,x_2;\Omega)\, d\Omega/\langle\omega\rangle} \tag{16}$$

$$\simeq \int \frac{dt}{2\pi} e^{int} \; e^{-\int_0^{\langle n\rangle/\beta} d\bar{n}(k)(1-e^{-ikt})}$$

The above function depends only upon β and $n/\langle n\rangle$. We propose that the KNO function is given by

$$\Psi(n/\langle n\rangle) = \langle n\rangle\, \mathbb{P}^{ch}(n,s) = \beta(s) \int \frac{dt}{2\pi} e^{i\beta \frac{n}{\langle n\rangle} t} \; e^{-\beta \int_0^1 \frac{dk}{k}(1-e^{-ikt})} \tag{17}$$

Defining the scaling variable $z = n/\langle n\rangle$, we have from Eq. (17) the result

$$\Psi(z) = \frac{1}{\gamma^{\beta}\Gamma(1+\beta)} (\beta z)^{\beta-1}\, p(\beta z, \beta) \tag{18}$$

with $\gamma = 1.781$ (Euler's constant). The function $p(y,\beta)$ obeys the integral equation

$$p(y,\beta) = p(y_0,\beta) - \beta \int_{y_0}^{y} \frac{dx}{x}\left(1-\frac{1}{x}\right)^{\beta-1} p(x-1,\beta) \tag{19}$$

with $p(y,\beta) = \varphi(y)$ for $y \leqslant 1$.

Our function obeys the two normalization conditions

$$\int z\,\varphi(z)\,dz = 1 \qquad (20a)$$

and

$$\int \varphi(z)\,dz = 1 \qquad (20b)$$

ant it can be used to calculate the moments $\langle n^q \rangle$ as well as the reduced cumulants γ_2, γ_3, γ_4 defined in refs. (1,2) and (3):

$$\gamma_2 = \frac{\langle (n-\langle n \rangle)^2 \rangle}{\langle n \rangle^2} \;; \qquad \gamma_3 = \frac{\langle (n-\langle n \rangle)^3 \rangle}{\langle n \rangle^3} \;; \qquad \gamma_4 = \frac{\langle (n-\langle n \rangle)^4 \rangle - 3\langle (n-\langle n \rangle)^2 \rangle^2}{\langle n \rangle^4}$$

It is interesting to observe that in our case these cumulants are directly related to the nth moment of the single gluon distribution function i.e.

$$\gamma_n = \frac{\int k^n \, d\bar{n}(k)}{\left[\int k \, d\bar{n}(k)\right]^n} \qquad (21)$$

For constant β, Eq. (21) gives simply

$$\gamma_n = \frac{1}{n\,\beta^{n-1}} \qquad (22)$$

It should be noted that, because of the approximation involved in using a constant β, we do not trust this equation for large values of n. On the other hand, we can use it for n=2 to directly relate γ_2 to the single gluon spectrum β, defined through Eq. (14). In the asymptotic freedom limit, one can write approximately (5)

$$\beta \sim \frac{16}{25} \ln\, \ln (s/\Lambda^2) \qquad (23)$$

Eqs. (22) and (23) show that γ_2 must decrease, albeit slowly, as s increases. Such a behaviour is experimentally observed in the very high energy region[2,3]. In Fig. 1 we compare γ_2 as given by

$$\gamma_2 = \left[\frac{32}{25} \ln\, \ln(s/\Lambda^2)\right]^{-1} \qquad \text{large s}$$

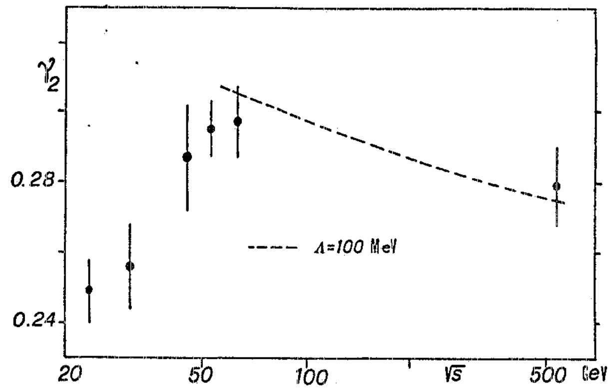

FIG. 1 - A plot of γ_2 vs. $\sqrt{s}$ is shown. The data points are from Thomé et al.[1] and UA1(CERN EP-134). The dashed line is our asymptotic expression, Eq. (24).

with the experimental data up to the collider energy. We believe that the decrease of γ_2 after $\sqrt{s} \simeq 60$ GeV signals the onset of asymptotic freedom.

We may estimate β at the superhigh collider energy $\sqrt{s}$ = 540 GeV, through Eq. (23). Using Λ=100 MeV, we find

$$\beta = 1.82$$

Having determined β, which was the only parameter, we can give an approximate formula for $\psi(z)$ through Eqs. (17-19). The result is shown in Fig. 2 along with the collider data for $\eta < 3.5$. For $\eta < 1.5$, the multiplicity curve can still be fitted, on the average, by our proposed $\psi(z)$ as one can see from Fig. 3. However one needs a smaller value of β, as one can heuristically expect from the fact that the available phase space in the second case is reduced. One should notice, in Figs. 2 and 3, that our proposed curve interpolates between the experimental points, as expected from an average behaviour.

We now compare our result, Eq. (17), with some other approaches to the problem[5,7,8]. If we perform an asympotic expansion of Eq. (17), valid for β "large", but β(z-1) small, we obtain

$$\psi_G(z) = \sqrt{\frac{\beta}{\pi}}\, e^{-\beta(z-1)^2} \qquad (25)$$

i.e. a Gaussian distribution. Lam and Yeung[5] define an elementary distribution given by Eq. (25). This distribution is folded with the impact parameter function $\Omega(b,s)$ to obtain the KNO distribution $\psi(z)$. For the particular value β=2, Eq. (25) should give their entire $\psi(z)$. These authors observe that this leads to an inconsistency since Eq. (25) is symmetrical about z=1, while the observed $\psi(z)$ is not. This difficulty is resolved in our case, since Eq. (25) is only an asymptotic form and indeed Eq. (17) is not symmetrical about z=1.

Also, for small z, Eqs. (17-19) lead to

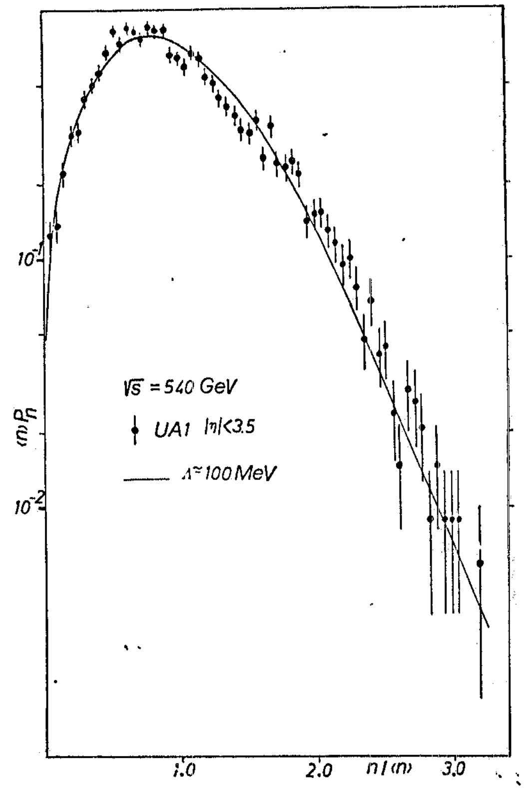

FIG. 2 - Ψ(z) vs. z is plotted. Data are from UA1 (CERN EP 82-134). The theoretical curve is from Eq. (17) with Λ=100 MeV i.e. β=1.82.

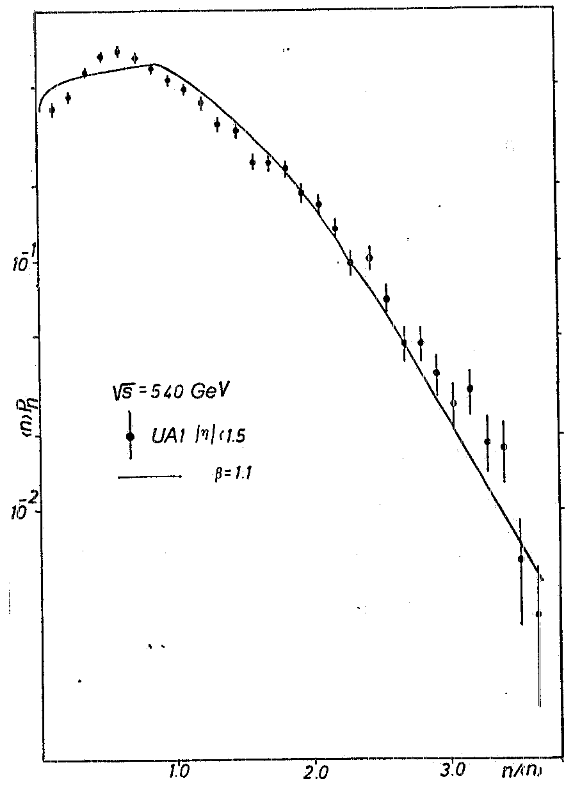

FIG. 3 - Ψ(z) vs. z is plotted for $|\eta| < 1.5$. Data are from UA1(CERN EP 82-134). The theoretical curve is from Eq. (17) with β=1.1.

$$\Psi(z) \underset{\beta z < 1}{\sim} z^{\beta-1} \qquad (26)$$

Thus for β=2, $\Psi(z) \sim z$ for small z, a result common to the de Groot and Barshay models as well as to the Slattery fit.

Eq. (26) also leads to the interesting result that if, for large s

(i) σ (n,s) has a finite limiting value, for $n \ll \langle n \rangle$,

(ii) $\langle n \rangle \sim \ln s$ and

(iii) $\sigma_{in}(s) \sim (\ln s)^r$

then KNO scaling demands that β=r. Thus the Froissart limiting cross-section corresponds to β=2. It is a pleasing result to find that $\beta \simeq (1.8 \div 2.0)$ gives the best description of the observed data. Choosing β=2, it is easy to verify a posteriori that indeed the mean energy per track is given by

$$\langle \omega (n,s) \rangle = \langle \omega \rangle + \frac{1}{n} \left(\frac{\langle \omega^2 \rangle - \langle \omega \rangle^2}{\langle \omega \rangle} \right)$$

This justifies our assumption in Eq. (9).

In conclusion, we have developed the KNO scaling behaviour as reflecting an average over the hadronic matter coordinates, i.e. over the variables describing the quark and gluon system. The charged multiplicity distribution appears well described quantitatively through the soft gluon distribution function. Asymptotic freedom limit ensures us that all the cumulants decrease with increasing energy. These results are well supported by the data.

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