ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-82/76(P)
27 Ottobre 1982

G. Martinelli: LATTICE QCD: RECENT RESULTS FOR HADRON SPECTROSCOPY

Invited talk at the "VI European Symposium on $\overline{N}N$ and $\overline{Q}Q$ Interactions", Santiago de Compostela (Spain), 1982

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ABSTRACT. - I report the results obtained in the last year in lattice QCD with fermions.

1. - INTRODUCTION

Recently many exciting progresses have been made in the computation of hadronic spectrum starting from lattice QCD. I will briefly report here the more relevant results.

The plan of the talk is the following: in section 2 I will recall the basic ingredients of lattice gauge theories; in section 3 I will discuss in some lenght the computation of the hadron mass spectrum and finally in section 4 I will present the relevant results obtained from Montecarlo simulations and compare them with the corresponding experimental quantities.

2. - WHY QCD ON THE LATTICE ?

We believe that the hadronic world can be described starting from a fundamental theory of interacting quarks and gluons : QCD. All the strong interaction physics should be contained in the following action :

$$S = \int d^4x \left\{ \left[-\frac{1}{2}\bar{q}(x)\gamma^{\mu}\overleftrightarrow{D}_{\mu}q(x) - m_q\bar{q}(x)q(x) \right] - \frac{1}{4}F_{\mu\nu}(x)^2 \right\} \tag{1}$$

$D_{\mu} = \partial_{\mu} + ig_o A^a_{\mu}\lambda^a$ is the covariant derivative , $F^a_{\mu\nu} = \partial_{\mu}A^a_{\nu} - \partial_{\nu}A^a_{\mu} - g_o f^{abc} A^b_{\mu}A^c_{\nu}$.

Unfortunately even such a simple action gives rise to a very complicated physics and, unlike for weak or electromagnetic interactions, it is possible to use perturbation theory only for deep inelastic (very high energy) phenomena.

Lattice QCD is at present the only formulation which is able to give quantitative predictions for hadron physics (masses, widths, ...) in the low energy domain where we cannot speak any more of quarks and gluons but only of pions, protons, etc. To put QCD on a lattice is a very natural way of introducing a gauge invariant ultraviolet cutoff : the continuum limit is obtained when the lattice spacing $a \to 0$ For the lattice action we require :

i) Local (lattice) gauge invariance;

ii) Formal $a \to 0$ continuum limit.

A possible action for a pure gauge field theory, first proposed by Wilson[1], has the form:

$$S_G(U) = \frac{1}{g_o^2} \sum_P \mathrm{tr}\,(U_P + U_P^+) \tag{2}$$

where U_P is the product of link matrices belonging to an elementary plaquette as shown in Figs. 1 and 2:

$$U_P = U_\mu(x) U_\nu(x+\hat{\mu}) U_\mu^+(x+\hat{\nu}) U_\nu^+(x) \,. \tag{3}$$

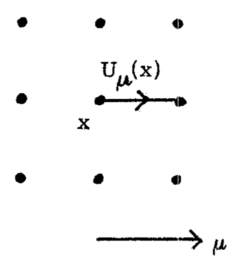

FIG. 1 - Definition of the link variable; the dots represents the lattice points.

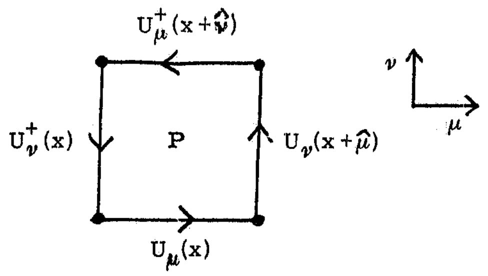

FIG. 2 - Definition of U_P (see text).

The link variable $U_\mu(x)$ is related to the gluon field $A^a_\mu(x)$ by the equation:

$$U_\mu(x) = e^{iag_o A^a_\mu(x) \lambda^a} \,. \tag{4}$$

It is very easy to show that:

$$S_G(U) \xrightarrow[a \to 0]{} -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) \tag{5}$$

and that $S_G(U)$ is invariant under local gauge transformations:

$$U_\mu(x) \to g(x)\, U_\mu(x)\, g^{-1}(x+\hat{\mu}) \,. \tag{6}$$

The only parameters which enter in the action are the lattice bare coupling constant g_o and implicitely the lattice spacing a. All masses and lenghts may be expressed as functions of these parameters:

$$m_i = \frac{1}{a} f_i(g_o) \,. \tag{7}$$

For $a \to 0$ the physics should become independent of a; this means for instance:

$$a \frac{dm_i}{da} = 0 \,. \tag{8}$$

In this limit, eq. (8) implies that all masses and lenghts become proportional to a single fundamental scale:

$$m_i = C_{m_i} \Lambda_{LATT} \tag{9}$$

where C_{m_i} are dimensionless constants. Using the renormalization group equation for g_o:

$$-a \frac{dg_o}{da} = -\beta_o g_o^3 - \beta_1 g_o^5 \tag{10}$$

and asymptotic freedom, one finds:

$$\Lambda_{LATT} = \frac{1}{a} \left(e^{-\frac{1}{2\beta_o g_o^2}} \right) (\beta_o g_o)^{-\frac{\beta_1}{2\beta_o^2}} \cdot \left[1 + O(g_o^2) \right] \,. \tag{11}$$

At Λ_{LATT} fixed (because it is a renormalization group invariant quantity) eq. (11) shows that the limit $g_o \to 0$ corresponds to the continuum $a \to 0$ limit. In this limit all the correlation lenghts $\xi_i / a = 1/m_i a$ go to infinity in units of the lattice spacing: we are in presence of a second order phase transition in the language of statistical physics.

All the relevant informations of a gauge theory are given by the expectation values of gauge invariant operators. For $t = ix_4$ we have:

$$\langle \hat{O}(U) \rangle = \frac{\int d[U]\, e^{-S_G(U)} O(U)}{\int d[U]\, e^{-S_G(U)}} \tag{12}$$

The exact computation of the functional integral in eq. (12) is a formidable task even on the lattice: to evaluate this functional integral several non perturbative methods of approximation are at disposition; the two methods which at present are able to give more accurate quantitative results are strong coupling expansions and Montecarlo simulations.

a) Strong coupling expansion (SCE):

Let us write the exponential of the action appearing in the functional integral of eq. (12) as:

$$W = e^{-S_G(U)} = e^{-\beta H} = e^{-\frac{\beta}{2N}\sum_P \mathrm{tr}\,(U_P + U_P^+)}$$

$$\beta = \frac{2N}{g_o^2} \qquad (13)$$

β plays the same role of the inverse temperature in statistical physics. In SCE one makes a Taylor expansion around $\beta = 0$ ($g_o \to \infty$) of W. Since at the end we are interested to extrapolate to the weak coupling region ($\beta \to \infty$) this expansion must be analytic in β. Due to singularities near the real β axis(2) or to the presence of an infinite order (roughening) phase transition(3) it is not obvious that informations on the interesting scaling region can be extracted by strong coupling expansion.

b) Montecarlo simulations:

One makes the direct integration of the gauge fields (eq. (12)) on a finite lattice.

One of possible algorithms is the following (Metropolis method):

1) Start with a trial gauge field configuration $\{U\}$; $N_{ex} = 1$.
2) Choose another link $U' \to \{U'\}$.
3) If $\exp\left[-(S_G(\{U'\}) - S_G(\{U\})\right] > x'$, with x' randomly choosen between $[0, 1]$ with a flat distribution, $\{U'\}$ is the new configuration.
4) $N_{ex} = N_{ex} + 1$; return to 2).

It can be shown that the probability distribution of U goes, for $N_{ex} \to \infty$, as:

$$P(U)\,dU \to \exp\left[-S_G(U)\right] dU . \qquad (14)$$

The expectation value of some operator becomes:

$$\langle \hat{O}(U) \rangle = \frac{\sum_{\{U\}} O\left[\{U\}\right]}{N_{ex}} \qquad N_{ex} \to \infty . \qquad (15)$$

The main limitations to this method come from the limitations on the size of the lattice imposed by computer memory and speed. The situation is described in Figs. 3.

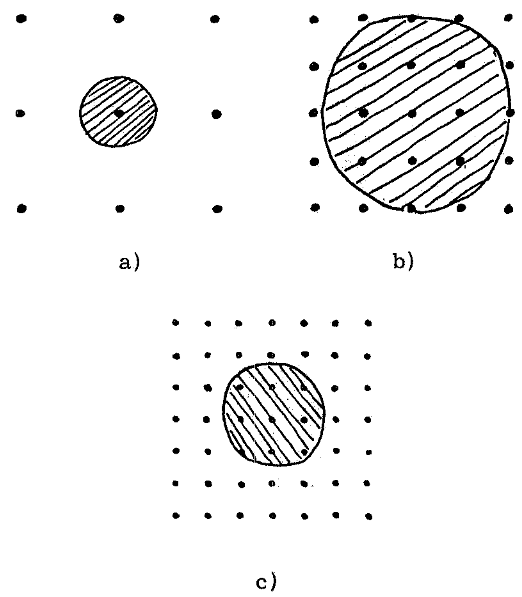

FIG. 3 - a) Hadron on the lattice for β small: the size of the hadron is smaller then the lattice spacing. b) The hadron for $\beta \to \infty$ becomes as large as the whole lattice. c) The optimal situation is obtained with a lattice made by a very large number of points: the hadron is large if compared to lattice spacing and small with respect to the lattice size.

In Fig. 3a (β small) the size of the hadron is small compared to lattice spacing: this is certainly not a good description of the continuum limit. In Fig. 3b (β large) the hadron contains many lattice points but because the maximum number of lattice points is small the hadron, put in a small box, feels very strong finite size effects. The ideal situation is described in Fig. 3c. However, for the actual computations, this situation is never reached, because it would require a lattice made by a huge number of lattice points.

To understand this problem it is useful to look at the behavior of the mass of some particle (let us say the glue ball O^{++} mass) as a function of β on a lattice of finite size.

In Fig. 4 a typical result from Montecarlo simulations is reported. The points with errors represent the measurements from computer simulations. The full curve is given by strong coupling expansion, the dashed lines are the expected renormalization group behavior (cfr. eq. (11)), and the dotted-dashed line is the spin-wave ($\beta \to \infty$ on a finite lattice) prediction.

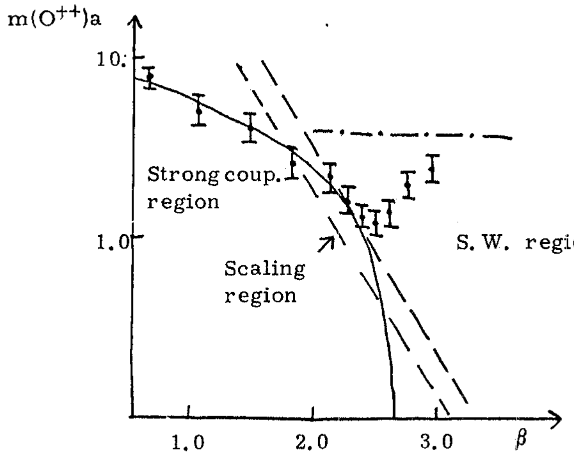

FIG. 4 - Schematic representation of the behavior of the O^{++} glue ball mass as a function of β (cfr. text). The scales of the plot are arbitrary.

For small β we are in the strong coupling region where the lattice physics is completely different from the continuum physics ; for large β finite size effects prevent us to obtain sensible results : Only in a very small window (in β not in lenght scale) we get results which can be extrapolated to the continuum using the renormalization group.

Note that the observation of the correct scaling behavior of the masses is crucial to give predictions for the continuum theory.

3. - LATTICE QCD WITH FERMIONS

In the preceding sect. we wrote one of the possible pure gauge field actions on the lattice. It is possible to add fermions in a gauge invariant way by writing the following expression for interacting quarks and gluons[4]:

$$S = \sum_f \left\{ \sum_x \left[-\frac{1}{2} \sum_\mu (\bar{\Psi}^f(x)(r-\gamma_\mu) U_\mu(x) \Psi^f(x+\hat{\mu}) + \bar{\Psi}^f(x+\hat{\mu})(r+\gamma_\mu) U^+_\mu(x) \Psi^f(x)) + (m_f + 4r)\bar{\Psi}^f(x)\, \Psi^f(x) \right] \right\} + S_G(U) = \bar{\Psi}^f \Delta_f(U)\, \bar{\Psi}^f + S_G(U) . \tag{16}$$

$S_G(U)$ is the pure gauge field action. Ψ^f is the quark field with flavour f. In eq. (15) for $r = 0$ we have the Kogut-Susskind[5] like action, plagued by the fermion doubling problem ; for $r = 1$ we obtain the action originally proposed by Wilson[1] which however does not have the same chiral properties of the continuum QCD action.

We are interested in the expectation value of some gauge invariant operator depending on the quark and gluon fields :

$$\langle \hat{O}(\Psi, \bar{\Psi}, U) \rangle = \frac{\int d[\Psi]\, d[\bar{\Psi}]\, d[U]\, e^{-S} O(\Psi, \bar{\Psi}, U)}{\int d[\Psi]\, d[\bar{\Psi}]\, d[U]\, e^{-S}} . \tag{17}$$

The formal functional integration over the fermionic degrees of freedom is possible because the action is quadratic in the fermion fileds[f1]. We obtain:

$$\langle \hat{O} \rangle = \frac{\int d[U]\, e^{-S_G(U)} \prod_f \det[\Delta_f(U)]\, \tilde{O}[U, \Delta_f^{-1}(U)]}{\int d[U]\, e^{-S_G(U)} \prod_f \det[\Delta_f(U)]} \tag{18}$$

The determinant of $\Delta_f(U)$ contains the effects of quark loops on the gluon Green functions (see Fig. 5); for a fixed gauge field configuration $\{U\}$, $\Delta_f^{-1}(U)$ is the quark propagator in presence of the external field $\{U\}$ as illustrated in Fig. 6. The main difficulty comes from the computation of the determinant and of the

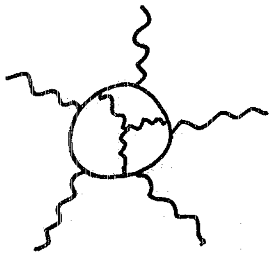

FIG. 5 - Typical diagram contributing to det $\Delta_f(U)$.

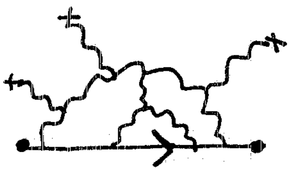

FIG. 6 - Quark propagator in presence of the field $\{U\}$.

inverse of $\Delta_f(U)$: for a typical lattice size used in Montecarlo simulations ($5^3 \times 10 \div 6^3 \times 12$) the number of matrix elements of $\Delta_f(U)$ is $\sim 10^9$.

Several methods have been proposed to compute the determinant and the inverse of $\Delta_f(U)$(6, 7); by using relaxation techniques(7), it came out that the computation of the quark propagator is much easier then the computation of the determinant. Most of the results for hadronic spectroscopy have been obtained by putting det $\Delta_f(U) = 1$ (quenched approximation) so that only the propagator must be computed: in this approximation we completely neglect the feedback of the quarks on the gluons. The approximation is expected to work reasonably well for several reasons: it is exact for $N_{colour} \to \infty$ at n_f fixed; Zweig rule is exactly satisfied and hadrons are made only by valence quarks and gluons which is not so far from what happens in the real world; the results obtained with the hopping parameter expansion(8) seem to confirm that quark loop effects only amount to a small changement for the relevant physical quantities. It is clear that it will be necessary in the future a complete computation of the functional integral of eq. (18) including the det $\Delta_f(U)$. In the quenched approximation eq. (18) becomes:

$$\langle O \rangle = \frac{\int d[U]\, e^{-S_G(U)}\, \tilde{O}\left[U, \Delta_f^{-1}(U)\right]}{Z} . \qquad (19)$$

And the integral over the gauge fields U can be made by usual Montecarlo methods.

To compute the hadron mass spectrum we start by defining operators carrying the same quantum numbers of the particles we want to measure the mass. For example for the pion, rho, proton and Δ^{++} particles we find:

$$\begin{aligned} \pi^+(x) &= \bar{u}^A(x)\gamma^5 d^A(x) , \\ \varrho^+_\mu(x) &= \bar{u}^A(x)\gamma_\mu d^A(x) , \\ P_\delta(x) &= (u^A(x)\mathbb{C}\,\gamma_5 d^B(x))\, u^C_\delta(x)\, \epsilon_{ABC} , \\ \Delta^{++}_{\mu,\delta}(x) &= (u^A(x)\mathbb{C}\,\gamma_\mu u^B(x))\, u^C_\delta(x)\, \epsilon_{ABC} . \end{aligned} \qquad (20)$$

It is straightforward to compute the correlation function for these operators. For example for the pion:

$$G(x) = \langle \pi(x)\pi^+(o)\rangle = \frac{\int d[U]\, e^{-S_G(U)}\, \mathrm{tr}\left[G^u(x,0)\gamma^5 G^d(0,x)\gamma^5\right]}{Z} \qquad (21)$$

$G^f(x,0)$ is the propagator of a quark with flavour f between 0 and x. For large time distances the pole corresponding to the lowest lying state of mass m will steam out and we expect:

$$G(t) = \sum_{\vec{x}} G(\vec{x},t) \xrightarrow[t\to\infty]{} A\, e^{-mt} . \qquad (22)$$

In principle, by fitting the various propagators we can obtain the hadron mass spectrum in lattice spacing units. In practice, because of the small size of available lattices (t small) the evaluation of hadron masses will be affected by systematic effects (for a detailed discussion see refs. (9, 10, 11)) due to the presence of many excited states propagating simultaneously in the same channel.

For mesons the coefficient A in eq. (22) is related to the determination of f_π, f_ϱ, ... The traslation of hadron masses in physical units is obtained by fixing a strong interaction scale (e. g. $m_\varrho^2 - m_\pi^2$) and a mass parameters for each quark flavour (which corresponds to fix for example m_π, m_K, etc.). β must be large enough so that all the masses scale in a well known computable way, e. g. :

$$\Lambda_{MOM} = \frac{\pi}{a}\left[\frac{8\pi^2}{33}\beta\right]^{51/121} \exp\left[-\frac{4\pi^2}{33}(\beta - 2.74)\right] . \tag{23}$$

No systematic analysis has been yet done on the scaling behavior of hadron masses, albeit the results (cfr. next section) within large statistical and systematic errors are compatible with scaling. The lack of scaling found for excited glueball states[12] indicates that a more carefull analysis on this point is needed.

4. - RESULTS FOR HADRON SPECTROSCOPY[f2]

The results for hadron spectroscopy (SU(3) of colour) obtained by several groups[9, 10, 15] are reported in Tables I, II and III. The different groups made their computations with different lattice sizes (ranging from $4^3 \times 8$ to $6^3 \times 14$), at different values of β (from 4.0 to 6.4) and using different expressions for the lattice action (r = 0, 1/2 and 1 ; cfr.

TABLE I - Results for strange hadrons.

STRANGE HADRONS	
m_{K^*}	890±70 MeV
m_ϕ	990±50 MeV
m_{Ω^-}	1.68±0.50 GeV
$m_\Sigma - m_\Lambda$	~20 MeV

TABLE II - Results for charmed hadrons.

CHARMED HADRONS	
$m_{\eta_c}(2980)$	3000±30 MeV
$J^{PC}=0^{++}$ m_{χ_o} (3414)	3400±50 MeV
$J^P=1^{++}$ m_{P_c} (3507)	3500±50 MeV
$J^{PC}=1^{+-}$	3600±100 MeV

TABLE III - List of the results for no-strange no-charmed hadrons. The value of r (cfr. eq.(16)), the lattice size (Volume), the number of gauge field configurations (N_{conf}) and of β are reported. Also the total computer time (t) spent in units of the CDC-7600 computer time is given. Then the values of physical hadronic quantities are reported. $m^B_{u,d}$ are the bare quark masses.

Ref. (9)	Ref. (10)	Ref. (10)	Ref. (15)	
1	0	1	0.5	r
5^3x10	6^3x12	5^3x8 6^3x12	4^3x8 4^3x12 6^3x14	Volume
32	8	28	8	$N_{conf.}$
6.0	6.0	5.6 6.0 6.4	4.0 5.55 5.7	β
~90 h	~150 h		~100 h	t
75±8 MeV	70 MeV	70 MeV	73 MeV	$\Lambda_{\overline{MS}}$
780 MeV	750 MeV	750±80 MeV	610^{+100}_{-120} MeV	m_ϱ
	970 MeV	950±100 MeV	730^{+240}_{-400} MeV	$m_{\delta(980)}$
	1120 MeV	1100±150 MeV		m_{A_1}
	1230 MeV	≳ 1100 MeV		m_B
200±15 MeV	180 MeV	150±50 MeV	81^{+13}_{-27} MeV	f_π
0.63±0.05		0.50±0.10	$0.45^{+0.15}_{-0.13}$	f_ϱ^{-1}
4.5±0.6 MeV		4.5 MeV	$7.0^{+4.2}_{-1.3}$ MeV	$m^B_{u,d}$
1.27±0.40 GeV	~ 1.70 GeV		$1.20^{+0.38}_{-0.43}$ GeV	m_p
1.37±0.60 GeV			$1.49^{+0.25}_{-0.30}$ GeV	$m_{\Delta^{++}}$
220±90 MeV		~ 200 MeV	290^{+180}_{-220} MeV	$m_{\Delta^{++}} - m_p$

eq.(16)). The results for s-wave mesons are compatible among different computations and in rather good agreement with the experimental values:

$$\begin{aligned} m_\rho &\sim 700\ \text{MeV}, \\ m_{K^*} &\sim 900\ \text{MeV}, \\ m_\phi &\sim 1000\ \text{MeV}, \\ m_{\eta_c} &\sim 3000\ \text{MeV}. \end{aligned} \qquad (24)$$

However the meson decay constants turn out to be too large when compared to their experimental values (e.g. $f_\pi \sim 150 \div 200$ MeV; experimentally $f_\pi \sim$ ~ 94 MeV). We believe that these systematic discrepancies are connected to the fact that the lattice spacing is still to large compared to the size of the hadrons and they should becomes less serious at larger values of β. The same kind of systematic effects are observed when one computes mass differences coming from spin-spin forces. For example:

$$\begin{aligned} m_{\Delta^{++}} - m_P &\sim 200\ \text{MeV}, \qquad (\exp \sim 290\ \text{MeV}) \\ m_\Sigma - m_\Lambda &\sim 20\ \text{MeV}. \qquad (\exp \sim 77\ \text{MeV}) \end{aligned} \qquad (25)$$

The situation for p-wave meson states (δ(980), A_1 and B mesons) is at present rather confused and the results from computer simulations should be further improved before making any comparison between Montecarlo results and experimental values. A surprising (and to me not understood) result has been obtained for

$m_B = \left(\frac{m_P + m_{\Delta^{++}}}{2}\right)$, that is the average lighest baryon mass, using Kogut-Susskind like fermions ($r =$ $= 0$ in eq. (16))[10]. They found:

$$m_B \sim 1700\ \text{MeV}. \qquad (26)$$

All the other computations of baryon masses give results in agreement, within rather large statistical errors, with experimental values, although one has to say that the average computed values turn out to be systematically higher than the experimental ones (typically $m_P \sim 1.2$ GeV). Probably this overextimation of baryon masses is due to a greater difficulty in isolating the lowest lying states for baryons because of the smallness of the lattice in the time direction (and perhaps to other finite volume and strong coupling effects).

My conclusion is the following. Many progresses have been done and many promising results have been obtained in the last year in lattice QCD : for the first time we see a way to compute and predict the hadronic world starting from basic principles. Many problems, which I hope will be solved in the next future, come from ultraviolet and infrared limitations imposed on the lattice and a carefull study of systematic errors is needed.

FOOTNOTES

(f1) - Note however that it does not exist any simple algorithm like that explained in the previous section to perform the functional integral over the fermion fields because of the anticommuting nature of the fermionic degrees of freedom.

(f2) - I have not enough time to discuss the computations of other important hadronic quantities that have been tried in the last few months as for example the proton and neutron anomalous magnetic moment(13) and G_A/G_V(14).

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