ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-82/74(P)
11 Ottobre 1982

A. Turrin: GENERALIZED FROISSART-STORA FORMULA

Presented at the "5th International Symposium on High Energy Spin Physics", Brookhaven National Laboratory, September 1982.

Servizio Documentazione
dei Laboratori Nazionali di Frascati
Cas. Postale 13 - Frascati (Roma)

GENERALIZED FROISSART-STORA FORMULA

A. Turrin *
Ist. di Fisica, Fac. d'Ingegneria, Univ. di Roma

In a previous paper we have developed an extension[1] of the Froissart-Stora formula[2], which allows one to calculate the effect, on polarization, of the finite betatron tune-jump magnitude, ΔQ, in crossing rapidly an intrinsic depolarization resonance during acceleration. In that paper, we have chosen a betatron tune-jump of the form $\dot{\chi}/\omega_0 = -(\Delta Q/2)\tanh(\alpha t)$, which is symmetric in time, thereby excluding the conditions encountered in cases where the spread in Q, $G\Delta\gamma$, due to the γ spread, $\Delta\gamma$, of the beam is comparable with the applied range, ΔQ. So, it is to the analysis with replacement of the symmetric Q -jump above by an asymmetric one, namely $\dot{\chi}/\omega_0 = G(\bar{\gamma}-\gamma_{res}) - (\Delta Q/2)\tanh(\alpha t)$, that we have addressed the present work. Here, $\bar{\gamma}$ is the relativistic energy factor of any (selected) group of off-momentum particles having the same energy.

It turned out that even in such a quite general case the problem can be solved analytically. The corresponding asymptotic solution for the polarization S_z is

$$S_{z\,(t\to+\infty)} = 2H_+H_- - 1, \qquad (1)$$

with

$$H_\pm = \sinh\{\pi(p \pm (1/2)(r_+ - r_-))\} / \sinh(\pi r_\pm), \qquad (1a)$$

$$p = (1/2)(\Delta Q/2)^2/Q', \qquad r_\pm = (p_\pm^2 + q^2)^{1/2}, \quad p_\pm = p \pm s, \qquad (2abc)$$

$$q = (1/2)(\Delta Q/2)\varepsilon/Q', \quad s = (1/2)(\Delta Q/2)G(\bar{\gamma}-\gamma_{res})/Q'. \qquad (2de)$$

Here, notations and nomenclature are the same as those adopted in ref. 1, which the reader is referred to for the meaning of the symbols. Since for the present case, $s \neq 0$, the whole calculation is mathematically identical with the one performed in ref. 1, it seemed superfluous to give it here in detail. We only mention that our old formulas[1] (4) - (21) have now to be replaced by the following new formulas:

---

*Mailing address: INFN, Lab. Nazionali, I-00044 Frascati

$$(S_-)_z = p_+/r_+ \quad (4) \quad , \quad S_z(t\to+\infty) = (S_{+av})_z\, r_-/p_- \quad (5)$$

$$\omega = \text{constant} \quad , \quad \dot{\chi} = d - \delta \tanh(\alpha t) \qquad (6ab)$$

$$\dot{\chi}_{t\to\pm\infty} = d \mp \delta = \omega_o G(\bar{\gamma} - \gamma_{res}) \mp \omega_o \Delta Q/2 \qquad (7a)$$

$$\ddot{g} + i\bigl(d - \delta \tanh(\alpha t)\bigr)\dot{g} + (\omega/2)^2 g = 0 \qquad (10)$$

$$x^2(1-x)g'' + x\bigl((1+ip_+) + (-1+ip_-)x\bigr)g' + (q/2)^2(1-x)g = 0 \qquad (11)$$

$$p = \delta/(2\alpha) \quad , \quad q = \omega/(2\alpha) \quad , \quad p_\pm = (\delta \pm d)/(2\alpha) \qquad (13abc)$$

$$2\lambda_\pm = -ip_+ \pm ir_+ \quad , \quad c_\pm = 1 \pm ir_+ \quad , \quad r_\pm = (p_\pm^2 + q^2)^{1/2} \qquad (16abc)$$

$$a_\pm = -ip \pm i(r_+ \pm r_-)/2 \quad , \quad b_\pm = -ip \pm i(r_+ \mp r_-)/2 \qquad (16defg)$$

$$g = D(-x)^{-i(p_+ + r_+)/2}\, F(b_-, a_-; c_-; x) \qquad (17)$$

$$DD^* = gg^*_{(t\to-\infty)} = (gg^*)_- = (1/2)\bigl(1 - (S_-)_z\bigr) = (1 - p_+/r_+)/2 \qquad (18)$$

$$g_{(t\to+\infty)} = D\Bigl[(-x)^{i(p_- + r_-)/2} A_+ + (-x)^{i(p_- - r_-)/2} A_-\Bigr] \qquad (19)$$

$$A_\pm = \frac{\Gamma(1 - ir_+)\ \Gamma(\pm ir_-)}{\Gamma\{-i(p + (r_+ \mp r_-)/2)\}\ \Gamma\{1 + i(p - (r_+ \mp r_-)/2)\}} \qquad (19a)$$

$$2(gg^*)_{+av} = 1 - (S_{+av})_z =$$

$$= \Bigl[-2(p_-/r_-)\sinh^2(\pi p) + B_+ - B_-\Bigr] / \Bigl[\sinh(\pi r_+)\sinh(\pi r_-)\Bigr] \qquad (20)$$

$$B_\pm = (1 \pm p_-/r_-)\sinh^2\bigl((\pi/2)(r_+ \pm r_-)\bigr) \qquad (20a)$$

$$(S_{+av})_z = (p_-/r_-)(2H_+H_- - 1) \quad . \qquad (21)$$

As a numerical example, we have considered, following References 3 and 4, the case $\Delta Q = .25$, $Q' = .0597$ and $\varepsilon = .0266$. For a symmetric jump we obtain a depolarization=.0915, and for an asymmetrical jump with $G(\bar{\gamma} - \gamma_{res}) = .05$ an additional depolarization=.0326 . Comparison of these figures with those evaluated[3] by the Courant-Ruth model[3,4] shows very good agreement.

REFERENCES

1. A.Turrin,IEEE Trans.Nucl.Sci. NS-26,3212 (1979)
2. M.Froissart and R.Stora,Nucl.Instr.&Meth.7,297 (I960)
3. R.D.Ruth,in "High-Energy Physics with Polarized Beams and Polarized Targets".Proc.of the 1980 International Symposium.Lausanne,1980.C.Joseph and J.Soffer,Editors Birkhäuser Verlag.Basel,Boston,Stuttgart,1981,p.472
4. E.D.Courant and R.D.Ruth,Brookhaven Formal Report,BNL 51270 (1980)