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## HOW TO EXAMINE KNO SCALING FOR HADRON-NUCLEUS COLLISIONS

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### ABSTRACT.

We present a method to examine KNO scaling for hadron-nucleus collisions in connection with measurements of the average charged multiplicity and the inelastic total cross-sections. It is shown that the assumption of the universal KNO scaling curve of the multiplicity distributions turns out to be inconsistent with experimental data concerning A-dependence of the absorption cross-section and the nuclear multiplicity ratio.

Experimental attempts have made to test whether the multiplicity distributions in hadron-nucleus interactions obey the same KNO (Koba-Nielsen-Olesen) scaling as in hadron-nucleon collisions. KNO scaling is one of the most brilliant phenomenological law in multiparticle production in h-N (hadron-nucleon) collisions up to ISR energies and it may reflect production mechanism of hadron dynamics. It is well accepted now the nucleus enable us to study the produced state before it reaches maturity; the nucleus is part of our laboratory wherein the details of hadron dynamics can be manifested. Therefore whether universal KNO scaling in h-A (hadron-nucleus) collisions is valid or should be modified is interesting question.

Analyses made to date<sup>(1-18)</sup> seem infer the same KNO scaling curve also in the case of nuclear targets. Those investigations were, however, incomplete to confirm the scaling hypothesis.

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sis for various nuclear targets over the wide energy range. Analyses so far were made only of the dispersion of the multiplicity or restricted within one or several nuclei. Universal KNO scaling requires, theoretically, stringent test: one should check whether all the higher moments of the multiplicity distribution are independent of both energy and mass number, which is difficult.

In this paper we propose a fruitful way to examine the KNO scaling hypothesis for h-A interactions on the basis of the KNO scaling law of h-N interactions. This is a simple way and we can deal with many data at the same time. It will be shown that the analysis based on this method casts strong doubt on the universal KNO hypothesis.

In order to construct the relation between the charged multiplicity distributions of h-N and h-A interactions, we shall require inelastic cross-section,  $\sigma_{inel}^{hA}$ , mean charged multiplicity,  $\langle n \rangle_{hA}$ , and n-prong cross-sections,  $\sigma_n^{hA}$ , for h-A collisions in comparison with those of h-N interactions. We first put<sup>(2)</sup>, making use of  $\sigma_{inel}^{hN}$  (inelastic cross-section for hadron-nucleon interactions),

$$\sigma_{inel}^{hA} = A^\alpha \sigma_{inel}^{hN}, \quad (1)$$

where we introduce a parameter  $\alpha$ , which is given by the model calculations<sup>(3)</sup> in the usual way. Eq. (1) yields the average number of inelastic collisions

$$\bar{\nu} = \frac{A \sigma_{inel}^{hN}}{\frac{hA}{\sigma_{inel}^{hA}}} = A^{1-\alpha}. \quad (2)$$

Subsequently, in accordance with the experimental observations, we may write<sup>(4-19)</sup>

$$R(A) = \frac{\langle n \rangle_{hA}}{\langle n \rangle_{hN}} = 1 + \beta(\bar{\nu} - 1), \quad (3)$$

where  $\beta$  is a dynamical parameter.

Now let us postulate<sup>(5)</sup> scaling of the multiplicity distributions for h-A collisions

$$\langle n \rangle_{hA} \frac{\sigma_n^{hA}}{\sigma_{inel}^{hA}} = \psi\left(\frac{n}{\langle n \rangle_{hA}}\right) \quad (4)$$

according to the KNO scaling law of h-N interactions. One can prospect the universal curve  $\psi$  for the respective targets, provided  $\psi$  does not depend on nuclear mass number  $A$ . With the help of the universal KNO scaling function we will get a relation between those reactions, and hence the choice of the dynamical parameters in Eqs. (1) and (3) by means of  $\chi^2$ -fit shall be examined in comparison with the experimental data. Thus we perform an accurate test for the universal KNO scaling hypothesis, i. e. Eq. (4).

By putting the ratio of n-prong cross-sections  $\sigma_n^{hA}/\sigma_n^{hN} = g(n, \langle n \rangle_{hN}, A)$ , we obtain from

Eq. (1) that

$$A^\alpha P_n^{hA} = g P_n^{hN}, \quad (5)$$

where  $P_n^{hA} = \sigma_n^{hA}/\sigma_{inel}^{hA}$  and  $P_n^{hN} = \sigma_n^{hN}/\sigma_{inel}^{hN}$ . It turns out from the universal KNO scaling function and the relations (3) and (5) that we can rewrite  $\psi(n/\langle n \rangle_{hA})$  in terms of  $A$  and  $z = n/\langle n \rangle_{hN}$  as

$$\psi\left(\frac{n}{\langle n \rangle_{hA}}\right) = A^{-\alpha} R(A) g(z, A) \psi(z). \quad (6)$$

The  $m$ -th moment is given by

$$M_{hA}^m = \frac{\langle n^m \rangle_{hA}}{\langle n \rangle_{hA}^m} = \frac{1}{A^\alpha R^m} \int_0^\infty dz z^m g(z, A) \psi(z). \quad (7)$$

We shall investigate the postulate that  $\psi(n/\langle n \rangle_{hA})$  shows the same functional form as  $\psi(n/\langle n \rangle_{hN})$ . Under this assumption, the function  $g(z, A)$  in Eq. (6) is determined with two appropriate parameters,  $\alpha$  and  $\beta$ . Hence we can choose automatically these parameters by comparing the experimental data of  $\sigma_n^{hN}/\sigma_n^{hA}$  with  $g(z, A)$ .

In Figs. 1, we show comparison of the data with our evaluations of  $g(z, A)$ . Here we have used Slattery's parametrization<sup>(6)</sup>

$$\psi(z) = (1.895z + 16.85z^3 - 3.32z^5 + 0.166z^7) \exp(-3.04z). \quad (8)$$

Our data fitting with  $\alpha = 0.6667$  and  $\beta = 0.3333$  gives the satisfactory value of  $\chi^2/NDF = 12.1/10$ . For comparison with other parametrizations we will show the following results, i.e.  $\chi^2 = 45.3$  for  $\alpha = 2/3$  and  $\beta = 1/2$ ,  $\chi^2 = 110$  for  $\alpha = 3/4$  and  $\beta = 1/2$  and  $\chi^2 = 307$  for  $\alpha = 3/4$  and  $\beta = 1/3$ .

Now let us turn to the investigation of the observed relative multiplicity which suggests  $\beta = 1/2$ <sup>(3, 4, 7)</sup> and accordingly seems to exclude  $\beta = 1/3$ <sup>(8)</sup>. Thus, our first assumption that the multiplicity distributions of hadron-hadron and hadron-nucleus collisions fall on the same universal KNO scaling curve leads to the inconsistent result as concerns observation of  $R(A)$ , while  $\alpha = 2/3$  is agreeable to the experimental studies. It should be noted<sup>(9)</sup> that we are apt to accept the feigned scaling behavior of  $\langle n \rangle_{hA} \sigma_n^{hA}/\sigma_{inel}^{hA}$  only because of the linear dependence of the dispersion on  $\langle n \rangle_{hA}$ .

It is astonishing that our assumption to put the moments  $M_{hA}^m$ , which come from the universal KNO scaling function, being independent of both energy and nuclear number yields curious tendency of  $R(A)$ . If  $\beta = 1/2$  is extensively suggested in various targets and wide energy region for the further experiments, it comes to that we must take  $A$ -dependence into consideration for  $\langle n \rangle_{hA} \sigma_n^{hA}/\sigma_{inel}^{hA}$  in the strict sense<sup>(20)</sup>.

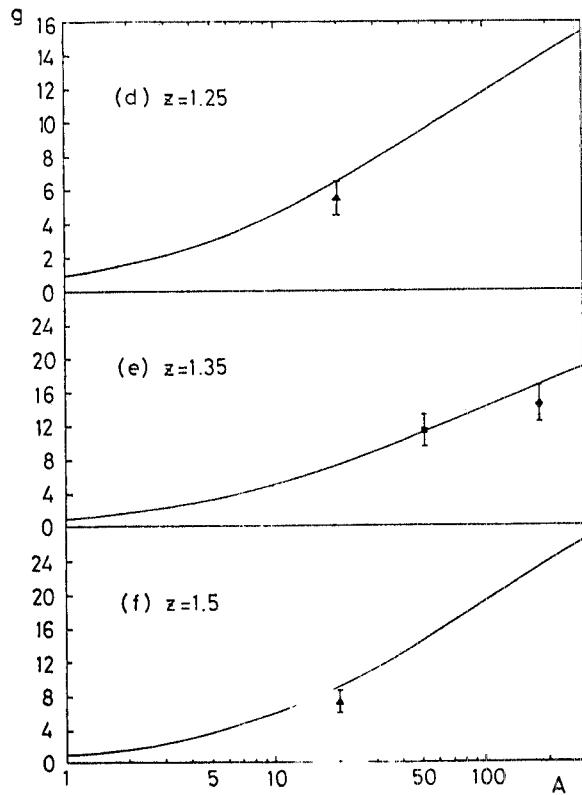
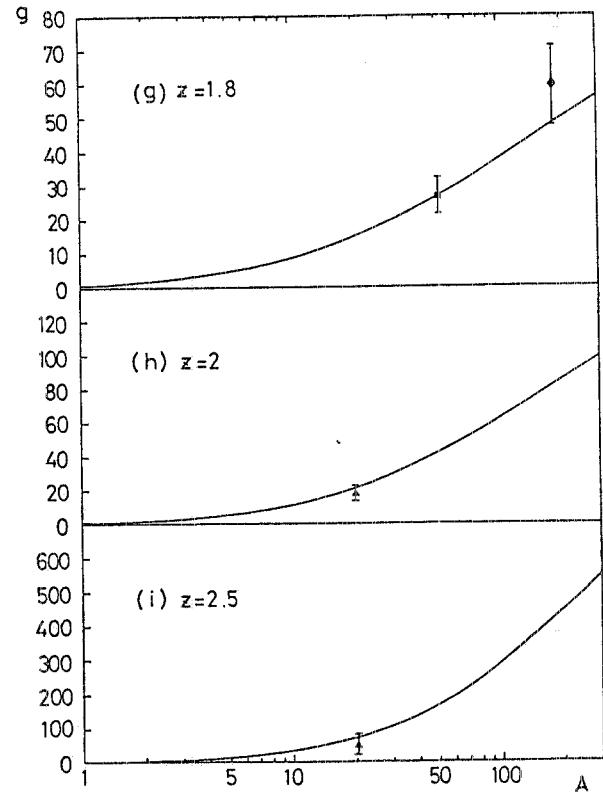
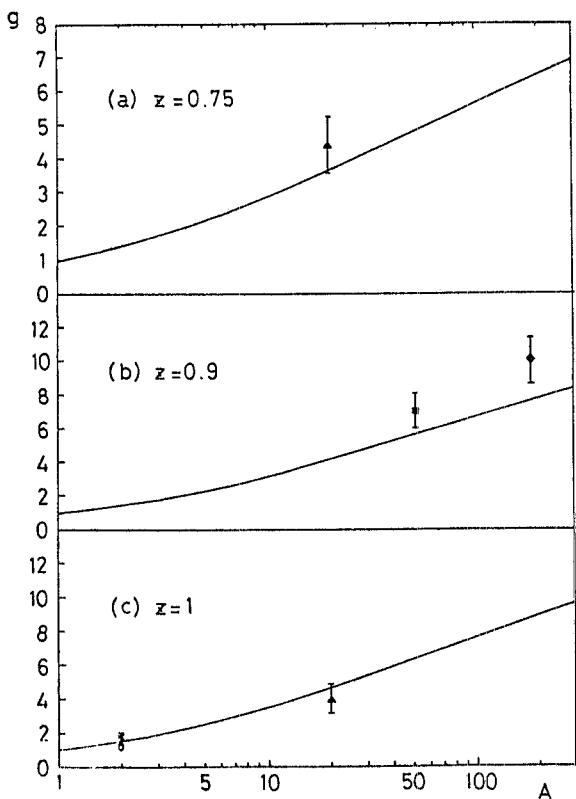


FIG. 1 - Plots of  $g = \sigma_n^{hA} / \sigma_n^{hN}$  versus  $A$  at the fixed value of  $z$ ;  
 • pd 19 GeV/pp 19 GeV (Refs. (10) and (13));  
 o pd 19 GeV/pp 102 GeV (Refs. (10) and (14));  
 x pd 19 GeV/pp 303 GeV (Refs. (10) and (15));  
 ▲  $\pi^-$ Ne 200 GeV/ $\pi^-$ p 205 GeV ((11) and (16));  
 ◆ pW 300 GeV/pp 303 GeV ((12) and (15));  
 ■ pCr 300 GeV/pp 303 GeV ((12) and (15)).  
 The solid line represents Slattery fit with  
 $\alpha = 0.6667$  and  $\beta = 0.3333$ .

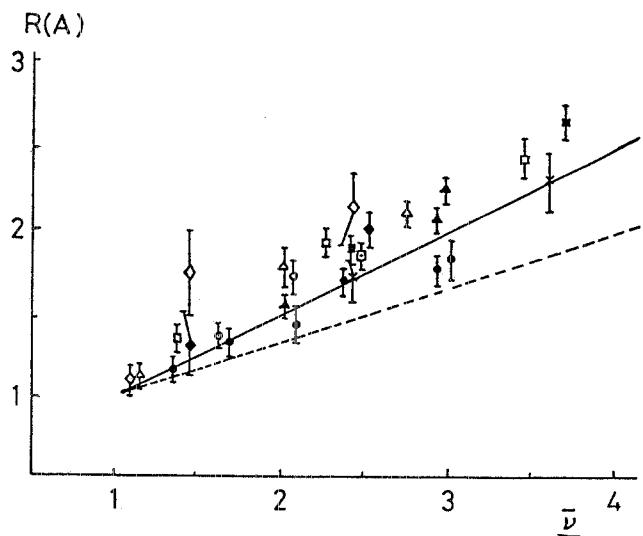


FIG. 2 - Plots of  $R = \langle n \rangle_{hA} / \langle n \rangle_{hN}$  versus  $\bar{\nu}$ .

Incident  $\pi^+$  (Ref. (4)):  $\Delta$  50 GeV;  $\blacktriangle$  100 GeV;  
Incident  $\pi^-$ :  $\circ$  60 GeV (Ref. (19));  $\bullet$  100 GeV (Ref. (18));  $\diamond$  200 GeV (Ref. (11));  
Incident  $K^+$  (Ref. (4)):  $\diamond$  50 GeV;  $\blacklozenge$  100 GeV;  
Incident  $p$  (Ref. (4)):  $\square$  50 GeV;  $\blacksquare$  100 GeV;  $\blacksquare$  200 GeV;  $\times$  300 GeV  
(W. Busza in Ref. (1)). The solid line (dashed line) represents Eq. (3) with  
 $\beta = 1/2$  ( $1/3$ ).

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