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SOFT GLUON EFFECTS IN HADRON COLLISIONS

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## SOFT GLUON EFFECTS IN HADRON COLLISIONS

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Most of the cross section for dilepton production in hadron collisions is concentrated at low transverse momentum  $p_{\perp}$ , where soft gluon effects in QCD play a very important role, as recently recognized by many authors<sup>(1)</sup>. Here we wish to give a short account of the present phenomenological status of the calculations, which are based on various techniques for summing leading and subleading logarithms of perturbative QCD<sup>(2)</sup>. I will mainly discuss  $p_{\perp}$  effects arising in Drell-Yan processes, giving finally some suggestions that similar mechanisms are also responsible for the large production of transverse energy observed in pure hadronic collisions at the SPS and collider energies.

Among the most spectacular QCD effects are the  $p_{\perp}$  effects which are caused by gluon bremsstrahlung. These have been extensively studied in  $e^+e^-$  annihilation and Drell-Yan processes and can be very easily measured in the latter reaction by observing the transverse momentum distribution of the lepton pair. In particular one expects, for the average  $p_{\perp}^2$ ,

$$p_{\perp}^2 = \alpha_s(Q^2) S f(\tau, \alpha_s(Q^2)) + \dots, \quad (1)$$

where the dots indicate terms which are constant with  $S$  and may be ascribed to an intrinsic transverse momentum of the partons. However the observation<sup>(1,3)</sup> of a linear increase with  $S$  of  $\langle p_{\perp}^2 \rangle$ , for fixed  $\tau$ , is in qualitative agreement only with first order results. Furthermore, the dependence of  $\langle p_{\perp}^2 \rangle$  predicted by the theory is wrong<sup>(1,3)</sup>. The comparison of the absolute distributions from first order diagrams with data is also not very satisfactory. To explain the data one needs quite a large value of the intrinsic transverse momentum  $\langle p_{\perp}^2 \rangle_{\text{int}} \sim 1 \text{ GeV}^2$ , which is also required to increase slowly with  $S$ <sup>(3)</sup>.

However this failure of perturbation theory in accounting for the observed features of the data is not dramatic. In fact one has to distinguish three different regions in  $p_{\perp}$ : (i)  $p_{\perp}^2 \sim 0(Q^2)$ , (ii)  $\Lambda^2 \ll p_{\perp}^2 \ll Q^2$  and (iii)  $\Lambda^2 \lesssim p_{\perp}^2$ . As well known, it is only in the first region that a detailed comparison with lowest order perturbation theory makes sense and even there  $O(\alpha_s^2)$  corrections can be sizeably large. This is the case for  $\pi N$  scattering at

high  $p_{\perp}$  and indeed a large correction factor  $K(p_{\perp}) \equiv \left[ \frac{d\sigma}{dp_{\perp}^2} \Big|_{O(\alpha_s) + O(\alpha_s^2)} \right] / \left[ \frac{d\sigma}{dp_{\perp}^2} \Big|_{O(\alpha_s)} \right] \approx 2$  has been found in Ref. (4) for the non singlet ( $\pi^+ - \pi^-$ ) cross sections. Of course the worry remains of the possible relevance of higher orders. In particular the question arises whether the resummation of the most important higher order corrections could be performed to all orders in  $\alpha_s$ .

This problem is directly related to the question of the relevance of the perturbation theory at intermediate and small  $p_{\perp}$ . This issue has received much theoretical attention in very recent years and has been discussed in great detail many times<sup>(5)</sup>. We sketch here the main lines. In the region  $\Lambda^2 \ll p_{\perp}^2 \ll Q^2$  one has two mass scales, and the large logarithms  $\alpha_s^n \ln^{2n-k}(Q^2/p_{\perp}^2)$ , ( $k=0, \dots, 2n-1$ ), which appear in the perturbative parton cross sections have to be resummed to all orders. The so called double leading logarithmic approximation (DLA), which resums in each order only the dominant terms of the expansion gives<sup>(6)</sup>, for fixed  $\alpha_s$ ,

$$\frac{1}{\sigma_0} \frac{d\sigma}{dp_{\perp}^2} \sim \frac{\alpha_s}{p_{\perp}^2} \ln \frac{Q^2}{p_{\perp}^2} \exp \left\{ - \frac{C_F \alpha_s}{2\pi} \ln^2 \frac{Q^2}{p_{\perp}^2} \right\}. \quad (2)$$

The exponential in the above Eq. corresponds to an effective quark form factor. It essentially gives the probability that the massive lepton pair is produced without emission of gluon having momenta  $k_{\perp} \gg p_{\perp}$ . When  $p_{\perp}^2 \ll Q^2$  this probability is very small, and indeed the DLA predicts a dip at  $p_{\perp}=0$ , which however is quite fictitious. In fact at small  $p_{\perp}^2$ , the subleading contributions from multigluon emission with  $k_{\perp i}^2 > p_{\perp}^2$  and which add vectorially to give a small  $p_{\perp}^2$ , become dominant and fill that dip.

It is therefore relevant to keep trace of exact momentum conservation. This can be most easily done by working in the impact parameter space and one finds<sup>(7)</sup>.

$$\frac{1}{\sigma_0} \frac{d\sigma}{dp_{\perp}^2} = \frac{1}{2} \int_0^{\infty} db J_0(b p_{\perp}) \exp \left[ \Delta(b, q_{\perp \max}) \right], \quad (3)$$

with

$$\Delta(b, q_{\perp \max}) = \frac{4C_F}{\pi} \int_0^{q_{\perp \max}} \frac{dq_{\perp}}{q_{\perp}} \ln \left( \frac{Q}{q_{\perp}} \right) \alpha(q_{\perp}) \left[ J_0(b q_{\perp}) - 1 \right], \quad (4)$$

and  $q_{\perp \max} \sim Q$  is the phase space limit for the emitted gluons.

The above result explicitly shows the relevance of a detailed understanding of the subleading corrections to Eq. (2). Thus a systematic investigation of such terms has been carried out recently<sup>(8)</sup>. The result<sup>(9)</sup> is essentially given by formula like (3), with next to leading corrections evaluated for the integrand of (4). We will come back to this point later. On the other hand, from a more phenomenological point of view, it has been shown<sup>(10)</sup> the relevant role played by the appropriate use of the exact kinematics in evaluating in Eq. (4) the emission of soft gluons. This observation has been motivated by analogous results obtained in deep inelastic scattering, and for the K factor in Drell-Yan<sup>(1,11)</sup>, where a careful treatment of the kinematics in multigluon emission accounts for the most important next to leading corrections. More explicitly, with  $z = \tau/x_1 x_2$  and  $y$  the dilepton rapidity, one gets for  $q_{\perp \max}$  in Eq. (4)

$$q_{\perp \max} \approx \frac{Q(1-z)}{2\sqrt{z}} \frac{1}{\sqrt{1+z \sin^2 y}}. \quad (5)$$

Then a simultaneous analysis<sup>(10)</sup> of the soft effects to all order in  $\alpha_s$  (Eqs. (3, 4, 5)) for  $p_{\perp} \leq \bar{p}_{\perp}$  and the hard terms to first and second order in  $\alpha_s$  for  $p_{\perp} < \bar{p}_{\perp}$ , with  $\bar{p}_{\perp} \sim 3$  GeV, gives an excellent description of all data available so far, both in  $\pi N$  and  $p N$  collision, after inclusion of an intrinsic  $\langle p_{\perp}^2 \rangle_{\text{intr}} \sim 0.4 \text{ GeV}^2$ . This last term accounts for the third region ( $p_{\perp}^2 \gtrsim \Lambda^2$ ) where non perturbative effects are expected to be important. It can be

simply included in the analysis by inserting in the r.h.s. of Eq. (3) a factor  $\exp. (-b^2/4 \langle p_{\perp}^2 \rangle_{\text{intr}})$ . The results<sup>(10)</sup> are shown in Figs. 1 and 2. The  $S$  dependence of  $\langle p_{\perp}^2 \rangle$  is also found in agreement with data. Thus the inclusion of multigluon effects improves considerably the agreement of the theory with the data at low  $p_{\perp}$ , without the need for an unnaturally large intrinsic  $p_{\perp}$ . The same analysis also suggests a sizeable  $\alpha_s^2$  corrections for the Compton term at large  $p_{\perp}$  in case of pN collisions.

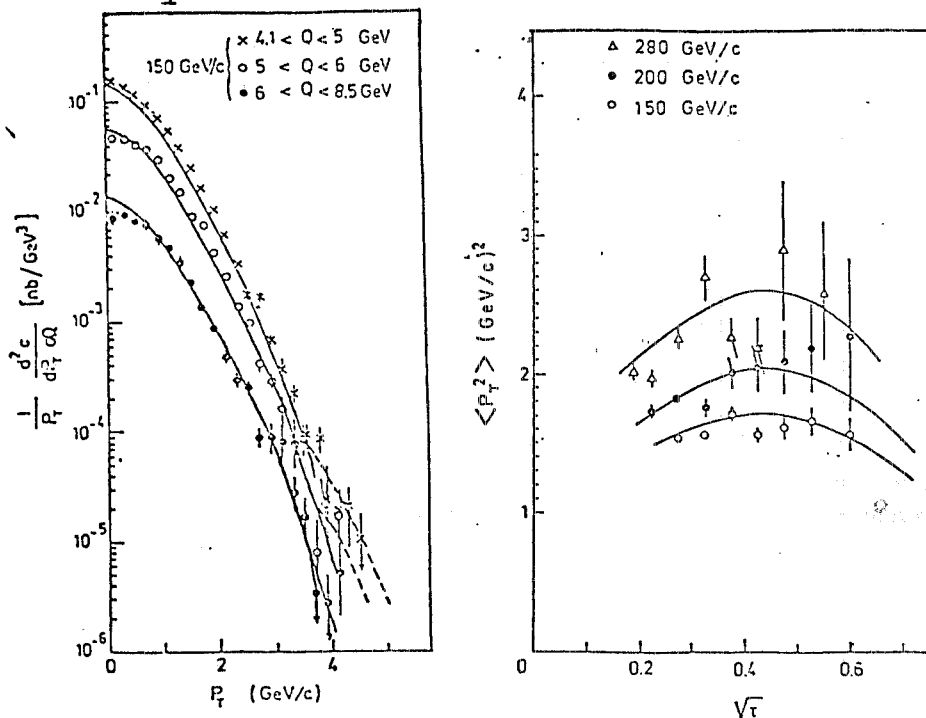


FIG. 1 -  $p_{\perp}$  distributions<sup>(10)</sup> for  $\pi N$  collisions: the full line represents the soft contribution including an intrinsic  $\langle p_{\perp}^2 \rangle_{\text{intr}} = 0.4 \text{ GeV}^2$  and the dashed one the annihilation hard term. The  $\langle p_{\perp}^2 \rangle$  curves are directly obtained from the distributions.

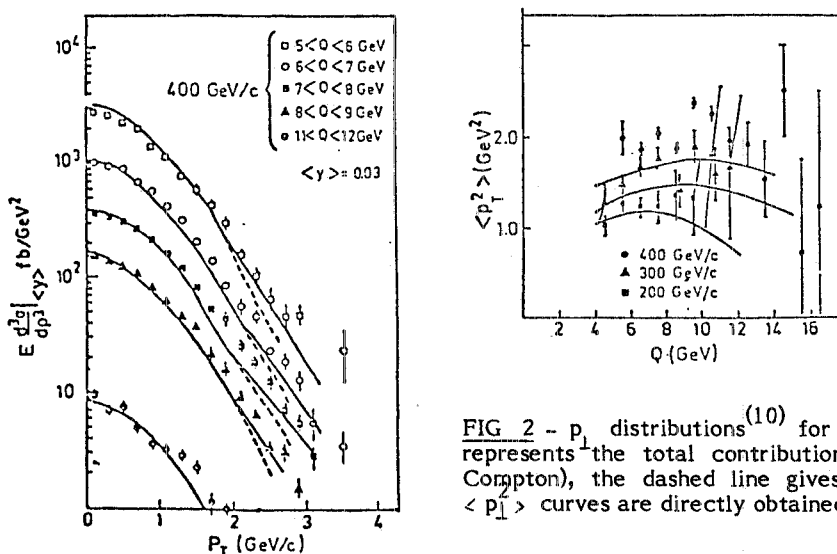


FIG. 2 -  $p_{\perp}$  distributions<sup>(10)</sup> for pN collisions: the full line represents the total contribution (soft + intrinsic + hard Compton), the dashed line gives the soft term only. The  $\langle p_{\perp}^2 \rangle$  curves are directly obtained from the distributions.

In view of this success it is important to understand better the role played by the previous kinematics considerations. As next step, we then compare the above analysis with the more rigorous results obtained in Ref. (9) in the framework of a systematic approach to the question of subleading corrections to Eq. (3). The final formula<sup>(9)</sup> reads as follows

$$\frac{Q^2 d\sigma}{dx_1 dx_2 dp_{\perp}^2} \sim \frac{1}{2} \int_0^{\infty} db J_0(b p_{\perp}) q(x_1, \frac{1}{b}) \bar{q}(x_2, \frac{1}{b}) e^{S(b, Q^2)} + Y \quad (6)$$

with

$$S(b, Q^2) = -\frac{C_F}{\pi} \int_{\frac{1}{b^2}}^{Q^2} \frac{dq^2}{q^2} \left[ \alpha_s(q^2) \ln\left(\frac{Q^2}{q^2}\right) + \frac{K}{2\pi} \alpha_s^2(q^2) \ln\left(\frac{Q^2}{q^2}\right) + 2 \ln \frac{e^{\gamma_E}}{2} \alpha_s\left(\frac{1}{b^2}\right) - \frac{3}{2} \alpha_s(q^2) \right], \quad (7)$$

where  $\gamma_E$  is the Euler constant and  $K = C_G \left( \frac{67}{18} - \frac{\pi^2}{6} \right) + N_F T_F \left( -\frac{10}{9} \right)$  is the group factor due to the inclusion of the two loop corrections. Finally  $Y$  is a correction term which will reproduce the standard perturbative result at large  $p_{\perp}$ .

Comparing Eqs. (6-7) with (3-4) one notice the appearance of subleading corrections to (4) as well as the  $b$  evolution in the parton densities, which was neglected in (3), in first approximations. In Figs. 3-4-5-6 we show the comparison<sup>(12)</sup> of various approximations to Eq. (3) with the full result of Eq. (6) and with recent NA3 data<sup>(13)</sup>, in the soft region  $p_{\perp} \lesssim 3$  GeV.

In Fig. 3 Eq. (3) is plotted in the leading kinematics approximation  $q_{\perp \max} \sim Q$ . In Fig. 4 the same equation is shown for  $q_{\perp \max}$  given by (5). Clearly the kinematical correction is very effective. A value  $\langle p_{\perp}^2 \rangle_{\text{intr}} \sim 0.4$  GeV<sup>2</sup> has been used, as in Ref. (10). Next the same formula is shown in Fig. 5, with the  $b$  evolution of the parton density includes, as in Eq. (6). This inclusion leads to a slight increase of  $\langle p_{\perp}^2 \rangle_{\text{intr}}$  to  $\sim 0.6$  GeV<sup>2</sup>. Finally the full result (6) is shown in Fig. 6. Again  $\langle p_{\perp}^2 \rangle_{\text{intr}} \sim 0.6$  GeV<sup>2</sup>. In all these fits a value  $\Lambda \simeq 0.25$  GeV has been used.

From inspection of Figs. 4-5-6 it follows that it is very hard to tell the difference between the kinematically improved Eqs.(3-5) and the full result (6). Essentially the same behaviour is found by raising the c.m. energy by a factor of ten. The conclusion of this analysis is therefore in close analogy with what has been previously found in electroproduction and for the  $K$ -factor in Drell-Yan, namely that the use of exact kinematics considerations takes into account the dominant corrections to the leading logarithmic analysis. Furthermore it is clear that the main features of the  $p_{\perp}$  effects observed so far are dominated by soft effects and these are quite well understood in perturbative theory.

Finally I will briefly discuss the production of transverse hadronic energy in high energy hadron collisions as the effect of multigluon soft emission from the hadron constituents. From the previous analysis of  $p_{\perp}$  effects in Drell-Yan processes, it is clear that one naturally expects the production of hadrons associated to the lepton pair and weak bosons as well.

It has been recently suggested<sup>(14)</sup> that the same mechanism is also responsible for the large production of transverse hadronic energy observed in  $pp$  and  $p\bar{p}$  collisions at SPS and collider energies. The idea is quite simple. In any hard scattering process among the constituents, which for simplicity we will take to be the valence quarks (antiquarks), a fraction of the initial quark's subenergy  $\sim \sqrt{s}$  is released in the form of soft QCD radiation, whose spectrum can be calculated to all orders in  $\alpha_s$ .

Furthermore, the emitted radiation being soft, the corresponding spectrum factorizes and, to leading order, is independent of the particular hard scattering process. Summing on the final states and integrating over the initial quark momenta, one obtains a transverse momentum distribution and a corresponding transverse energy flow which should be observable in all high-energy experiments with larger transverse energy triggers. Of course,

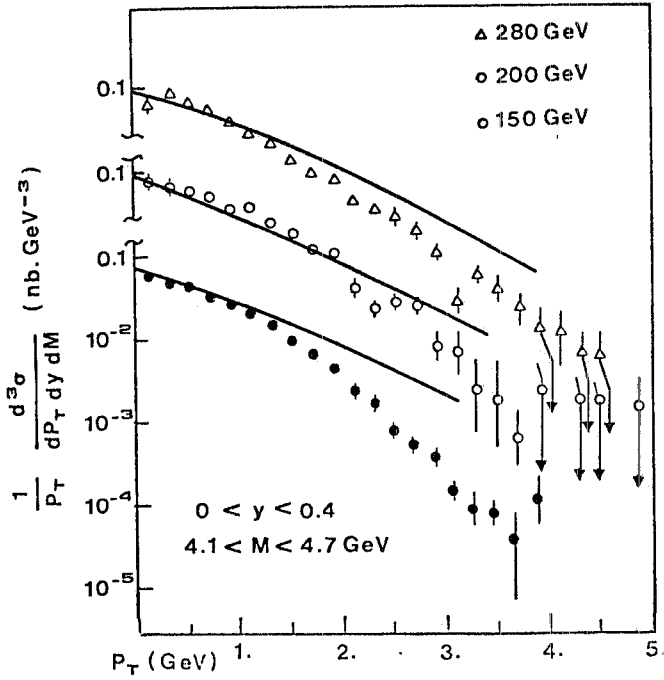


FIG. 3 -  $p_T$  distributions<sup>(12)</sup> for  $\pi N$  collisions: the curves represent Eq. (3) with  $q_{1\max} \sim Q$ . The data are from Ref. (13).

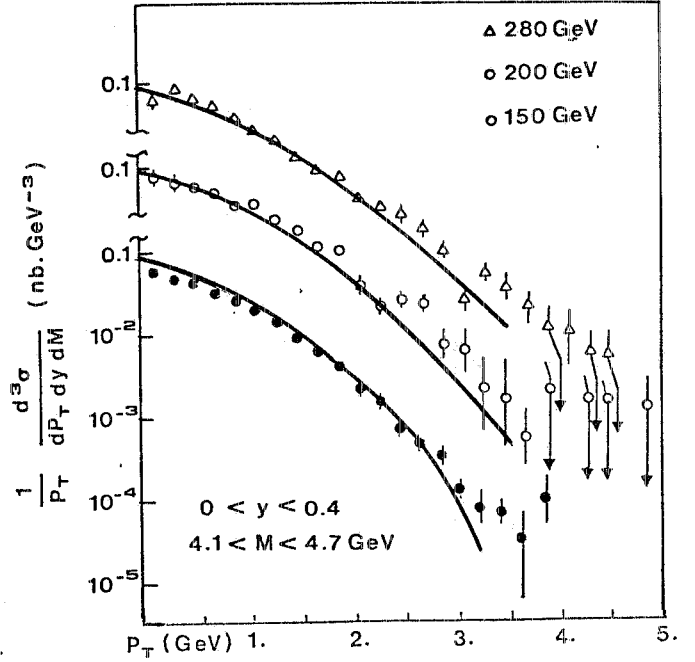


FIG. 4 -  $p_T$  distributions<sup>(12)</sup> for  $\pi N$  collisions: the curves represent Eq. (3) with  $q_{1\max}$  given by Eq. (5). The data are from ref. (13).

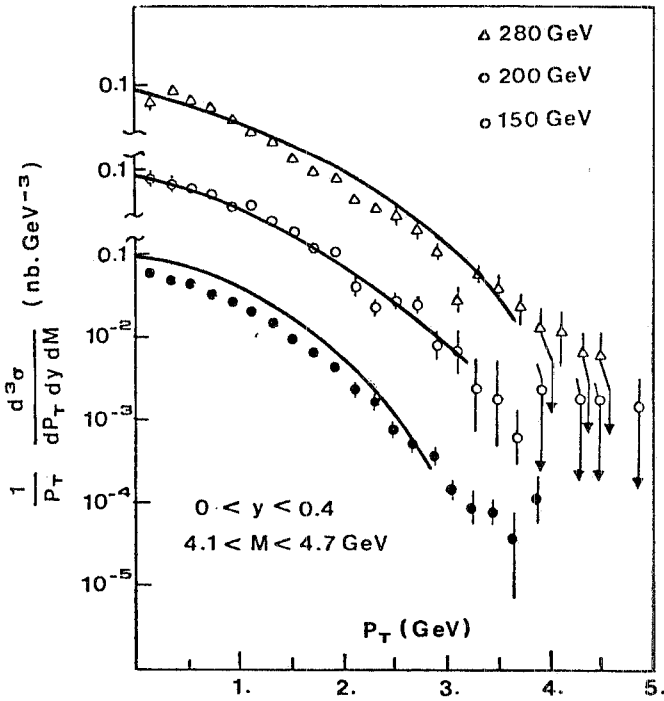


FIG. 5 -  $p_T$  distributions<sup>(12)</sup> for  $\pi N$  collisions: the curves represent Eq. (3) with  $q_{1\max}$  given by Eq. (5) and the parton densities evolved in  $b$ . The data are from Ref. (13).

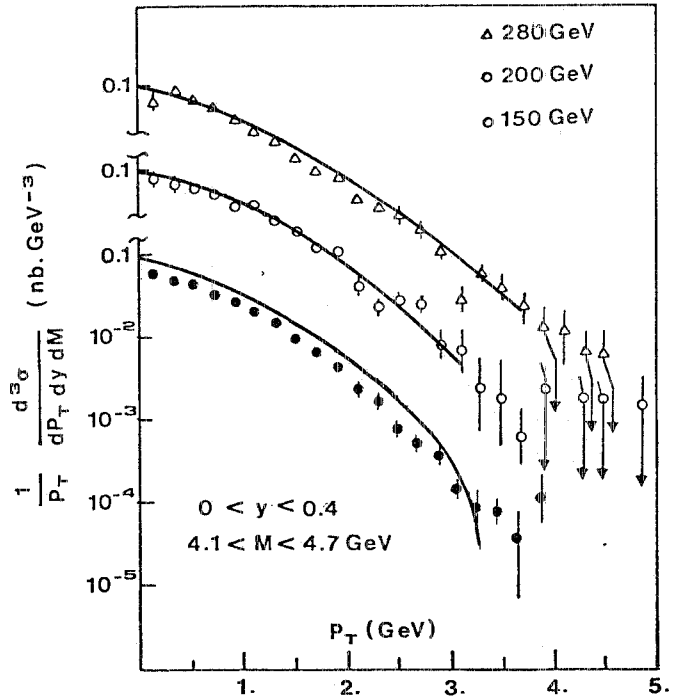


FIG. 6 -  $p_T$  distributions for  $\pi N$  collisions. The curves represent Eq. (6). The data are from Ref. (13).

the very tail of the spectrum might well be modified by the detailed dynamics of a particular hard scattering process, similar to what happens in the lepton pair production when  $p_T \sim 0(Q)$ .

Then the transverse energy distribution, defined as

$$\frac{d\sigma}{dE_T}(S) \sim \sum_{ij} \int dx_1 dx_2 p_i(x_1) p_j(x_2) \frac{d\sigma_{ij}^{\text{hard}}}{dE_T}(\hat{s} = x_1 x_2 S), \quad (8)$$

where the sum is over the initial partons, is shown in Fig. (7) normalized to the UA1 data<sup>(15)</sup>. The low energy region is regulated by some energy threshold  $\sqrt{s_0}$  large enough to justify the factorization of the distribution and

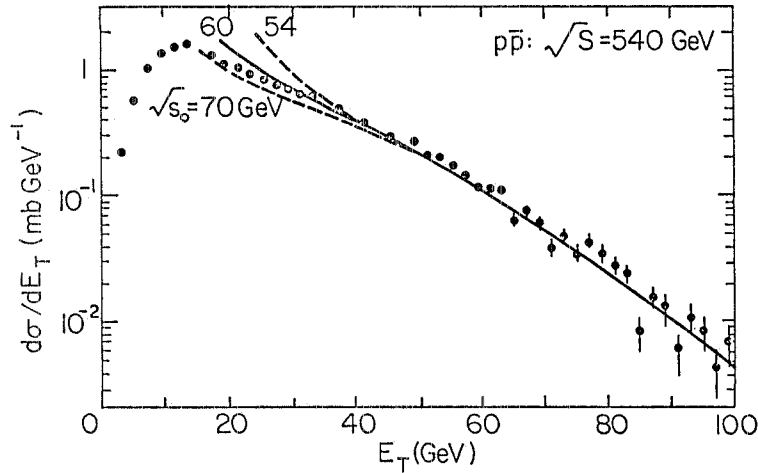


FIG. 7 - Transverse hadronic energy distributions<sup>(14)</sup> in  $p\bar{p}$  collisions at  $\sqrt{S} = 540$  GeV. The data are from Ref. (15).

the assumption that a hard scattering is taking place. The shape at large  $E_T$  is an absolute prediction. In spite of the simplified assumption of taking only valence quarks into account the model reproduces qualitatively the behaviour of a more complete analysis<sup>(16)</sup> which includes also  $gg$  and  $qg$  interactions. Comparison with  $pp$  data at 400 GeV shows that the strong  $S$  dependence observed in the range  $\sqrt{S} \approx 27-540$  GeV is also well reproduced. Of course these results cannot be trusted at very large  $E_T$  ( $2E_T \lesssim \sqrt{S}$ ) where both the soft approximation breaks down and genuine hard scattering effects are expected to take over. Indeed evidence for jet structure at very large  $E_T$  have very recently suggested<sup>(17)</sup>.

In summary it can be said that our present knowledge of the techniques for summing leading and subleading logarithms of perturbative QCD provides a successful framework for understanding many of the observed features in high energy hard collisions.

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