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CONSTANTS

THE CONNECTION BETWEEN LOCAL OPERATORS ON THE LATTICE AND IN THE
CONTINUUM AND ITS RELATION TO MESON DECAY CONSTANTS

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ABSTRACT

We have computed, at first order in perturbation theory, the relation between lattice and continuum local operators used in Montecarlo simulations to evaluate the meson decay constants. These corrections, although with the correct sign, are too small to compensate the discrepancy between existing lattice calculations and experimental values.

Recently several groups have attempted the computation of the hadron mass spectrum in SU(3) colour lattice QCD with encouraging results^(1,2,3,4). A byproduct of these Montecarlo simulations has been the computation of the bare quark masses and meson decay constants on the lattice. In order to compare the Montecarlo results for these quantities with the experimental values we must know the relation between certain operators on the lattice and the corresponding operators in the continuum. At first order in perturbation theory to achieve this program it is only necessary to compute the complete one loop perturbative corrections to the relevant operators on the lattice and in some continuum renormalization scheme^(f1).

The relation between the mass of the quarks in the continuum and on the lattice has been computed elsewhere⁽⁵⁾, here we present the result of a computation to find the values of the meson decay constants starting from Montecarlo results. This computation became even more relevant since the values of the lattice meson decay constants are about a factor two larger than their experimental values - and one would like to know if perturbative corrections can take into account for these discrepancies.

The relevant local operators^(f2) used in Montecarlo simulations to measure the meson masses and decay constants are of the form:

$$O_i(x) = \bar{\psi}_1(x) \Gamma_i \psi_2(x) \quad (1)$$

where $\psi_{1,2}(x)$ are the quark fields of flavour 1,2 and Γ_i is one of the 16 Dirac matrices.

We define the zero-momentum correlation function for these operators as:

$$G(t) = \sum_{\vec{x}} \langle O_i(\vec{x}, t), O_i(0,0) \rangle \quad (2)$$

t fixed

At large time distances ($t \rightarrow it$) we expect:

$$G(t) \xrightarrow[t \rightarrow \infty]{} \frac{A}{2m} e^{-mt} \quad (3)$$

Where m is the mass of the lowest lying state. The coefficients A 's are related to the meson decaying constants through the equations⁽⁸⁾:

$$\begin{aligned} (m_1 + m_2) \langle 0 | \bar{\psi}_2 \gamma_5 \psi_1 | P \rangle &= \sqrt{2} f_P m_P^2 \\ (m_1 - m_2) \langle 0 | \bar{\psi}_2 \psi_1 | S \rangle &= f_S m_S^2 \\ \langle 0 | \bar{\psi}_2 \gamma_\mu \psi_1 | V \rangle &= f_V^{-1} m_V^2 \epsilon_\mu \\ \langle 0 | \bar{\psi}_2 \gamma_5 \gamma_\mu \psi_1 | A \rangle &= f_A^{-1} m_A^2 \epsilon_\mu \end{aligned} \quad (4)$$

$m_{1,2}$ are the quark masses. P, S, V and A denote the pseudoscalar, scalar, vector and axial vector mesons respectively.

The terms on the r.h.s. in Eqs. (4) depend on the regularization procedure, at one loop level one has:

$$\begin{aligned} & \left[(m_1 + m_2) \langle 0 | \bar{\psi}_2 \gamma_5 \psi_1 | P \rangle \right]_{\text{CONT}} = \\ & \left(1 + \frac{\alpha_S}{4\pi} C_F \Delta_P \right) \left[(m_1 + m_2) \langle 0 | \bar{\psi}_2 \gamma_5 \psi_1 | P \rangle \right]_{\text{LATT}} \\ & \langle 0 | \bar{\psi}_2 \gamma_\mu \psi_1 | V \rangle_{\text{CONT}} = \left(1 + \frac{\alpha_S}{4\pi} C_F \Delta_V \right) \cdot \\ & \cdot \langle 0 | \bar{\psi}_2 \gamma_\mu \psi_1 | V \rangle_{\text{LATT}} \end{aligned} \quad (5)$$

and analogously for the scalar and axial vector mesons (Δ_S and Δ_A). "CONT" stands for some regularization scheme in the continuum (e.g. $\overline{\text{MS}}$ scheme). $\Delta_{S,P,V,A}$ are perturbatively computable coefficients.

They are found through the calculation of the $O(\alpha_S)$ diagrams for the vertex correction and quark self energy shown in Figs. 1 and 2 on the lattice and in the continuum.

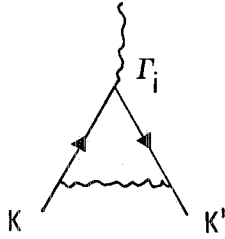


FIG. 1 - Feynman diagram for the vertex corrections at order α_S .

FIG. 2 - Feynman diagrams for the self-energy corrections at order α_S .



The vertex function between arbitrary external quark states with momentum K and K' can be written at one loop level as:

$$\begin{aligned} \Gamma_i^{\text{LATT,CONT}} &= \Gamma_i + \frac{\alpha_S}{4\pi} C_F \tilde{\Gamma}_i^{\text{LATT,CONT}}(K, K') \\ \Delta \Gamma_i &= \tilde{\Gamma}_i^{\text{CONT}}(K, K') - \tilde{\Gamma}_i^{\text{LATT}}(K, K') \end{aligned} \quad (6)$$

In the limit in which the lattice spacing $a \rightarrow 0$, $\Delta \Gamma_i$ is independent of K and K' .

The quark self energy has the form:

$$\Sigma^{\text{LATT,CONT}}(K) = iK \left[1 - \frac{\alpha_S}{4\pi} C_F \Sigma_1^{\text{LATT,CONT}}(K) \right] + m \left[1 - \frac{\alpha_S}{4\pi} C_F \Sigma_2^{\text{LATT,CONT}}(K) \right] \quad (7)$$

$$\Delta_{\Sigma_{1,2}} = \Sigma_{1,2}^{\text{CONT}} - \Sigma_{1,2}^{\text{LATT}}$$

On the lattice, beside Σ_1^{LATT} and Σ_2^{LATT} there is in general a linear divergent term in the quark self energy; such a term is however totally inessential for all the following discussion^(5,9).

The finite $O(\alpha_S)$ corrections of Eqs. (5) are found using Eqs. (6) and (7):

$$\Delta_{S,P} = \Delta_{\mathbb{1}, \gamma_5} + \Delta_{\Sigma_2} \quad ; \quad \Delta_{V,A} = \Delta_{\gamma_\mu, \gamma_5 \gamma_\mu} + \Delta_{\Sigma_1} \quad (8)$$

The explicit expressions for Δ_{Σ_1} and Δ_{Σ_2} using in the continuum the $\overline{\text{MS}}$ regularization can be found in Ref. (5). For the operators $\mathbb{1}$, γ_5 , γ_μ , $\gamma_5 \gamma_\mu$ and $\sigma_{\mu\nu}$ we obtain (relating the lattice computation with the continuum computation in the $\overline{\text{MS}}$ scheme):

$$\begin{aligned} \Delta_{\mathbb{1}, \gamma_5} &= 4(\gamma_E - F_{0001} - 1) + 4\pi^2 (I_1 + I_2 - I_3) \\ \Delta_{\gamma_\mu, \gamma_5 \gamma_\mu} &= \gamma_E - F_{0001} - 2 + 4\pi^2 \left(\frac{I_1}{4} + \frac{I_2}{2} - I_3 \right) \\ \Delta_{\sigma_{\mu\nu}} &= -4\pi^2 I_3 \end{aligned} \quad (9)$$

γ_E is the Euler-Mascheroni constant and the constant $F_{0001} \sim 1.31$ is defined in Ref. (10). $I_{1,2,3}$ are given by:

$$\begin{aligned} I_1 &= \int_{-\pi}^{\pi} \left(\frac{dq}{2\pi} \right)^4 \left\{ \frac{\left[\frac{4}{3} (\Delta_1 \Delta_4 - \Delta_5) + 2r^2 \Delta_4^2 + 4r^4 \Delta_1^2 \Delta_4 \right]}{\Delta_1 \Delta_2^2} + \frac{4 \Delta_4 (\Delta_3^2 - 2 \Delta_1 \Delta_3)}{\Delta_1^3 \Delta_2^2} \right\} \\ I_2 &= r^2 \int_{-\pi}^{\pi} \left(\frac{dq}{2\pi} \right)^4 \left\{ \frac{[4 \Delta_1 (4 - \Delta_1) - \Delta_4]}{\Delta_2^2} \right\} \\ I_3 &= \int_{-\pi}^{\pi} \left(\frac{dq}{2\pi} \right)^4 \left\{ \frac{-4 \Delta_5}{3 \Delta_1 \Delta_2^2} + \frac{\left(\frac{\Delta_4}{3} + 2r^2 \Delta_4 + 4r^4 \Delta_1^2 \right)}{\Delta_2^2} \right\} \end{aligned} \quad (10)$$

where r is the parameter which defines the fermionic action on the lattice^(5,9) and $\Delta_1, \dots, \Delta_5$ are:

$$\begin{aligned} \Delta_1 &= \sum_{\mu} \sin^2(q_{\mu}/2); \quad \Delta_2 = \sum_{\mu} \sin^2(q_{\mu}) + 4r^2 \left[\sum_{\mu} \sin^2(q_{\mu}/2) \right]^2 \\ \Delta_3 &= \sum_{\mu} \sin^4(q_{\mu}/2); \quad \Delta_4 = \sum_{\mu} \sin^2(q_{\mu}); \quad \Delta_5 = \sum_{\mu} \sin^2(q_{\mu}) \sin^2(q_{\mu}/2) \end{aligned} \quad (11)$$

From Eqs. (9), (10) and (11) and the expressions for Σ_1 and Σ_2 given in Ref. (5), we found by numerical integration the results listed in the Table^(f3).

Then we can compare the results from Montecarlo simulations for f_π and f_0^{-1} with their experimental values. On the lattice, taking for example the results of Ref. (3), one has:

TABLE I - We give $\Delta_{\Sigma_{1,2}}, \Delta_{\mathbb{1}}, \gamma_5, \gamma_\mu, \gamma_5 \gamma_\mu, \sigma_{\mu\nu}$, computed by numerical integration, for different values of r ⁽⁵⁾. The error in the numerical integration is $\sim 5-10\%$.

Δ_{Σ_1}	Δ_{Σ_2}	$\Delta_{\mathbb{1}}$	Δ_{γ_5}	Δ_{γ_μ}	$\Delta_{\gamma_5 \gamma_\mu}$	$\Delta_{\sigma_{\mu\nu}}$	r
-6.03	33.24	-33.24	-33.24	-8.74	-8.74	0.76	0.0
-7.15	25.39	-18.64	-26.74	-8.97	-4.92	-0.37	0.2
-8.90	16.52	-8.34	-19.77	-8.76	-3.05	-1.85	0.4
-10.48	10.41	-3.84	-15.13	-8.37	-2.72	-2.90	0.6
-11.74	6.20	-1.52	-12.12	-8.06	-2.75	-3.60	0.8
-12.82	3.07	-0.26	-9.78	-7.75	-3.00	-4.16	1.0

$$f_\pi^{\text{LATT}} = 150 \pm 50 \text{ MeV} \quad (\text{exp} \sim 93 \text{ MeV})$$

(12)

$$(f_0^{-1})^{\text{LATT}} = 0.5 \pm 0.1 \quad (\text{exp} \sim 0.19)$$

for $r=1$ and $\beta=6$ ($\alpha_S = \frac{3}{2\pi\beta}$).

From Eqs. (5), (8) we get:

$$f_\pi = (1 + \frac{\alpha_S}{4\pi} C_F \Delta_\pi) f_\pi^{\text{LATT}} \sim \left[1 + \frac{-6.7}{2\pi^2\beta} \right] f_\pi^{\text{LATT}} \sim 0.94 f_\pi^{\text{LATT}}$$

$$f_0^{-1} = (1 + \frac{\alpha_S}{4\pi} C_F \Delta_0) (f_0^{-1})^{\text{LATT}} \sim \left[1 + \frac{-20.6}{2\pi^2\beta} \right] (f_0^{-1})^{\text{LATT}} \sim 0.83 (f_0^{-1})^{\text{LATT}}$$

One sees that first order perturbative corrections turn out to be too small and cannot explain the difference between the results from Montecarlo simulations and the experimental values (Eqs. (12)). These discrepancies are probably due to $O(a^2)$ effects in the lattice action and could be probably partially corrected by systematically constructing a lattice action differing from the continuum one by terms at least of order $O(a^4)$ ⁽¹¹⁾.

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FOOTNOTES

- (1) It can be eventually necessary to know the two loops anomalous dimension for these operators in the continuum.
- (2) An analog computation for non local operators (also used in Montecarlo simulations⁽⁶⁾) will be presented elsewhere⁽⁷⁾.
- (3) Note that, on the lattice, even for $r=0$ and $m_1=m_2$ the vector current $\bar{\psi}_2 \gamma^\mu \psi_1$ is not conserved (cfr. the Table)⁽⁹⁾ and that, as for the \overline{MS} scheme for $r=0$ the $O(a_s)$ term of Σ_2 exactly cancels the perturbative correction to $\Delta_{1, 5}$.

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