To be submitted to
Phys. Letters B

ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-82/68(P)
14 Settembre 1982

G. De Franceschi, N. Lo Iudice and F. Palumbo:

TWO-ROTOR MODEL OF DEFORMED NUCLEI

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ABSTRACT.

The two-rotor model of deformed nuclei, already studied for N = Z, is reformulated using a more appropriate quantization procedure and extended to the case N ≠ Z. While the neutron excess effects are negligeable, the different quantization procedure has the effect of halving the excitation energy of the collective M1 state.

A few years ago a two-rotor model of deformed nuclei has been considered[(1)]. In such a model protons and neutrons are assumed to form separate rigid bodies of elliptical shape. The restoring force generated by their relative displacement can give rise, classically, either to relative rotational oscillations or to a configuration where the nucleus rotates as a whole while the proton neutron symmetry axes stay at a fixed angle.

The model which has been studied for simplicity only for N = Z, predicts collective states only in the region of heavy deformed nuclei. In such a region for A = 180 and deformation parameter $|\delta| \sim 0.25$, there are two K = 1 states with I = 1, 2 at about 12 MeV with $B(M1)\uparrow \sim 15(\frac{e}{2m})^2$ and $B(E2)\uparrow \sim 0.6$ W.u. respectively.

In Ref. (1) (to be referred to as I), the classical Hamiltonian has been quantized after the transformation to intrinsic frame variables (to be defined later). Now it is known that quantiza

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tion in different systems of coordinates gives rise in general to different results[2]. We find in fact, quantizing in the fixed-frame coordinates a different Hamiltonian, whose predictions are qualitatively similar but quantitatively different from the previous ones.

The two quantization procedures are both legitimate, but since the model is to be considered an approximation to a Nuclear Hamiltonian expressed in terms of the fixed-frame nucleon coordinates, the present procedure appears more appropriate from a physical point of view.

We consider the general case $N \neq Z$. The classical Hamiltonian is

$$H = \frac{1}{2\mathcal{I}_p} I^{p^2} + \frac{1}{2\mathcal{I}_n} I^{n^2} + V \ , \tag{1}$$

where $\vec{I}^p$, $\vec{I}^n$, $\mathcal{I}_p$, $\mathcal{I}_n$ are the angular momenta and moments of inertia of protons, neutrons, and V is their potential energy.

Having in mind the quantization of the model, we add the constraints appropriate to rigid bodies with axial symmetry

$$I^p_{\zeta_p} = I^n_{\zeta_n} = 0 \ , \tag{2}$$

where $\hat{\zeta}^p$ and $\hat{\zeta}^n$ are the proton and neutron symmetry axes.

It is now convenient to introduce the total angular momentum $\vec{I}$ via the transformation

$$\vec{I} = \vec{I}^p + \vec{I}^n, \qquad \vec{S} = \vec{I}^p - \vec{I}^n \ , \tag{3}$$

and rewrite H as

$$H = \frac{1}{2\mathcal{J}}(I^2 + S^2) + \frac{\mathcal{I}_n - \mathcal{I}_p}{4\mathcal{I}_n \mathcal{I}_p} \vec{I}\vec{S} + V \ , \tag{4}$$

where

$$\mathcal{J} = \frac{4\,\mathcal{I}_p \mathcal{I}_n}{\mathcal{I}_p + \mathcal{I}_n} \ . \tag{5}$$

In the second term we recognize the familiar Coriolis coupling.

We assume the potential V to depend on the angle θ between $\hat{\zeta}^p$, $\hat{\zeta}^n$

$$\cos 2\theta = \hat{\zeta}^p \hat{\zeta}^n \ . \tag{6}$$

It is therefore natural to introduce this variable along with the other variables necessary to identify $\hat{\zeta}^p$ and $\hat{\zeta}^n$. These variables may be the Euler angles α, β, γ of the intrinsic frame defined by

$$\hat{\xi} = \frac{\hat{\zeta}^p \hat{\zeta}^n}{\sin 2\theta} \ , \qquad \hat{\eta} = \frac{\hat{\zeta}^p - \hat{\zeta}^n}{2\sin\theta} \ , \qquad \hat{\zeta} = \frac{\hat{\zeta}^p + \hat{\zeta}^n}{2\cos\theta} \ . \tag{7}$$

The transformation

$$(\hat{\zeta}^p, \hat{\zeta}^n) \longrightarrow (\theta, \alpha, \beta, \gamma) \tag{8}$$

is one to one and regular for $0 < \theta < \pi/2$.

We have now two options. As in I, we can first express the Hamiltonian in terms of the classical components of $\vec{I}$ and $\vec{S}$ along the intrinsic axes, and then perform the quantization by replacing such components by their operator realizations.

The alternative procedure, which we follow here for the mentioned reasons, is to quatize by replacing in Eq. (4) the fixed-frame components of $\vec{I}$ and $\vec{S}$ with their operator realization, and then perform transformation (8). The details of the calculations involved will be given elsewhere.

We report here the results. The Hamiltonian is

$$H = \frac{1}{2\mathcal{J}} I^2 + H_I \ , \tag{9}$$

$$\begin{aligned} H_I = & \frac{1}{2\mathcal{J}} \left[\operatorname{ctg}^2\theta\, I_\zeta^2 + \operatorname{tg}^2\theta\, I_\eta^2 - \frac{\partial^2}{\partial\theta^2} - 2\operatorname{ctg}^2 2\theta \frac{\partial}{\partial\theta} \right] + \\ & + \frac{\mathcal{J}_p - \mathcal{J}_n}{4\mathcal{J}_p\mathcal{J}_n} \left[\frac{1}{2} (\operatorname{ctg}\theta + \operatorname{tg}\theta)(I_\zeta I_\eta + I_\eta I_\zeta) - i I_\xi \frac{\partial}{\partial\theta} \right] . \end{aligned} \tag{10}$$

For N = Z, Eq. (10) differs from the Hamiltonian derived in I by the term $-\frac{1}{2\mathcal{J}} 2\operatorname{ctg}^2 2\theta \frac{\partial}{\partial\theta}$.
The general expression for the eigenfunctions is

$$\Psi_{IM\sigma} = \sqrt{\frac{2I+1}{8\pi^2}} \sum_K D^I_{MK}\, \Phi_{IK\sigma}(\theta) \ , \tag{11}$$

where σ stands for all the additional quantum numbers. The scalar product is defined by

$$\langle\Psi|\Psi'\rangle = \int 2\sin 2\theta\, d\theta \sin\beta\, d\beta\, d\alpha\, d\gamma\, \Psi^* \Psi' \ , \qquad 0 \le \theta \le \pi/2 \ . \tag{12}$$

Since the versus of $\hat{\zeta}^p$, $\hat{\zeta}^n$ is unobservable, we must impose on the w. f. the constraints

$$R^p_\xi(\pi)\Psi = R^n_\xi(\pi)\Psi = \Psi \ , \tag{13}$$

$R^{p,n}_\xi(\pi)$ being a rotation by π around $\hat{\xi}$ of protons, neutrons resp. The other symmetries considered in I are absent for $Z \neq N$.

Taking into account the above constraints we get

$$\Psi_{IM\sigma} = \sqrt{\frac{2I+1}{8\pi^2}} \sum_{K \ge 0} \frac{1}{1+\delta_{KO}} \left[D^I_{MK} + (-1)^I D^I_{M-K} \right] \Phi_{IK\sigma}(\theta) \ , \tag{14}$$

and a set of relations which for the states coupled by e. m. transitions ($I = 0$, $I = 1, 2$ with $|K| = 1$) can be written

$$\Phi_{IK\sigma}(\theta) = (-1)^I \, \Phi_{IK\sigma}(\frac{\pi}{2} - \theta) . \tag{15}$$

It is therefore sufficient to solve the eigenvalue problem for $0 \leq \theta \leq \pi/2$. The solution is simplified by the following transformation

$$U\Phi_{IK\sigma}(\theta) = \sqrt{\sin 2\theta}\, \Phi_{IK\sigma}(\theta) \overset{\text{def.}}{=} \varphi_{IK\sigma}(\theta) , \tag{16}$$

$$\begin{aligned} U H_I U^{-1} \overset{\text{def}}{=} H' &= \frac{1}{2\mathcal{J}} \left[\operatorname{ctg}^2\theta \, I_\zeta^2 + \operatorname{tg}^2 I_\eta^2 - \frac{\partial^2}{\partial\theta^2} - 2 - \operatorname{ctg}^2 2\theta \right] + \\ &+ \frac{\mathcal{J}_n - \mathcal{J}_p}{4\mathcal{J}_n \mathcal{J}_p} \left[\frac{1}{2} (\operatorname{ctg}\theta + \operatorname{tg}\theta)(I_\zeta I_\eta + I_\eta I_\zeta) - i I_\xi \frac{\partial}{\partial\theta} \right] + V(\theta) . \end{aligned} \tag{17}$$

We assume the harmonic approximation for V

$$V = \frac{1}{2} C \theta^2 , \qquad 0 \leq \theta \leq \pi/4 , \tag{18}$$

and consistently expand H' in powers of θ. Introducing the parameters

$$\theta_o = (\mathcal{J} C)^{-1/4} , \qquad \omega = \sqrt{\frac{C}{\mathcal{J}}} , \qquad x = \frac{\theta}{\theta_o} \tag{19}$$

we get

$$H' \simeq \frac{1}{2} \omega \left[- \frac{\partial^2}{\partial x^2} + \frac{1}{x^2} (I_\zeta^2 - \frac{1}{4}) + x^2 \right] , \tag{20}$$

where we have neglected the constant term and terms of order $\omega\theta_o^4$, $\omega\theta_o \frac{\mathcal{J}_n - \mathcal{J}_p}{\mathcal{J}_n + \mathcal{J}_p}$. This is justified by our estimate (see I) $\theta_o \sim 0.03$.

In this approximation the only dependence on the neutron excess is contained in ω. Eq. (20) differs from the intrinsic Hamiltonian in I by the term $-\frac{1}{2}\omega\,(-\frac{1}{4x^2})$.

The eigenvalues are

$$\varepsilon_{|K|\,n} = \omega\,(2n + |K| + 1) , \tag{21}$$

and the corresponding eigenfunctions

$$\varphi_{I|K|n}(\frac{\theta^2}{\theta_o^2}) = \sqrt{\frac{n!}{2\,\Gamma(n + |K| + 1)}} \, (\frac{\theta}{\theta_o})^{|K| + \frac{1}{2}} \, e^{-\frac{1}{2}\frac{\theta^2}{\theta_o^2}} \, L_n^{|K|}(\frac{\theta^2}{\theta_o^2}) , \tag{22}$$

where $L_n^{|K|}$ are generalized Laguerre polynomials.[3]

Finally the total eigenfunctions are

$$\varphi_{IM|K|n} = \sqrt{\frac{2I+1}{8\pi^2}}\ \frac{1}{1+\delta_{KO}} \left[D^I_{MK} + (-1)^I D^I_{M-K} \right] \frac{1}{\sqrt{2\theta}}\ \varphi_{|K|n} \ . \tag{23}$$

For a numerical estimate we determine the restoring force constant according to the procedure described in I (App. B) with the only difference that rather than equating the energies we impose the generally adopted condition

$$C\,\theta_c = \left.\frac{d\varepsilon}{d\theta}\right|_{\theta=\theta_c} , \tag{24}$$

where ε is the energy increase due to the fact that at an angle θ_c a certain number of nucleon pairs no longer interact. Assuming equal neutron and proton radii we obtain

$$C = 26\,\delta^2 A^{4/3}\ \mathrm{MeV} , \tag{25}$$

which is smaller by a factor 2 than the value obtained in I. Taking the rigid body values for $\mathcal{J}_p$ and $\mathcal{J}_n$ we obtain for the frequency

$$\omega = \sqrt{\frac{A^2}{4NZ}\ \frac{C}{\mathcal{J}_n + \mathcal{J}_p}} \simeq \sqrt{\frac{C}{\mathcal{J}_n + \mathcal{J}_p}} \simeq 42\,|\delta|\,A^{-1/6}\ \mathrm{MeV} , \tag{26}$$

the factor $\sqrt{A^2/4NZ}$ due to neutron excess being practically 1.

For $|\delta| \sim 0.25$, $A = 188$, $\omega \simeq 4.4$ MeV.

The e.m. radiation excites only the $I = 1, 2$, $|K| = 1$ states. The M1 transition strength (entirely due to orbital motion) is

$$B(M1)\uparrow \simeq \frac{3}{16\pi}\ \frac{1}{\sqrt{\theta_o}} \left(\frac{e}{2mc}\right)^2 \sim 0.035\,|\delta|\,A^{3/2} \left(\frac{e}{2mc}\right)^2 , \tag{27}$$

showing no effect of the neutron excess. For $A = 188$, $|\delta| = 0.25$, $B(M1)\uparrow \sim 22\left(\frac{e}{2mc}\right)^2$.

The E2 transition strength is given by

$$B(E2)\uparrow \simeq 0.32\,\sqrt{\frac{Z^3}{N}}\,|\delta|\,A^{5/6}\,e^2 f_m^4 , \tag{28}$$

and depends on the neutrons excess. For $Z = 78$, $A = 188$ and $|\delta| = 0.25$, $B(E2)\uparrow \sim 413\,e^2 f_m^4 \simeq$ $\simeq 1.3$ W.u.

Summarizing, only the state $I = |K| = 1$ results to be a truly collective state, as in I. However, its excitation energy is reduced from 12 MeV to 4.4 MeV.

In this connection we would like to mention that in the framework of the VPM[4] and the IBM[5] a state at about 2-3 MeV has been predicted, which according to the authors can be interpreted in terms of our model.

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