To be submitted to
Physics Letters B

ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-82/62(P)
31 Luglio 1982

G. Pancheri and Y. N. Srivastava: SOFT GLUON COMPONENT OF DIPHOTON AND DILEPTON TRANSVERSE MOMENTUM.

Servizio Documentazione
dei Laboratori Nazionali di Frascati
Cas. Postale 13 - Frascati (Roma)

INFN - Laboratori Nazionali di Frascati
Servizio Documentazione

LNF-82/62(P)
31 Luglio 1982

G. Pancheri and Y. N. Srivastava SOFT GLUON COMPONENT OF DIPHOTON AND DILEPTON TRANSVERSE MOMENTUM.

ABSTRACT

The rise with energy in the mean transverse momentum of diphotons and dileptons produced in hadronic scattering is discussed through an expression obtained by summing soft gluons. Some numerical tests and predictions are presented.

In this paper we study the contribution of soft gluon emission[1] to the mean square transverse momentum in dilepton and diphoton production in $\pi^{\pm}p$, pp and $p\bar{p}$ scattering. The standard parton model has been very successful in analyzing current-hadron processes. However, by construction, it has no transverse momentum spread, i. e. the spectrum in $q_{\perp}$ should be $\delta(q_{\perp}^2)$. Even allowing for an intrinsic transverse momentum, the parton model does not account for the rise with energy of the mean square transverse momentum[2,3]. Numerical estimates based on first order QCD calculations[4,5] fall short of describing the actual data. It has been suggested by a number of authors[6,7] that emission of soft gluons from the quark legs can account for the observed transverse momentum distribution in the small to medium energy region for a variety of processes. Likewise, it also accounts for most of the observed scaling violations in deep inelastic scattering[8].

In this paper we shall discuss the soft gluon component of the mean square transverse momentum of lepton or photon pairs produced in hadron-hadron scattering. We propose the following formula:

$$\langle q_\perp^2(x_1,x_2;s)\rangle = \frac{4s}{3\pi}\,\frac{\Sigma e_i^{2n}\int_{x_1}^1 dy_1\int_{x_2}^1 dy_2\,\alpha_s\left[s(y_1-x_1)(y_2-x_2)\right]\left\{I_i(y_1,\beta)\bar{I}_i(y_2,\beta)+(I_i\leftrightarrow\bar{I}_i)\right\}}{\Sigma e_i^{2n}\left\{I_i(x_1,\beta)\,\bar{I}_i(x_2,\beta)+(I_i\leftrightarrow\bar{I}_i)\right\}} \qquad (1)$$

where $n=1,2$ for $\mu^+\mu^-$ and $\gamma\gamma$ production respectively. In eq. (1), α_s is the running coupling constant,

$$\beta = \frac{4}{3\pi}\int\frac{dk_\perp^2}{k_\perp^2}\,\alpha_s(k_\perp^2) \sim \frac{16}{25}\ln\left[\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}\right]$$

and the Q^2-dependent parton densities I_i are related to the integrated μ-pair cross-section through

$$s\,\frac{d\sigma^{hh\to\mu^+\mu^-+X}}{dx_1dx_2} = \frac{4\pi\alpha^2}{9x_1x_2}\,\Sigma e_i^2\left\{I_i(x_1,\beta)\bar{I}_i(x_2,\beta)+(I_i\leftrightarrow\bar{I}_i)\right\}. \qquad (2)$$

The interest in examining $\mu^+\mu^-$ and $\gamma\gamma$ production[9] side by side is that the dominant contribution should come from quark-antiquark annihilation graphs and from higher order QCD corrections to these graphs. Concerning the latter, one can separate the "hard" corrections from the soft ones, which factorize and which do not distinguish between $q\bar{q}\to\mu^+\mu^-$ + gluons and $q\bar{q}\to\gamma\gamma$ + gluons. For

$$h_1(p_1)+h_2(p_2)\longrightarrow a(k_1)+b(k_2)+X$$

let the basic cross-section be[10]

$$d\sigma^{hadronic} = \sum_i e_i^{2n}\int dy_1\int dy_2\;\mathcal{G}_i(y_1,y_2)\int d^4\mathcal{P}(K)\,\delta^4(p_1y_1+p_2y_2-k_1-k_2-K)\,d\sigma_i^{q\bar{q}\to ab+hard}$$

where $\mathcal{G}_i(y_1,y_2)$ is related to the bare parton densities and emission of soft gluons of total momentum K is described by the function $d^4\mathcal{P}(K)$, while the short distance aspects of the process are included in $d\sigma_i^{q\bar{q}\to ab+hard}$. To zeroeth order in α_s, one has

$$\frac{d\sigma_i^{q\bar{q}\to\mu^+\mu^-}}{dz} = \frac{\pi\alpha^2}{6Q^2}\,e_i^2(1+z^2) \qquad \text{and} \qquad \frac{d\sigma_i^{q\bar{q}\to\gamma\gamma}}{dz} = \frac{2\pi\alpha^2}{3Q^2}\,e_i^4\,\frac{(1+z^2)}{(1-z^2)}$$

with $Q^2=q^2\equiv(k_1+k_2)^2$ and z is the scattering angle in the c.m. of the final pair. The Bloch Nordsieck approach for summing the massless quanta to all orders, gives

$$d^4\mathcal{P}(K) = \frac{d^4K}{(2\pi)^4}\int d^4x\,e^{iK\cdot x}\,\exp\left\{-\int_0^{\epsilon} d^3\bar{n}_k(1-e^{-ik\cdot x})\right\} \qquad (3)$$

where the average number of gluons $d^3\bar{n}_k$ can be calculated in the one loop approximation using a fixed quark source, i. e.

$$d^3\bar{n}_k \simeq \frac{4}{3\pi} \alpha_s(k_\perp^2) \frac{dk_+ dk_-}{k_\perp^2} , \qquad k_\pm = k_o \pm k_3 .$$

The distribution in eq. (3) is normalized to 1. When integrated in the energy and longitudinal momentum variables it gives an eikonal type formula for the transverse momentum distribution(6), i. e.

$$\int \frac{d^4\mathscr{P}(K)}{dK_o dK_3} dK_o dK_3 = \frac{d^2\vec{K}_\perp}{(2\pi)^2} \int d^2\vec{x}_\perp\, e^{-i\vec{K}_\perp\cdot\vec{x}_\perp} \exp\left\{- \int_0^\epsilon d^3\bar{n}_k (1 - e^{i\vec{k}_\perp\cdot\vec{x}_\perp})\right\} .$$

Likewise, it can be integrated in the transverse momentum variable. Defining

$$\mathbb{P}(K) = \int \frac{d^4\mathscr{P}(K)}{d^4K} d^2\vec{K}_\perp = \int \frac{dt\, dx_3}{(2\pi)^2} e^{iK_o t - iK_3 x_3} \exp\left\{- \int_0^\epsilon d^3\bar{n}_k (1 - e^{-ikt + ik_3 x_3})\right\}$$

one can then write for the differential cross-section (μ-pairs)

$$\frac{d\sigma}{dq_o dq_3 dz} = \frac{\pi\alpha^2}{6Q^2} (1+z^2) \sum_i e_i^2 \int_{x_1}^1 dy_1 \int_{x_2}^1 dy_2\, \mathscr{G}_i(y_1, y_2)\, \mathbb{P}(p_1 y_1 + p_2 y_2 - q) .$$

Taking the limit of zero quark mass, the function $\mathbb{P}(K)$ separates in a product of two independent probabilities in the light cone variables $K_\pm = K_o \pm K_3$. We then have

$$\mathbb{P}(K) \simeq \frac{2d\mathscr{P}(K_+)}{dK_+} \frac{d\mathscr{P}(K_-)}{dK_-} = \frac{1}{2\epsilon^2} \left[\frac{\gamma^{-\beta}}{\Gamma(\beta)}\right]^2 \left(\frac{K_+K_-}{4\epsilon^2}\right)^{\beta - 1} \qquad (4)$$

$$\text{for} \quad K_o < \epsilon$$

The parameter ϵ represents the energy scale which we shall take to be sy_1y_2 and $K_+ = \sqrt{s}(y_1 - x_1)$, $K_- = \sqrt{s}(y_2 - x_2)$. The power law expression thus obtained for $\mathbb{P}(K)$ allows to write

$$\frac{s}{2} \frac{d\sigma^{\text{hadronic}}_{\mu\text{ pairs}}}{dq_o dq_3 dz} \simeq \frac{\pi\alpha^2}{6Q^2} (1+z^2) \sum_i e_i^2 \int_{x_1}^1 \frac{dy_1}{y_1} \int_{x_2}^1 \frac{dy_2}{y_2} \mathscr{G}_i(y_1, y_2) \left[\frac{\gamma^{-\beta}}{\Gamma(\beta)}\right]^2 \left(1 - \frac{x_1}{y_1}\right)^{\beta - 1} \left(1 - \frac{x_2}{y_2}\right)^{\beta - 1}$$

Comparing with the usual parametrization for the cross-section one sees that we have defined Q^2-dependent parton densities :

$$I_i(x,\beta) = \frac{\gamma^{-\beta}}{\Gamma(\beta)} \int_x^1 \frac{dy}{y} f_i(y)(1-\frac{x}{y})^{\beta-1} \xrightarrow[\beta\to 0]{} f_i(x)$$

where $f_i(y)$ are the bare parton densities. Similarly, for $\gamma\gamma$ production one has

$$\frac{s}{2}\frac{d\sigma^{hadronic}_{\gamma\gamma}}{dq_o dq_3 dz} \simeq \frac{2\pi\alpha^2}{3Q^2}\frac{(1+z^2)}{(1-z^2)}\sum_i e_i^4 \left\{ I_i(x_1,\beta)\bar{I}_i(x_2,\beta) + (I_i \leftrightarrow \bar{I}_i)\right\} .$$

Having thus defined tha basic differential cross-section in our formalism, we can now proceed to calculate the mean square transverse momentum from

$$\langle q_\perp^2 \rangle \frac{d\sigma}{dq_o dq_3 dz} = \int q_\perp^2 d^2\vec{q}_\perp \frac{d\sigma}{d^4q\, dz} . \qquad (5)$$

In our approximation the only $q_\perp$-dependence is through the function $d^4\mathscr{P}(K)$ for which the following formula can be easily derived:

$$\int d^2K_\perp K_\perp^2 \frac{d^4\mathscr{P}(K)}{d^4K} \equiv \Pi(K) = \int \frac{dt\, dx_3}{(2\pi)^2} e^{iK_o t - iK_3 x_3} \int_0^\epsilon k_\perp^2 d^3\bar{n}_k e^{-ikt + ik_3x_3} \cdot$$
$$\cdot \exp\left\{ -\int_0^\epsilon d^3\bar{n}_k (1 - e^{-ikt+ik_3x_3}) \right\}$$

i. e.

$$\Pi(K) = \int_0^\epsilon k_\perp^2 d^3\bar{n}_k\, \mathbb{P}(K-k) \simeq \qquad (6)$$
$$\simeq \frac{4}{3\pi}\left[\frac{\gamma^{-\beta}}{\Gamma(\beta)}\right]^2 2^{1-\beta}\epsilon^{-2\beta} \int_0^{K_+} dk_+(K_+-k_+)^{\beta-1} \int_0^{K_-} dk_-(K_- - k_-)^{\beta-1}\alpha_s(k_+k_-)$$

having used the factorized form of eq. (4) and the property that $\mathbb{P}(K) = 0$ for $K_\pm \leqslant 0$. Inserting the above expression in eq. (4), for fixed values of z, q_o and q_3, will give

$$\langle q_\perp^2 \rangle_{soft} = \frac{\Sigma e_i^{2n} \int_{x_1}^1 dy_1 \int_{x_2}^1 dy_2\, \mathscr{G}_i(y_1,y_2)\, \Pi(p_1y_1 + p_2y_2 - q)}{\Sigma e_i^{2n} \int_{x_1}^1 dy_1 \int_{x_2}^1 dy_2\, \mathscr{G}_i(y_1,y_2)\, \mathbb{P}(p_1y_1 + p_2y_2 - q)} . \qquad (7)$$

Finally, using eqs. (4) and (6) into (7), one obtains the equation we presented at the beginning.

A number of interesting predictions can be obtained from eq. (1). One notices that, as far as the soft gluon component is concerned, μ-pair and diphoton production differ only in the charge factor e_i^{2n}. For processes like $p\bar{p}$ and $\pi^- p$ where a single quark term is dominant (for most x values), we have the rather general result that $\langle q_\perp^2(x_1, x_2; s)\rangle$ should be the same for $\mu^+\mu^-$ or $\gamma\gamma$, provided $x_{1,2}$ are large enough that Compton[11] and other processes can be ignored.

For the numerical work which follows, and because of our ignorance of the behaviour of $\alpha_s(k_\perp^2)$ in the infrared region, we shall use a constant $\bar{\alpha}_s$, averaged over the $x_{1,2}$ range under consideration. As for the intrinsic transverse momentum $\langle q_\perp^2\rangle_{int}$, we are as yet unable to calculate it. Thus our prediction concerns the quantity

$$B(x_1, x_2; s) = \frac{\langle q_\perp^2\rangle - \langle q_\perp^2\rangle_{int}}{s} \quad . \tag{8}$$

For the parton densities occurring in eq. (1), we have employed the parametrization given in ref. (3). It should be noticed that at the present level of accuracy, there is no Q^2-dependence in these densities.

With the above approximations, B in eq. (8) depends on x_1 and x_2 explicitly and on s implicitly through $\bar{\alpha}_s$. In Fig. 1, we have plotted B vs. $\tau = M^2/s$, for zero ra-

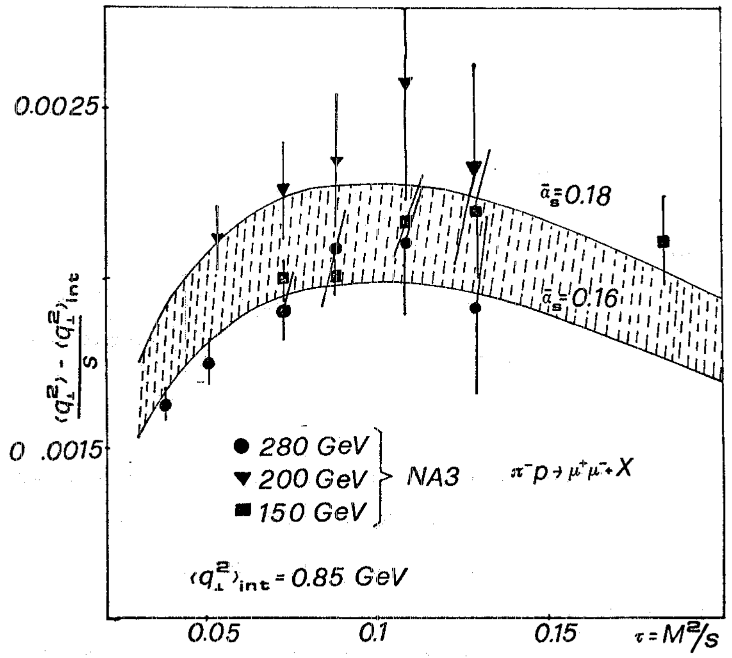

FIG. 1 - The shaded region is our prediction for $B = \frac{\langle q_\perp^2\rangle - \langle q_\perp^2\rangle_{int}}{s}$ vs τ (rapidity y = 0) with $\bar{\alpha}_s = 0.17 \pm 0.01$ for $\pi^- p \to \mu^+\mu^- + X$. The data are from ref. (12) averaged over rapidity.

pidity, $y = \frac{1}{2}\ln(x_1/x_2) = 0$ for μ-pair production in $\pi^- p$ scattering. The shaded region represents our prediction for this quantity for $\bar{\alpha}_s = (0.17 \pm 0.01)$. For comparison, a plot made by Michelini[12] using the NA3 data[3] is also presented. It should be mentioned that the first order QCD result[5] is off by a factor (2÷3) and also its dependence on τ is wrong. A second order calculation[13] ameliorates this discrepancy. We also predict a significant variation of $B(x_1, x_2; s)$ with rapidity. In Fig. 2, we have illustrated this point for μ-pair production in $p\bar{p}$ scattering by plotting B vs. τ for two different values of $x_F = x_1 - x_2$. At collider energies, $\langle q_\perp^2 \rangle$ must be sizable since it is roughly linear with s. Thus the intrinsic momentum can be neglected there and $sB(x_1, x_2; s)$ measures directly the mean square transverse momentum. Taking into account the logarithmic variation of α_s with energy, one obtains

$$\langle q_\perp^2 \rangle^{p\bar{p}}_{\mu^+\mu^-} \simeq 190\ \mathrm{GeV}^2 \qquad \text{at } \sqrt{s} = 540\ \mathrm{GeV},\ x_F = 0 .$$

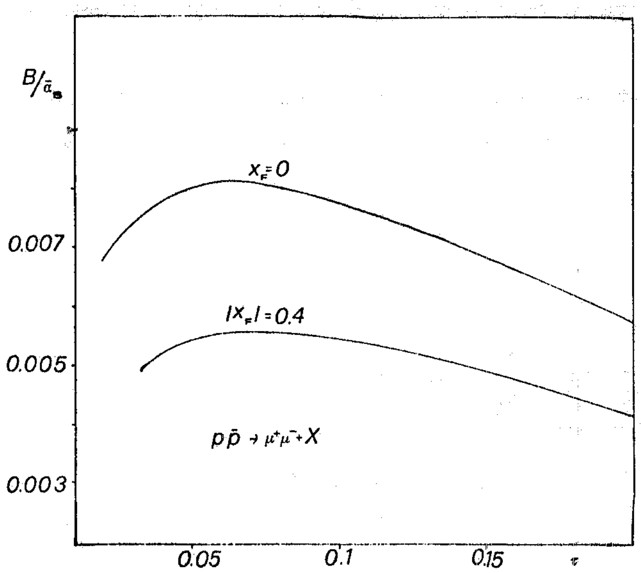

FIG. 2 - $(B/\bar{\alpha}_s)$ vs. τ at $x_F = 0$ and $x_F = 0.4$ for $pp \to \mu^+\mu^- + X$.

For diphoton production, to be measured some time in the future at the Tevatron, one can estimate the quantity $B(x_1, x_2; s)$ for fixed τ as a function of x_F. The same numerical estimates also apply, to a very good approximation, for μ-pair production. In Fig. 3, we show our prediction.

Our last point concerns the QCD coupling constant in eq. (1). Although we have used an average constant value, this is by no means mandated by our formalism. On the contrary, it could be very interesting to use our equation to test various models of α_s in the infrared region probed by the integrals at the numerator. Such tests

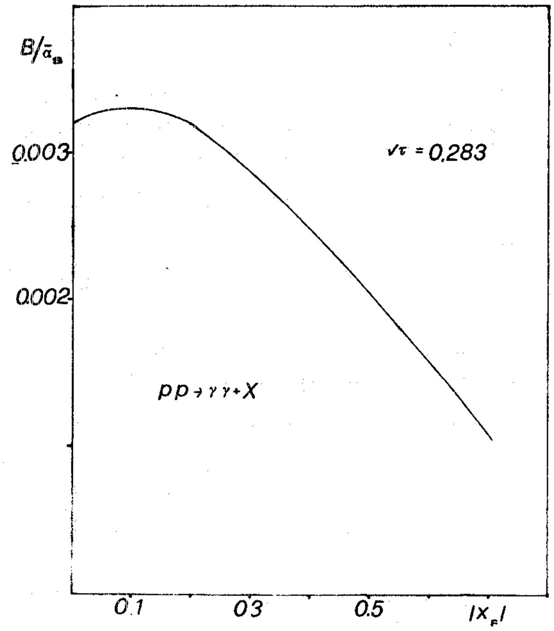

FIG. 3 - $(B/\bar{\alpha}_s)$ vs. F_x at $\tau = 0.08$ for pp $\rightarrow \gamma\gamma$ + X.

may even throw some light on the nature of the intrinsic momentum and its relation to the $k_\perp \rightarrow 0$ limit of $\alpha_s(k_\perp^2)$.

In conclusion, specific predictions have been presented for the transverse momentum spectrum of $\mu^+\mu^-$ and $\gamma\gamma$ given by summing soft gluons using the Bloch Nordsieck method. The variation of the slope parameter B with $\tau = M^2/s$ was compared with $\pi^- p$ data (for μ-pair production). For a reasonable average value of α_s ($\simeq 0.17 \pm 0.01$) good agreement was found. Collider data will be crucial in testing these predictions more accurately since for such high energies the contribution from the intrinsic $\langle q_\perp^2 \rangle_{int}$ would be negligible. In our model, B varies significantly with rapidity. Again it should be relatively clean to make an experimental check of the above predictions at collider energies.

REFERENCES.

(1) - For a discussion of soft gluon effects, see recent reviews by M. Greco and G. Pancheri in "Moriond Workshop on Lepton Pair Production", Edited by J. Tran Thanh Van (Frontieres, France, 1981).

(2) - J. K. Yoh et al., Phys. Rev. Letters 41, 684 (1978); A. S. Ito et al., Phys. Rev. D23, 604 (1981).

(3) - J. Badier et al., Phys. Letters 89B, 145 (1979); G. Matthiae, Rivista del Nuovo Cimento 4, 1 (1981).

(4) - F. Halzen and D. Scott, Phys. Rev. D18, 3378 (1978); K. Kajantie and R. Raito, Nuclear Phys. B139, 72 (1978).

(5) - G. Altarelli, G. Parisi and R. Petronzio, Phys. Letters 76B, 351 and 356 (1978).

(6) - G. Pancheri and Y. Srivastava, Phys. Rev. Letters 43, 11 (1979); G. Parisi and R. Petronzio, Nuclear Phys. B154, 427 (1979); J. C. Collins and D. E. Soper, in Proceedings of the XX Intern. High Energy Physics Conference, Madison, Wisconsin, 1980.

(7) - P. Chiappetta and M. Greco, Phys. Letters B106, 219 (1981); Marseille Preprint CPT-81/P. 1335 (1981).

(8) - D. Amati, A. Bassetto, M. Ciafaloni, G. Marchesini and G. Veneziano, Nuclear Phys. B173, 429 (1980); G. Pancheri, Y. Srivastava and M. Ramon-Medrano, Phys. Rev. D23, 2533 (1981).

(9) - For a review on direct photon production see I. Mannelli, in Proceedings of the Bonn Conference, 1981.

(10) - G. Pancheri, M. H. Friedman and Y. N. Srivastava, Northeastern University Preprint NUB 2486 (1981).

(11) - J. Kubar, M. Le Bellac, J. L. Meunier and G. Plaut, Nuclear Phys. B175, 251 (1980).

(12) - A. Michelini, Rapporteur talk at the EPS Intern. Conference on High Energy Physics, Lisbon, 1981.

(13) - R. K. Ellis, G. Martinelli and R. Petronzio, Phys. Letters 104B, 45 (1981), and Preprint CERN-TH/3079 (1981).