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V. Bellini, E. De Sanctis and V. Lucherini: PHOCHA: A MONTE CARLO  
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FROM ANNIHILATION AND BREMSSTRAHLUNG OF RELATIVISTIC  
POSITRONS

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**PHOCHA: A MONTE CARLO PROGRAM TO CALCULATE THE PHOTON BEAM CHARACTERISTICS FROM  
ANNIHILATION AND BREMSSTRAHLUNG OF RELATIVISTIC POSITRONS**

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**PROGRAM SUMMARY**

Title of Program: PHOCHA

Computer: CDC 7600; Installation: CINECA Bologna. The program has also been run on the computer DEC VAX-11/780, with a word lenght of 32 bits

Operating system: CDC Scope 2.1.5 and NOS/BE

Programming Language used: FORTRAN IV EXTENDED

High speed storage required: 3000 Kws

No. of bits in a word: 60

Overlay structure: none

Peripherals used: mass storage, line printer

No. of cards in combined program and test deck: 1481

Typical running time: In the test program set-up (positron energy 140 MeV, positron beam incidence angle on the target 0.75°, No. of collimators 8, about 25 photon stories for each of the 5000 sampled positrons) 318 s of CDC-7600 CPU time are required to reach a statistical accuracy better than 2%, on the photon spectra.

Unusual features of the program: Use of the CERN program library:

subroutines: MXMPY, POLROT, VMATR

functions: RNDM

Restrictions on the complexity of the problem:

- a) the equations of the radial and the vertical phase-space ellipses of the positron beam are used in the canonical form;
- b) the incoming and outgoing target surfaces are assumed to be plane;
- c) the bremsstrahlung straggling is not considered.

Keywords: Intermediate energy photon beam; in flight positron annihilation and bremsstrahlung; Monte Carlo calculation.

Nature of the physical problem: The program PHOCHA calculates the absolute energy spectra, the radial and vertical profiles and the angular distributions of annihilation and bremsstrahlung photons from intermediate energy positrons. The energy spread, the energy loss and the multiple scattering of the positrons in the annihilation target are taken into account. Moreover the positron emittance, the positron incidence angle and the finite angular acceptance of the photon collimation channel are explicitly considered.

Method of solution: A positron, whose energy is extracted according to a given energy distribution, impinges on the target, flying in a given direction. The radiation point is determined according to the positron beam emittance and by a uniform extraction in the effective target thickness. Positron multiple scattering and energy loss are then sampled according to the appropriate laws. In order to minimize photon losses, the photon emission angle is extracted, for each positron, within a variable solid angle whose size and orientation are calculated by taking into account the positron flight direction and the photon production point in the target; moreover a number of photons proportional to the selected solid angle is always sampled.

LONG WRITE-UP

1. - INTRODUCTION

The measurements of photonuclear cross-section, using the quasi-monochromatic photon beam from the in-flight annihilation of positrons, are actually carried on in several laboratories (1).

In order to determine the shape and absolute value of different photonuclear cross sections an accurate knowledge of the photon flux is required. The annihilation and bremsstrahlung processes involved when an intermediate energy positron beam crosses the target are very sensitive to the photon collection angle. Moreover the energy spread and the emittance of the positron beam, the energy loss and the multiple scattering of the positron in the target strongly affect the photon flux. The aim of this Monte Carlo program is to take properly into account all the above effects in order to evaluate a reliable photon flux, in a reasonable computing time.

The program structure enables its use in the widely different conditions typical of the positron annihilation facilities. The program calculates the absolute energy spectra, the radial and vertical profiles, the angular distribution of photons on selectable planes (in the following called "view planes") set at different distances from the target.

All the relevant formulae concerning the involved processes and used in the calculation are given in ref. (2) and are reported in Appendix, for convenience

The results of the calculation have been successfully compared with the measurements of the LEALE annihilation photon beam of the Frascati Laboratories.

## 2. - PROGRAM LOGIC AND ORGANIZATION

As it is known the annihilation peak is associated with an unwelcome continuous bremsstrahlung spectrum. It is possible to improve the annihilation to bremsstrahlung ratio, at the expense of intensity and peak resolution, by increasing the collection angle of the photon beam respect to the positron beam. Usually one varies the photon to positron angle by changing, with dipole magnets, the positron impinging angle on the annihilation target. For this reason in the PHOCHA program we use several right-handed reference frames. Before going to describe in detail the program, let us define them. Fig. 1:

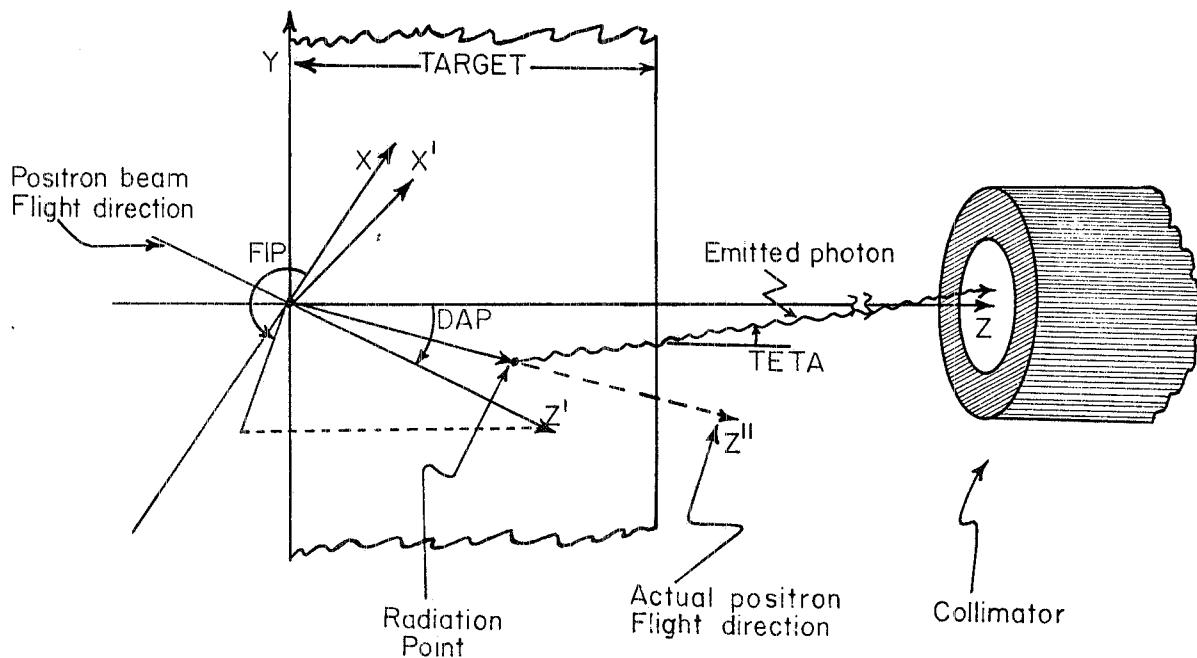


FIG. 1 - Right handed reference frames used in handling the radiation events. (X-Y-Z) - Lab frame; (X'-Y'-Z') - positron beam frame; (X''-Y''-Z'') - positron frame; (FIP, DAP) - positron beam flight direction in the Lab frame; TETA - polar angle of the photon flight direction in the Lab frame.

- the Lab reference frame has its origin at the incoming of the surface target ( $x,y$  plane); the  $y$ -axis points upward; the  $z$ -axis is coincident with the photon collimation axis;
- the positron beam reference frame has its  $z'$ -axis in the direction defined by the polar angles DAP, FIP with respect to the Lab one. The  $x'$ -axis coincides with the  $x$ -axis projection on the  $x'-y'$  plane;
- the positron reference frame has as its  $z''$  axis the actual positron flight direction; its  $x''$ -axis is oriented coherently with the use of the CERN polar angle rotation subroutine POLROT.

Different standard sampling techniques are used in the PHOCHA program, an exhaustive review of them may be found in ref (3).

A given number of positrons, NPOS, is let to impinge on the target at an angle calculated from the chosen photon collection angle and from the positron emittance. The positron energy is extracted according to a given distribution. Each positron undergoes multiple scattering and energy loss in the target, then it emits a number of photons, NCYCL1, uniformly distributed within an appropriate solid angle, DOM, whose orientation and size are calculated taking into account the radiation point inside the target and the actual positron flight direction, in order to minimize the loss of photons on the walls of the collimators. NCYCL1 is determined so that the number of photons emitted per unit of solid angle keeps always equal to the given value, PHDENS. All the photons gone beyond the required collimators are considered both as annihilation and bremsstrahlung photons, and are recorded, after weighting them for the appropriate differential cross section values, Avogadro number and atomic factors depending on the Z and A of the target.

The absolute value of the photon distributions per incident positron are finally obtained by multiplying by the target density and thickness, and by dividing by the extracted numbers of positrons and photons per unit of solid angle.

### 3. - PROGRAM DESCRIPTION

A flowchart depicting the organization of the program is given in Fig. 2.

The main program PHOCHA controls and organizes the overall calculation.

All input data are read by the subroutine DATIN and stored in the COMMON blocks or passed as arguments: see Table I for names, meaning and unit of measure of all input data.

Before entering the Monte Carlo loop the following quantities are calculated:

- the absolute normalizing constant, CDIINC=RO\*TAT/(COS(DAP)\*PHDENS\*NPOS);
- the transformation matrix from positron beam reference frame to the Lab one (subroutines ROTZ, ROTY, MXMPY);
- the minimum geometric half angle defined by the collimators for pointlike source TELIM.

At this point the Monte Carlo loop starts.

The positron total energy, PE, is sampled, according to its energy distribution (subroutine GAUS), and the energy loss in the media before the target is subtracted (subroutine MAXWE). The positron radiation depth inside the target, Z0, is uniformly extracted (function RNDM) in the effective target thickness, TAT/COS(DAP). If Z0 exceeds a minimum cutoff distance, THCUT, the energy loss, DE, and the multiple scattering angles, TESC and FISC, of the positron in the target are sampled (subroutines LANDAU, SCATT and MAXWE). If the proper index is activated, IEMIT=1, the positron beam emittance is taken into account through the subroutines BEDIM, POSDIV and the function PROJEC. At this point the transformation from the x', y', z frame to the x, y, z one is done (subroutine VMATR). Then, the subroutine PREROT samples the photon emission angles, with respect to the actual positron flight direction, within the solid angle DOM and calculates the number of photons to be emitted, NCYCL1, for the given positron. The subroutine PREROT calls the subroutines POLROT, PHDISE, CROSS, UNEXSA and the function ADJFI: POLROT is the CERN-Library routine which performs polar angle rotations; PHDISE selects the photon flight direction (TETEP, FIPIU), with respect to the actual positron flight direction, certainly accepted by the collimators, taking into account the photon production point (X0, Y0, Z0) and the positron azimuthal angles; CROSS checks if a photon flying in the Lab polar direction (TEG, FIG) crosses all the collimators until ICUT (index of the most selective collimator); ADJFI carries back an azimuthal angle inside the

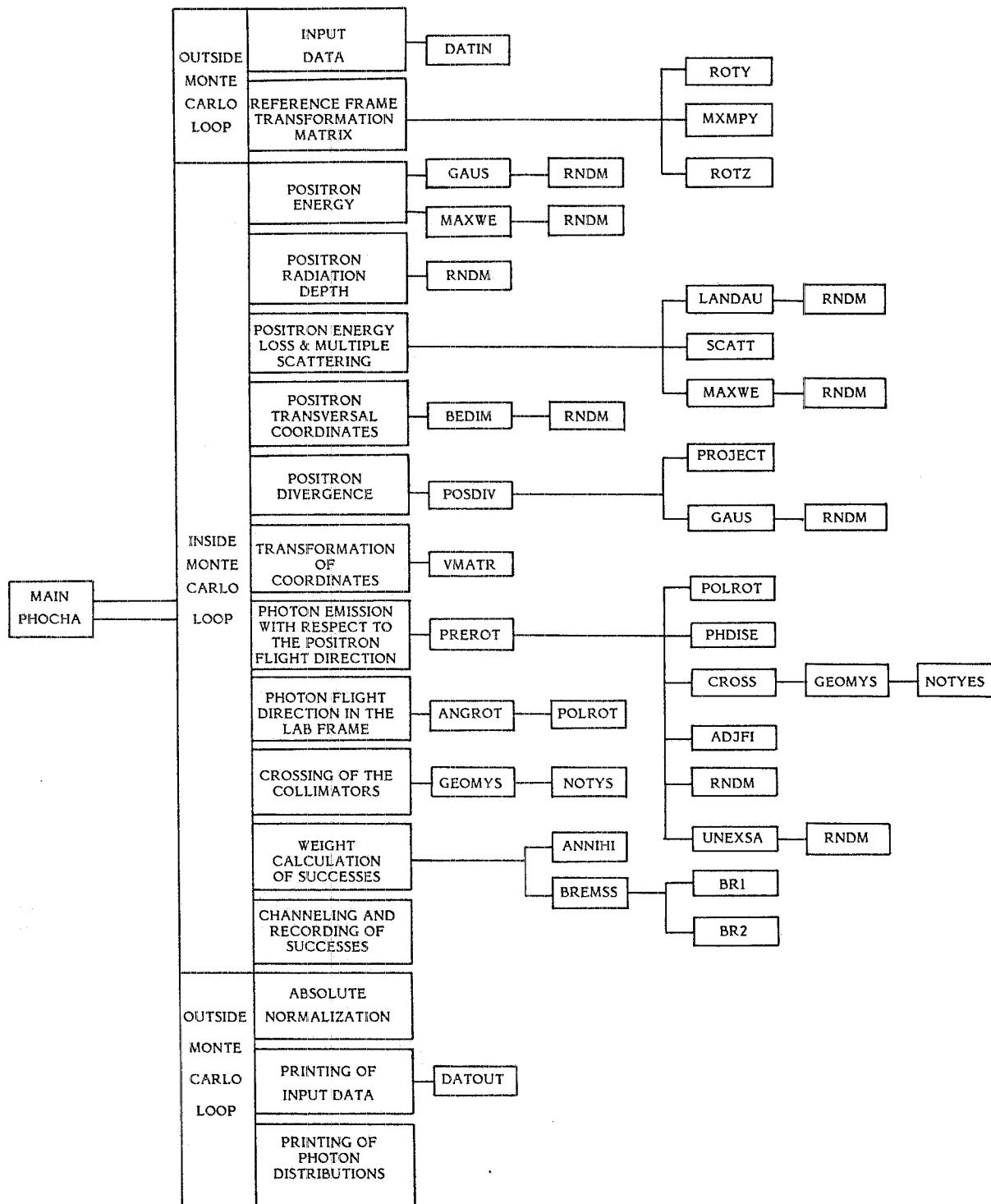


FIG. 2 - Flowchart of the PHOCHA program.

**TABLE I:** Input data in reading order.

Card	Variable	Format	Meaning
1-2-3	NOURE(I)	10I6	Array of 30 numbers for the random extractions.
4	IEMIT	3I6	Index to determine if the positron beam is point-like divergenceless (IEMIT=0, or it is a real positron beam (IEMIT=1).
	NCYCLE		Number of photons used to determine the number of photons to be emitted per unit of solid angle.
	NPOS		Number of sampled positron stories.
5	PE	5F10 5	Positron mean total energy (MeV).
	PEFWHM		F.W.H.M. of the positron energy distribution (MeV).
	APEL		Positron energy loss before entering the target (MeV).
	DAP		Mean deflection angle of the positrons with respect to the collimators axis (Rad).
	FIP		The azimuthal angle of the positron deflection direction (Rad).
6	RPBR	5F10.5	Radial positron beam radius (cm).
	RDIV		Radial positron beam divergence (Rad).
	VPBR		Vertical positron beam radius (cm).
	VDIV		Vertical positron beam divergence (Rad).
	TESØ		Mean multiple scattering angle of the positrons before entering the target (Rad).
7	RO	E10.3	Target density ( $\text{g}/\text{cm}^3$ ).
	TAT	F10.5	Target thickness (cm).
8	FZ1	6F10.5	$(\sum_i Z_i)^{4/3}$
	FZ2		$\sum_i Z_i (Z_i+1) / \sum_i A_i$
	FZ3		$(\sum_i Z_i^2)^{1/3}$
	FZ4		$Z_i / \sum_i A_i$
	FZ5		$Z_i^2 / \sum_i A_i$
	FZ6		$(\sum_i Z_i)^{1/3}$
			$(\text{g}^{-1})$
			Z and A are the atomic number and the atomic weight, respectively. The index i runs over all the elements of the target chemical formula.
9	NCOL	I2	Number of photon collimators.
10+10+NCOL	CIR(I)	4F10.5	Collimator internal radius (cm).
	CTH(I)		Collimator thickness (cm).
	CDI(I)		Distance between the incoming surface of the collimator and the incoming target surface (cm).
	DECDD(I)		Distance from the outgoing surface of the collimator to the selected photon view plane (cm).
11+NCOL	ICUT	3I2	Index of the most selective collimator.
	ICLIM		Index of the first collimator after which one wants to know the photon beam characteristics.
	ISCR		Index associated with the screening in the positron-nucleus bremsstrahlung; if ISCR > 1 the screening is taken into account.

TABLE I - Continued

Card	Variable	Format	Meaning
12+NCOL	NBSTEP	I3	Bremsstrahlung energy channel after which the channel width is changed from BSTEP1 into BSTEP2.
	BSTEP1	4F10.5	First energy step of the bremsstrahlung spectrum (MeV).
	BSTEP2		Second energy step of the bremsstrahlung spectrum (MeV).
	ASTEP		Energy step of the annihilation spectrum (MeV).
	ANE0		Starting energy of the annihilation spectrum (MeV).
13+NCOL	TESTEP	4F10.5	Angular step for the photon angular distribution (Rad).
	XYSTEP		Step of the radial and vertical distributions (cm).
	XY0		Starting point of the radial and vertical photon distributions (cm).
	FPHD		Distance from the target to the last photon view plane (cm).

limits  $0 \rightarrow 2\pi$ ; UNEXSA finally, provides the extraction of an angle  $\theta$ , with  $\sin\theta$  law, between given  $\Omega$  limits.

Thereafter the subroutine ANGROT performs all the angular rotations that give the photon flight direction ( $TETA, FI$ ) in the Lab frame. By means of a DO loop over all the collimators, the subroutine GEOMYS, using the function NOTYES decides if the photon reaches the selected view plane: in this case, the photon weights are calculated (subroutines ANNIHI, BREMSS and functions BR1, BR2) and photon channeling is made in the appropriate distributions.

In the annihilation spectrum the energy channel arising from the one to-one correspondence with the angle of the photon is increased by a quantity equal to the cross section of the event. In the bremsstrahlung spectrum, being any energy available at a given angle, all the energy channels are increased of the proper cross section values.

When a number NCYCL1 of photons has been extracted, another positron story starts: when the desired number of positron stories, NPOS, has been reached, the Monte Carlo loop stops. All the photon distributions are then absolutely normalized (constant CDIINC), and are printed with their errors, after the printout of the Monte Carlo input data (DATOUT).

For convenience in Table II are shown all the COMMON blocks, with their variables and meaning while Table III shows all the SUBROUTINES and FUNCTIONS used in the program, with their arguments and a short description of their action.

#### 4. - INPUT DATA

In order to run the program it is enough to feed the input data whose names meaning and unit of measure are given in Table I.

TABLE II COMMON Variables in alphabetical order of label.

Label	Variable	Meaning
ANGFU	COSF	
	SINF	
	COST	Trigonometric function values stored to save computation time (for internal use only).
	TANT	
	ICUT	see Table I
ANGLE	TELIM	Minimum geometric half angle defined by the collimators for a pointlike source.
	TEMAX	Minimum among the maximum geometric angles accepted by the collimators for the actual source.
	DAP	see Table I
	FIP	see Table I
COLLI	CIR(I)	see Table I
	CTH(I)	see Table I
	CDI(I)	see Table I
	DECDD(I)	see Table I
COUNT	NPOS	see Table I
	NCOL	see Table I
	ITOT	Total number of photon stories sampled
	ICLIM	see Table I
EMIT	RPBR2	RPBR2 = RPBR***2 (see Table I)
	RDIV	see Table I
	VPBR2	VPBR2 = VPBR***2 (see Table I)
	VDIV	see Table I
ENBR	NBSTEP	see Table I
	BSTEP1	see Table I
GBR*	BSTEP2	see Table I
	GAMMA	
NORM	GAMMA2	
	TEA2	for internal use only, to save computation time
	YPS	
	ZSA	
	BN	
PARAM	PHDENS	Number of photons, per unit of solid angle (held constant for all the program)
	NCYCL1	Number of photons to be extracted in the selected solid angle to give the chosen PHDENS
PARAM	RO	see Table I
	TESØ	see Table I
	APEL	see Table I
	TAT	see Table I

TABLE II - (Continued)

Label	Variable	Meaning
PDAT	PE	see Table I
	PEFWHM	see Table I
	RPBR	see Table I
	VPBR	see Table I
RANDOM	NOURE(I)	see Table I
SCREEN	ISCR	see Table I
SIGM2	SPR2	
	SPV2	Parameters used in the Subroutine BEDIM (for internal use only)
	FACTOR	
STEP	ANEØ	see Table I
	ASTEP	see Table I
	TESTEP	see Table I
	XYSTEP	see Table I
	XYØ	see Table I
STIME	DIMAS	
	RMAX	
	RMIN	Parameters determined by the first and the ICUT-th collimators, used in the subroutine PHDISE (for internal use only)
	RMAS	
	DIMIN	
ZTARG	FZ1	see Table I
	FZ2	see Table I
	FZ3	see Table I
	FZ4	see Table I
	FZ5	see Table I
	FZ6	see Table I

\*COMMON linking only the SUBROUTINES ANNIHI, BREMSS, and the FUNCTIONS BR1 and BR2

TABLE III SUBROUTINES and FUNCTIONS (in alphabetical order)

Name	Arguments	Action
ADJFI	A	Function which brings back the A angle inside the limits $0 \pm 2\pi$
ANGROT	TETE FIE TESC FISC TETD FID TETA FI	Subroutine which performs the angular rotations through which it is possible to obtain the photon direction (TETA,FI) in the Lab Frame. Its arguments are: the divergence (TETD FID), the multiple scattering angles (TESC,FISC) of the positron and the photon emission angles (TETE,FIE) with respect to the actual positron flight direction.
ANNIHI	PE DE TETE SECAN EGA	Subroutine which calculates the $2\gamma$ annihilation cross section, SECAN, for a positron of initial total energy PE, energy loss DE, photon emission angle TETE, and photon energy EGA.
BEDIM	XØ YØ	Subroutine which performs the extraction of the positron position in a plane normal to the positron mean flight direction, XØ, YØ, according to a given bidimensional Gaussian distribution.
BREMSS	PE DE TETE SECBR(I) NCH	Subroutine calculating the total bremsstrahlung cross sections: it adds the bremsstrahlung cross section on nucleus to the bremsstrahlung cross section on electrons. The differential cross sections SECBR(I) are calculated for all the NCH energy channels. PE is the total initial positron energy; DE is the positron energy loss; TETE is the photon emission angle.
BR1	E1 EG	Function which calculates the cross section for positron-nucleus bremsstrahlung, differential in the photon energy, EG, and angle. E1 is the kinetic energy of the radiating positron.
BR2	EG	Function which calculates the positron-electron bremsstrahlung cross section, differential in the photon energy, EG, and angle.
CROSS	TEG FIG XØ YØ ZØ IPAS	Subroutine which decides if a photon, flying in the direction TEG FIG with starting point XØ, YØ, ZØ, crosses all the collimators until ICUT (IPAS=-1), or not (IPAS=1).
DATIN	ICUT NCYCLE IEMIT FPHD	Subroutine which reads all input data. See Table I for meaning of all input data
DATOUT	ICUT NCYCLE IEMIT FPHD	Subroutine which prints all input data.
GAUS	AM S V	Subroutine which samples a variable V from a Normal distribution whose mean value and standard deviation are AM and S, respectively.
GEOMYS	XØ YØ ZØ M MM	Subroutine which decides if the M-th collimator is crossed, MM=1, or not, MM=0, by the photon leaving the target at the point XØ, YØ, ZØ.

TABLE III - Continued

Name	Arguments	Action
LANDAU	PE PALAND HNP2 VALOG ROTH DE	Subroutine which samples, the energy loss, DE, of a positron of incident total energy PE, which has crossed ROTH ( $\text{g}/\text{cm}^2$ ) of target thickness. PALAND, HNP2, VALOG are parameters of the calculation.
MAXWE	TES2 TESE	Subroutine which extracts a variable TESE, according to a Maxwellian distribution whose peak value is at $\text{SQRT}(\text{TES2}/2)$ .
MXMPY	CERN-LIBRARY	Subroutine which multiplies two matrices.
NOTYES	X Y R	Function which sets itself to the value 0 if $(X^2+Y^2) > R$ , otherwise NOTYES=1
PHDISE	TETEP FIPIU X $\emptyset$ Y $\emptyset$ Z $\emptyset$ FID FISC IST	Subroutine which calculates the photon flight direction (TETEP, FIPIU) certainly accepted by the collimators, when the photon production point X $\emptyset$ , Y $\emptyset$ , Z $\emptyset$ is known, and the azimuthal angles of the positron divergence (FID) and multiple scattering (FISC) are given. IST is a flag: when it is set equal to 1 it means that, from the given position, it is impossible to cross the collimators.
POLROT	CERN-LIBRARY	Subroutine which performs rotations in polar coordinates.
POSDIV	X $\emptyset$ Y $\emptyset$ TETD FID	Subroutine which extracts, according to a Normal law, the positron radial and vertical divergence, for a positron position X $\emptyset$ , Y $\emptyset$ ; then it calculates the corresponding polar angles TETD FID.
PREROT	TETD FID TESC FISC TETE FIE X $\emptyset$ Y $\emptyset$ Z $\emptyset$ IST IZO IEMIT	Subroutine which, in the first part, calculates for every positron the size and the orientation of photon emission solid angle that gives the minimum photon losses through the collimators. It calculates also the number of photons to be emitted in order to keep constant the number of photons per unit of solid angle. In the second part it extracts uniformly the photon flight direction, (TETE,FIE). inside the selected solid angle. TETD FID are the positron divergence angles, TESC FISC the positron multiple scattering angles; X $\emptyset$ , Y $\emptyset$ , Z $\emptyset$ are the coordinates of the photon production point. IZO is a partial photon counter: the first part of the subroutine is executed, only once (IZO=1). The flag IEMIT determines the calculations for ideal point like, divergenceless positron beam (IEMIT=0), or for an actual positron beam (IEMIT=1). The meaning of IST is the same as in the subroutine PHDISE.
PROJEC	A B C	Function which calculates $ Y /3$ . Y and C are the ordinate and the abscissa, of a point belonging to a canonical ellipse, with semiaxes $\text{SQRT}(A)$ and B, respectively.
RNDM	CERN-LIBRARY	Function to perform random numbers extractions.
ROTY	TETA A	Subroutine which calculates the rotation matrix A of argument TETA around the y-axis in a right-handed ref. frame.
ROTZ	FI A	Subroutine which calculates the rotation matrix A of argument FI around the z-axis in a right-handed ref. frame.
SCATT	PE T TES2	Subroutine which computes the mean square multiple angle TES2 of a positron of total energy PE, after it has crossed a thickness T( $\text{g}/\text{cm}^2$ ) of target.
UNEXSA	YMAS YMIN TETE	Subroutine which samples uniformly an angle TETE within given $\theta$ limits, with a $\sin \theta$ law. YMIN is the cosine of the $\theta$ lower edge, YMAS is the cosine of the $\theta$ upper edge.
VMATR	CERN-LIBRARY	Subroutine to perform the product of a matrix times a vector.

### 5. - TEST RUN

The included example of run shows the structure of the output and the typical information obtainable from the program. The calculation refers to the experimental conditions of the annihilation photon beam facility of the Frascati Laboratory ( 2 ).

The first output page consists of the printout of all the input data, with their unit of measure, except for the dummy arguments used for the random extractions. In this page the total number of photon stories sampled, ITOT is also printed. This page is subdivided in five parts: a) run data, b) positron beam characteristics, c) target characteristics, d) collimator characteristics, e) photon parameters.

In the next pages, the calculated photon beam characteristics, with their unit of measure, are printed; the order is the following:

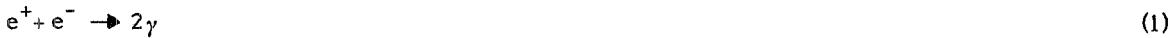
- i) total photon Lab angular distributions at the selected view planes, with relative number of successes;
- ii) total radial and vertical photon profiles at the selected view planes, with relative number of successes;
- iii) annihilation photon spectra (with errors), total number of annihilation photons and intensities at the selected view planes, with relative number of successes;
- iv) bremsstrahlung photon spectra (with errors), total number of bremsstrahlung photons and intensities at the selected view planes, with relative number of successes.

### REFERENCES

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APPENDIX: FORMULAE USED IN THE CALCULATION OF THE PHOTON SPECTRUM (2)

The predominant reactions when an intermediate energy positron ( $\gtrsim 5$  MeV) collides on a target are the following:



Reactions (2) and (3) constitute an unavoidable background source to the monochromatic photon from the two-body final state (1)

When a positron of energy  $E$  annihilates, the energy  $k$  of the photon emitted at a laboratory angle  $\theta$  is given by

$$k = mc^2 / (1 - \delta \cos \theta),$$

where  $\delta = \sqrt{(\gamma-1)/(\gamma+1)}$ , with  $\gamma = E/mc^2$  ( $mc^2 = 0.51$  MeV is the electron mass). When  $\theta \ll 1$  and  $\gamma \gg 1$ , the two photon differential cross section (reaction (1)) is given by

$$\frac{d\sigma}{d\Omega} = \frac{r_e^2}{2} \cdot \frac{1}{(1+z)^2} \left( \frac{2\gamma}{1+l} + \frac{1+l}{2\gamma} + \frac{4l}{(1+l)^2} \right),$$

where:  $r_e = e^2/mc^2 \approx 2.82 \cdot 10^{-13}$  cm is the classical radius of the electron,  $z = \gamma\theta^2/2$  and  $l = \gamma^2\theta^2$ .

For the process (3) the Schiff cross section, which is very accurate for  $\theta \ll 1$ , has been used

$$\frac{d\sigma}{d\Omega dk} = \frac{2\alpha r_e^2}{\pi} \frac{\gamma^2}{k} z^2 \left\{ \frac{16 l(1-\eta)}{(1+l)^4} - \frac{(2-\eta)^2}{(1+l)^2} + \left[ \frac{2-2\eta+\eta^2}{(1+l)^2} - \frac{4l(1-\eta)}{(1+l)^4} \right] \ln \left[ \frac{2\gamma(\eta^{-1}-1)}{(1+S^2)} \right] \right\},$$

where  $\eta = k/E$ ,  $\alpha$  is the fine structure constant and  $S$  is the screening factor:

$$S = \frac{2\gamma(\eta^{-1}-1) z^{1/3}}{111(1+l)}.$$

For the process (2) the Schiff cross section with  $Z=1$  has been used when  $\theta \ll \sqrt{2/\gamma}$ . Otherwise the following formula has been used

$$\frac{d\sigma}{d\Omega dk} = \frac{2\alpha r_e^2}{\pi} \frac{2}{k} F \left[ \frac{1}{y^4 \theta^4} + \frac{1}{4\gamma^2} \right] \left\{ 2(2-2y+y^2) \ln \left[ 2\gamma(y^{-1}-1) \right] - 3+3y-y^2 \right\},$$

where  $y=k/k_m$ ; with  $k_m=mc^2\gamma/(1+z)$  and  $F$  is a scaling function which reduces the cross section to 0 for  $k=k_m$ :

$$F = \left\{ (1-y) \exp \left[ -3(1-y)^{1/3} \right] \right\} \left\{ 1 - 0.1 \sin \left[ \cos^{-1} \left( \frac{1-z}{1+z} \right) \right] \right\}$$

We have used the following formula for the probability  $P(\theta_s)d\theta_s$  for a positron to be multiply scattered through a cone  $\theta_s$ ,  $\theta_s + d\theta_s$ :

$$P(\theta_s)d\theta_s = \frac{2\theta_s}{\langle\theta_s^2\rangle} \exp\left(-\frac{\theta_s^2}{\langle\theta_s^2\rangle}\right) d\theta_s,$$

where:

$$\langle\theta_s^2\rangle = \chi^2(-0.17 + 1.13 \ln \Omega)$$

with

$$\chi^2 = 0.157 \frac{Z(Z+1)}{A} \frac{t}{E^2} \quad \Omega = \chi^2 / \left[ 0.595 \times 10^{-4} \frac{Z^{2/3}}{E^2} \right]$$

where  $E$  is the positron energy in MeV,  $t$  and  $A$  are the thickness (in g/cm<sup>2</sup>) and the atomic weight of the target.

For energy collision losses it has been used the theory of Landau, with corrections to take into account the density effect for relativistic positrons. If we call  $F(E, t, \Delta) d\Delta$  the probability that a positron of incident energy  $E$  has suffered, after traversing a thickness  $t$  of matter, an energy loss between  $\Delta$  and  $\Delta+d\Delta$ , we can write, apart from a normalization factor:

$$F(E, t, \Delta) d\Delta = \exp\left[-\alpha^+(\lambda + \ln \alpha^+)\right] \varphi(\lambda) d\lambda$$

where

$$\alpha^+ = \left[ 2 - (\gamma + 1)^{-2} \right], \quad \lambda = \frac{\Delta}{\zeta(E-mc^2)} - \left[ \ln \frac{\zeta(E-mc^2)mc^2}{(h\nu_p)^2} + 1.114 \right]$$

$$\zeta = \frac{2\pi r_e^2 mc^2 N_o Z}{(E-mc^2) A} t$$

with  $N_o$  = Avogadro number,

$$\nu_p = \left( \frac{N_o \rho Z e^2}{\pi m A} \right)^{1/2} \text{ plasma frequency of medium of density } \rho.$$

$\varphi(\lambda)$  is the Landau universal function.

## TEST RUN OUTPUT

- 15 -

RUN DATA									
TEMIT =	1	NCYCLE=	25	NPOS	=5000	ITOT	=	157529	
POSITRON BEAM CHARACTERISTICS									
PE =	140.51000 MEV	PEFWHM=	3.00000 MEV	APEL =	*15950 MEV	BAP =	.01300 RAD	FIP =	*4.71239 RAD
RBR =	1.60000 CM	RDIV =	.00150 RAD	VBR =	1.30000 CM	VDIV =	*.00150 RAD	TESO =	*.00307 RAD
TARGET CHARACTERISTICS									
RO=	*710E-01 G/CM***3	TAT =	10.50000 CM						
FZ1 =	1.00000	FZ2 =	1.98447	FZ3 =	1.00000	FZ4 =	*.99224	FZ5 =	*.99224
									FZ6 = 1.00000
COLLIMATOR CHARACTERISTICS (CM)									
ICUT	NCOL	CIR	CTH	CDI	DECDI				
1		*45000	9.90000	106.25000	0.00000				
2		*50000	9.90000	116.15000	0.00000				
3		*52500	9.90000	126.05000	0.00000				
4		*57500	9.90000	135.95000	0.00000				
5		*61000	9.90000	145.85000	0.00000				
6	6	*69000	12.00000	160.75000	0.00000				
7		2.00000	50.00000	213.15000	50.50000				
8		1.25000	10.00000	313.65000	0.00000				
PHOTON PARAMETERS									
ICLIM =	7	ISCR = 0	TELIM = *00418 RAD	TEMAX = *01453 RAD					
NSTEP=	47	BSTEP1= 3.00000 MEV	BSTEP2= *700000 MEV	ASTEP = *50000 MEV	ANE0 = 120.00000 MEV				
TESTEP=	*00050 RAD	XSTEP= *20000 CM	XY0 = -5.00000 CM	FPHD = 420.25000 CM					

PHOTON ANGULAR DISTRIBUTIONS WITH RESPECT TO THE COLLIMATOR AXIS AT VARIOUS DISTANCES FROM THE TARGET

## PHOTON RADIAL(X) AND VERTICAL(Y) DISTRIBUTIONS AT VARIOUS DISTANCES FROM THE TARGET

X OR Y (CM)	PHOTX/POS	PHOTY/POS	PHOTX/ POS	PHOTY/ POS	PHOTX/POS	PHOTY/POS
-4.9000	0.	0.	0.	0.	0.	0.
-4.7000	0.	0.	0.	0.	0.	0.
-4.5000	0.	0.	0.	0.	0.	0.
-4.3000	0.	0.	0.	0.	0.	0.
-4.1000	0.	0.	0.	0.	0.	0.
-3.9000	0.	0.	0.	0.	0.	0.
-3.7000	0.	0.	0.	0.	0.	0.
-3.5000	0.	0.	0.	0.	0.	0.
-3.3000	0.	0.	0.	0.	0.	0.
-3.1000	0.	0.	0.	0.	0.	0.
-2.9000	0.	0.	0.	0.	0.	0.
-2.7000	0.	0.	0.	0.	0.	0.
-2.5000	*11069E-07	*11377E-05	0.	0.	0.	0.
-2.3000	*44655E-06	*48551E-05	0.	0.	0.	0.
-2.1000	*33658E-05	*27354E-04	0.	0.	0.	0.
-1.9000	*49971E-05	*50445E-04	0.	0.	*58524E-06	*34659E-05
-1.7000	*16877E-04	*10138E-03	0.	0.	*14308E-04	*44540E-04
-1.5000	*34579E-04	*16947E-03	0.	0.	*35860E-04	*10368E-03
-1.3000	*63966E-04	*22224E-03	*48005E-05	*10598E-04	*52864E-04	*13184E-03
-1.1000	*10173E-03	*26881E-03	*39546E-04	*12981E-03	*71395E-04	*13055E-03
-90000	*13893E-03	*26778E-03	*8351E-04	*18174E-03	*76246E-04	*13335E-03
-70000	*16889E-03	*25515E-03	*10244E-03	*18544E-03	*91450E-04	*12538E-03
-50000	*19891E-03	*23155E-03	*13070E-03	*17582E-03	*10522E-03	*12212E-03
-30000	*22841E-03	*20430E-03	*14981E-03	*16985E-03	*11733E-03	*12367E-03
-10000	*26028E-03	*19138E-03	*17446E-03	*16147E-03	*11853E-03	*10927E-03
.10000	*25930E-03	*15280E-03	*16801E-03	*12796E-03	*12498E-03	*93361E-04
.30000	*25182E-03	*11572E-03	*1688E-03	*95410E-04	*11564E-03	*75375E-04
.50000	*21151E-03	*84839E-04	*13998E-03	*65701E-04	*10595E-03	*59302E-04
.70000	*19021E-03	*67978E-04	*11066E-03	*52247E-04	*98689E-04	*46983E-04
.90000	*15724E-03	*46317E-04	*89764E-04	*29217E-04	*87143E-04	*38885E-04
1.1000	*11300E-03	*24661E-04	*46442E-04	*13971E-04	*73048E-04	*24713E-04
1.3000	*64937E-04	*15283E-04	*29352E-05	*95825E-06	*59554E-04	*19871E-04
1.5000	*33082E-04	*64427E-05	0.	0.	*36605E-04	*10157E-04
1.7000	*13486E-04	*50892E-05	0.	0.	*14199E-04	*34997E-05
1.9000	*43742E-05	*10477E-05	0.	0.	*58644E-06	*18012E-06
2.1000	*94062E-06	*22477E-06	0.	0.	0.	0.
2.3000	*10500E-06	*57807E-07	0.	0.	0.	0.
2.5000	*53658E-08	*72439E-09	0.	0.	0.	0.
2.7000	0.	0.	0.	0.	0.	0.
2.9000	0.	0.	0.	0.	0.	0.
3.1000	0.	0.	0.	0.	0.	0.
3.3000	0.	0.	0.	0.	0.	0.
3.5000	0.	0.	0.	0.	0.	0.
3.7000	0.	0.	0.	0.	0.	0.
3.9000	0.	0.	0.	0.	0.	0.
4.1000	0.	0.	0.	0.	0.	0.
4.3000	0.	0.	0.	0.	0.	0.
4.5000	0.	0.	0.	0.	0.	0.
4.7000	0.	0.	0.	0.	0.	0.
4.9000	0.	0.	0.	0.	0.	0.

AT DISTANCES  
SUCCESSES

313.65  
79938

323.65  
48139

420.25  
48139

IN CM

PHOTON ANNIHILATION SPECTRA AND THEIR ERRORS AT VARIOUS DISTANCES FROM THE TARGET IN PHOTOFOS, MEV

PHOTON BREMSSTRAHLUNG SPECTRA WITH THEIR ERRORS AT		VARIOUS DISTANCES FROM THE TARGET				IN PHOT/POS/MEV	
PHEM (MEV)	PHOT	PHOTER	PHOT	PHOTER	PHOT	PHOT	PHOTER
1.5000	*38597E-03	*44649E-05	*214449E-03	*32464E-05	*214449E-03	*32464E-05	*32464E-05
4.5000	*11211E-03	*12966E-05	*62280E-04	*94218E-06	*62280E-04	*94218E-06	*94218E-06
7.5000	*6197E-04	*71638E-06	*34422E-04	*52069E-06	*34422E-04	*52069E-06	*52069E-06
10.5000	*41515E-04	*47987E-06	*23052E-04	*34874E-06	*23052E-04	*34874E-06	*34874E-06
13.5000	*30557E-04	*35322E-06	*16936E-04	*25666E-06	*16936E-04	*25666E-06	*25666E-06
16.5000	*23794E-04	*27504E-06	*13204E-04	*19983E-06	*13204E-04	*19983E-06	*19983E-06
19.5000	*19239E-04	*22235E-06	*10670E-04	*16153E-06	*10670E-04	*16153E-06	*16153E-06
22.5000	*15965E-04	*18856E-06	*13412E-04	*13412E-06	*13412E-04	*13412E-06	*13412E-06
25.5000	*13524E-04	*15649E-06	*75004E-05	*11363E-06	*75004E-05	*11363E-06	*11363E-06
28.5000	*11635E-04	*13467E-06	*64529E-05	*97806E-07	*64529E-05	*97806E-07	*97806E-07
31.5000	*10144E-04	*11743E-06	*82824E-05	*85260E-07	*85260E-05	*85260E-07	*85260E-07
34.5000	*89335E-05	*10345E-06	*49501E-05	*75104E-07	*49501E-05	*75104E-07	*75104E-07
37.5000	*79371E-05	*91962E-07	*39346E-05	*66746E-07	*43966E-05	*66746E-07	*66746E-07
40.5000	*71056E-05	*82357E-07	*39340E-05	*59765E-07	*39340E-05	*59765E-07	*59765E-07
43.5000	*64012E-05	*74238E-07	*75430E-05	*53865E-07	*35435E-05	*53865E-07	*53865E-07
46.5000	*58001E-05	*67304E-07	*32089E-05	*48823E-07	*32088E-05	*48823E-07	*48823E-07
49.5000	*52818E-05	*61328E-07	*29210E-05	*44792E-07	*29210E-05	*44792E-07	*44792E-07
52.5000	*48316E-05	*56138E-07	*27072E-05	*40702E-07	*26707E-05	*40702E-07	*40702E-07
55.5000	*44377E-05	*51599E-07	*24518E-05	*37407E-07	*24518E-05	*37407E-07	*37407E-07
58.5000	*40905E-05	*47650E-07	*22590E-05	*34505E-07	*22590E-05	*34505E-07	*34505E-07
61.5000	*37838E-05	*40970E-07	*32084E-05	*31934E-07	*32084E-05	*31934E-07	*31934E-07
64.5000	*35105E-05	*40924E-07	*19364E-05	*29648E-07	*19364E-05	*29648E-07	*29648E-07
67.5000	*32665E-05	*38117E-07	*18006E-05	*27606E-07	*18006E-05	*27606E-07	*27606E-07
70.5000	*30465E-05	*35595E-07	*16786E-05	*25773E-07	*16786E-05	*25773E-07	*25773E-07
73.5000	*28488E-05	*33323E-07	*15688E-05	*24122E-07	*15688E-05	*24122E-07	*24122E-07
76.5000	*26697E-05	*31270E-07	*14689E-05	*22628E-07	*14689E-05	*22628E-07	*22628E-07
79.5000	*25056E-05	*29405E-07	*12783E-05	*21272E-07	*13783E-05	*21272E-07	*21272E-07
82.5000	*23581E-05	*27704E-07	*12956E-05	*20336E-07	*12956E-05	*20336E-07	*20336E-07
85.5000	*22214E-05	*26149E-07	*12199E-05	*18905E-07	*12199E-05	*18905E-07	*18905E-07
88.5000	*20964E-05	*24720E-07	*11501E-05	*17866E-07	*11501E-05	*17866E-07	*17866E-07
91.5000	*19807E-05	*23402E-07	*10857E-05	*16908E-07	*10857E-05	*16908E-07	*16908E-07
94.5000	*18735E-05	*22181E-07	*10259E-05	*16019E-07	*10259E-05	*16019E-07	*16019E-07
97.5000	*17728E-05	*21045E-07	*97004E-06	*15192E-07	*97004E-06	*15192E-07	*15192E-07
100.50	*16784E-05	*19977E-07	*91751E-06	*14146E-07	*91751E-06	*14146E-07	*14146E-07
103.50	*15892E-05	*18971E-07	*86793E-06	*13668E-07	*86793E-06	*13668E-07	*13668E-07
106.50	*15040E-05	*18013E-07	*82058E-06	*12988E-07	*82058E-06	*12988E-07	*12988E-07
109.50	*14219E-05	*17093E-07	*77494E-06	*12318E-07	*77494E-06	*12318E-07	*12318E-07
112.50	*13418E-05	*16197E-07	*73041E-06	*11665E-07	*73041E-06	*11665E-07	*11665E-07
115.50	*12624E-05	*15312E-07	*68630E-06	*11022E-07	*68630E-06	*11022E-07	*11022E-07
118.50	*11821E-05	*14420E-07	*64174E-06	*10372E-07	*64174E-06	*10372E-07	*10372E-07
121.50	*10982E-05	*13498E-07	*50721E-06	*15794E-06	*34274E-06	*15794E-06	*34274E-06
124.50	*10097E-05	*12511E-07	*54605E-06	*89794E-08	*54605E-06	*89794E-08	*89794E-08
127.50	*90915E-06	*11403E-07	*49022E-06	*81683E-08	*49022E-06	*81683E-08	*81683E-08
130.50	*81632E-06	*10060E-07	*68830E-06	*11665E-07	*68830E-06	*11665E-07	*11665E-07
133.50	*71369E-06	*82084E-08	*32742E-06	*58058E-08	*32742E-06	*58058E-08	*58058E-08
136.50	*50721E-08	*15794E-06	*34274E-06	*15794E-06	*34274E-06	*15794E-06	*34274E-06
138.35	*10248E-06	*26047E-08	*48572E-07	*16760E-08	*48572E-07	*16760E-08	*16760E-08
139.05	*52812E-07	*17741E-08	*23690E-07	*10677E-08	*23690E-07	*10677E-08	*10677E-08
140.45	*21687E-07	*10759E-08	*90671E-08	*58787E-09	*90671E-08	*58787E-09	*58787E-09
PHOT/FOS	.24898E-02 +/-	.14313E-04	.13815E-02 +/-	.10406E-04	.13815E-02 +/-	.10406E-04	.10406E-04
MEV/FOS	.33794E-01		*18636E-01		*18636E-01		*18636E-01
AT DISTANCES	313.65		323.65				420.25
SUCCESSES	79938		48139				48139
IN CM							