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P. Chiappetta and M. Greco :  
 $p_\perp$  EFFECTS FOR DRELL-YAN PAIRS IN QCD

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## **$p_\perp$ EFFECTS FOR DRELL-YAN PAIRS IN QCD**

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The transverse momentum properties of Drell-Yan pairs produced in hadronic collisions are analyzed in QCD by combining the soft gluon description at low  $p_\perp$  with hard parton effects at high  $p_\perp$ . Soft gluon effects, implemented by exact kinematics considerations, play a very important role in the analysis and describe the main features of the data. Recently calculated  $\alpha_s^2$  corrections to the annihilation terms improve the agreement with data at large  $p_\perp$  for  $\pi^- N$  collisions. Predictions are also made for  $p\bar{p}$  annihilation at collider energies.

### **1. Introduction**

The process of lepton pair production in hadronic collisions has been the subject of considerable interest and development since the pioneering experiment of Christenson et al. [1] and the subsequent suggestion by Drell and Yan [2] to explain the dilepton continuum in terms of annihilation of a quark-antiquark pair into the virtual photon. Large amounts of data [3, 4] have definitively established the validity of the quark parton mechanism and, more recently, confirmed some specific predictions of QCD, as the absolute normalization of the cross section and the broadening with energy of the  $p_\perp$  distributions [4].

The first issue, namely the presence of a large correction [5] to the basic Drell-Yan cross section – the so-called  $K$  factor – which is approximately constant in the kinematical region explored so far, has been impressively confirmed by the data in all measured channels. The close agreement of experiments with the first-order QCD predictions can be explained by the observation that the largest contributions to this effect can be resummed [6] and the exponential formula [7] slightly differs from the first-order result in the kinematical regions reasonably accessible to experiments. This assumes, however, that the residual terms in the expansion are well behaved also at higher orders, which remains a conjecture to be proved.

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The observation [3] of a linear increase with  $s$  of  $\langle p_{\perp}^2 \rangle$  for fixed  $\tau$ , is also in fairly good agreement with QCD predictions. More quantitatively however, the full transverse momentum properties of the lepton pairs produced in pN and  $\pi$ N collisions have not yet found a completely satisfactory description, although much progress has been recently achieved on the theoretical side.

First-order results [8] have been shown [9] to be inadequate in describing the data at large  $p_{\perp} \sim O(Q)$ , where  $Q$  is the dilepton invariant mass, suggesting sizeable corrections to the leading-order calculations. An improvement in this direction comes from the study by Ellis et al. [10] of the next to leading terms of order  $\alpha_s^2$  at high transverse momenta, who have indeed found large and positive corrections – a new  $K(p_{\perp})$  factor – which brings theory into better agreement with data at large  $p_{\perp}$ . For a quantitative agreement in  $\pi$ N collisions one still needs, however, quite a large value of the intrinsic transverse momentum  $\langle p_{\perp}^2 \rangle_{\text{intr}} \sim 1 \text{ GeV}^2$ , a result also required by first-order analyses. Similar corrections seem to be required also in pN collisions [9], although the gluon distributions which enter in the Compton terms are not so well determined as the quark densities.

On the other hand, most of the cross section for dilepton production is at low  $p_{\perp}$ . In this region ( $\Lambda^2 \ll p_{\perp}^2 \ll Q^2$ ) at each order of perturbation theory the dominant corrections to the naive Drell-Yan process are of the form  $\alpha_s^n \ln^{2n}(Q^2/p_{\perp}^2)$  arising from the emission of  $n$  soft and collinear gluons. These terms have to be summed to all orders and important progress has been achieved since the work of Dokshitzer et al. [11], concerning, in particular, the study of the subleading corrections [12, 13].

In a recent paper [14] we have shown the relevant role played by the appropriate use of the exact kinematics in evaluating the emission of the soft gluons, consistent with what has been found elsewhere [6]. Thus the absolute theoretical soft distributions for lepton pairs produced both in  $\pi$ N and NN collisions have been shown to be in very good agreement with data, with a more reasonable value of the intrinsic  $p_{\perp}$ , i.e.  $\langle p_{\perp}^2 \rangle_{\text{intr}} \sim 0.4 \text{ GeV}^2$ .

The aim of the present paper is to implement the treatment of soft effects discussed above with the  $\alpha_s^2$  results of Ellis et al. [10], and compare the resulting expressions with data. Furthermore, we extend our soft formalism to dilepton rapidities  $y \neq 0$  and give new predictions for  $\bar{p}N$  and  $p\bar{p}$  reactions at collider energies.

The main features of our results are in agreement with our first analysis [14]. The gross properties of all data are described by soft effects, the hard component playing a role in a region where the actual data are not very precise. In particular, the hard annihilation terms of order  $\alpha_s$  and  $\alpha_s^2$  have important effects in  $\pi^- N$  and  $p\bar{p}$  collisions, while at present energies and in the  $p_{\perp}$  range explored so far they seem to be negligible for pN collisions. Here  $\alpha_s^2$  corrections to Compton terms can play a role at  $p_{\perp} \gtrsim 3 \text{ GeV}$ . From our analysis it follows that both soft and hard effects are necessary to have a rather accurate description of all existing experimental results. We confirm a low value of the global  $\langle p_{\perp}^2 \rangle_{\text{intr}} \sim 0.4 \text{ GeV}^2$ .

The paper is organized as follows. In sect. 2 we discuss the soft gluon contributions and give the relevant formulae. In sect. 3 we recall various results which are needed for our analysis of high  $p_\perp$ . Our results are then presented and discussed in sect. 4. Sect. 5 contains our final conclusions.

## 2. Soft gluon effects

The kinematical regime of interest is  $\Lambda^2 \ll p_\perp^2 \ll Q^2$ , where two mass scales are involved in the theory. There has been considerable interest in extending the perturbative analysis into such a region where strict perturbation theory breaks down due to the appearance of dominant corrections of the form  $\alpha_s^n \ln^{2n}(Q^2/p_\perp^2)$ , which have to be summed to all orders [11]. The structure of non-leading corrections has been investigated [12, 13] in great detail and is the subject of current research. By restricting the non-leading logarithmic contributions to those associated with an exact treatment of transverse momentum one obtains [12]

$$\frac{dP}{dp_\perp^2} = \frac{1}{2} \int_0^\infty b db J_0(bp_\perp) \exp[\Delta(b, q_{\perp \max})], \quad (1)$$

with

$$\Delta(b, q_{\perp \max}) = \frac{16}{3\pi} \int_0^{q_{\perp \max}} \frac{dq_i}{q_\perp} \ln\left(\frac{Q}{q_\perp}\right) \alpha(q_\perp) [J_0(bq_\perp) - 1], \quad (2)$$

and  $q_{\perp \max} \sim \frac{1}{2}Q$  is the phase-space limit for the emitted gluons.

Some corrections to eq. (1) have been calculated by Collins and Soper [13] in their systematic program for the subleading corrections to the DDT formula [11]. Because the full analysis is not completed yet we will restrict ourselves to eq. (1). It is worth mentioning that this resummed formula has been found to be in good agreement with the transverse momentum distributions of  $e^+e^-$  jets at PETRA energies, measured by the PLUTO collaboration [15].

In a previous letter [14] we have suggested that an important improvement of eqs. (1), (2), which brings the theoretical predictions in good agreement with data, is obtained by using exact kinematics considerations in the definition of  $q_{\perp \max}$  in eq. (2). This is in close analogy with what has been found in lepton production [6] and for the  $K$  factor [7] in Drell-Yan, as the dominant corrections to leading logarithmic analyses. In these cases, however, there is only one large mass scale and one can use more rigorous arguments to resum these next-to-leading corrections.

To be more precise, let us write the full differential cross section, in the leading soft approximation, in the factorized form (for a detailed derivation, see ref. [16])

$$\frac{d\sigma}{dQ dy dp_\perp^2} = \frac{d\sigma}{dQ dy} \frac{dP}{dp_\perp^2}, \quad (3)$$

with

$$\frac{d\sigma}{dQ dy} = \frac{8\pi\alpha^2}{9Qs} K \sum_i e_i^2 \left[ q_i^{(1)}(\sqrt{\tau} e^y) \bar{q}_i^{(2)}(\sqrt{\tau} e^{-y}) + (1 \leftrightarrow 2) \right], \quad (4)$$

where  $\sqrt{s}$  is the hadronic invariant energy,  $\tau = Q^2/s$ ,  $K$  is the  $K$  factor and  $dP/dp_\perp^2$  is given by eq. (1). So far we have neglected the effect of an intrinsic  $p_\perp$  in the parton distributions. We will come back to this point later.

From simple kinematical considerations, for fixed dilepton rapidity  $y$  and invariant energy  $\sqrt{\hat{s}} = \sqrt{x_1 x_2 s}$  of the subprocess  $q\bar{q} \rightarrow \gamma(Q^2)g$ , it follows that the upper limit of the transverse gluon momentum is

$$q_{\perp \max} = \frac{Q(1-z)}{2\sqrt{z}} \frac{1}{\sqrt{1+z \sinh^2 y}}, \quad (5)$$

with  $z \equiv Q^2/\hat{s} = \tau/x_1 x_2$ . Then a phenomenological improvement of eqs. (1)–(3) follows from the substitution of (5) in eq. (2), where now the effective invariant energy in (5) is weighted with the parton densities, namely

$$\begin{aligned} \langle \sqrt{z} \rangle &= \frac{\int \frac{dx_1}{x_1} \frac{dx_2}{x_2} \sqrt{z} \delta(x_1 - x_2 - x_F) \theta(x_1 x_2 - \tau) q(x_1) \bar{q}(x_2)}{\int \frac{dx_1}{x_1} \frac{dx_2}{x_2} \delta(x_1 - x_2 - x_F) \theta(x_1 x_2 - \tau) q(x_1) \bar{q}(x_2)} \\ &= \frac{\int_{x_{1\min}}^1 \frac{dx_1}{[x_1(x_1 - x_F)]^{3/2}} \sqrt{\tau} q(x_1) \bar{q}(x_1 - x_F)}{\int_{x_{1\min}}^1 \frac{dx_1}{x_1(x_1 - x_F)} q(x_1) \bar{q}(x_1 - x_F)}, \end{aligned} \quad (6)$$

with  $x_{1\min} = \frac{1}{2} \left( x_F + \sqrt{x_F^2 + 4\tau} \right)$  and  $x_F \simeq 2\sqrt{\tau} \sinh y$ . At  $y = 0$  eq. (6) reduces to the result reported in ref. [14].

The first moment of distribution (1) is known [12]:

$$\langle p_\perp^2 \rangle_{\text{soft}} = \frac{4}{3\pi} \int_0^{q_{\perp \max}^2} dq_\perp^2 \ln \left( \frac{Q^2}{q_\perp^2} \right) \alpha(q_\perp), \quad (7)$$

which leads to  $\langle p_\perp^2 \rangle_{\text{soft}} \propto s F(\alpha(Q^2), \tau)$ . In comparing with data at present energies one has to include an intrinsic  $p_\perp$  parton transverse momentum. This can be simply

achieved by modifying eq. (1) to

$$\frac{dP'}{dp_\perp^2} = \frac{1}{2N} \int_0^\infty b db J_0(bp_\perp) e^{-b^2/4A} \exp[\Delta(b, q_{\perp \max})], \quad (8)$$

corresponding to an average value  $\langle p_\perp^2 \rangle_{\text{intr}} = 1/A$ . The factor  $N$  properly normalizes the new distribution, namely  $\int (dP'/dp_\perp^2) dp_\perp^2 = 1$ . This completes the discussion of soft gluon effects.

### 3. Hard gluon effects

In this section we will discuss the contributions coming from hard subprocesses – gluon bremsstrahlung in  $q\bar{q}$  annihilation and Compton scattering – which are relevant at high  $p_\perp$ . The full cross sections to first order in  $\alpha_s$  have been calculated earlier by many authors [8] and we report them here for the reader's convenience.

The pair annihilation terms give [17]

$$\begin{aligned} \frac{d\sigma^A}{dQ^2 dy dp_\perp^2} &= \left( \frac{4\pi\alpha^2}{9Q^4} \right) \left( \frac{2\alpha_s}{3\pi} \right) \frac{\tau}{p_\perp^2} \int_{x_{1\min}}^1 dx_1 \sum_i [q_i^{(1)}(x_1) \bar{q}_i^{(2)}(x_2) + (1 \leftrightarrow 2)] \\ &\times \left\{ \frac{1 + \tau^2/x_1^2 x_2^2 - x_T^2/2x_1 x_2}{x_1 - \frac{1}{2}\bar{x}_T e^y} \right\}, \end{aligned} \quad (9)$$

where

$$\begin{aligned} x_T^2 &= 4p_\perp^2/s, \quad \bar{x}_T^2 = x_T^2 + 4\tau, \\ x_2 &\equiv \frac{\frac{1}{2}\bar{x}_T e^{-y} x_1 - \tau}{x_1 - \frac{1}{2}\bar{x}_T e^y}, \quad x_{1\min} = \frac{\frac{1}{2}\bar{x}_T e^y - \tau}{1 - \frac{1}{2}\bar{x}_T e^{-y}}. \end{aligned} \quad (10)$$

These  $O(\alpha_s)$  annihilation terms have been shown [9] to be in disagreement with experimental data at large  $p_\perp$  for  $\pi N$  collisions in spite of the introduction of an intrinsic transverse momentum of the order of 1 GeV. Similarly the Compton terms lead to [17]

$$\begin{aligned} \frac{d\sigma^C}{dQ^2 dy dp_\perp^2} &= \left( \frac{4\pi\alpha^2}{9Q^4} \right) \left( \frac{\alpha_s}{2\pi} \right) \frac{\tau^2}{Q^2} \int_{x_{1\min}}^1 \frac{dx_1}{x_1 - \frac{1}{2}\bar{x}_T e^y} \frac{1}{x_1^2 x_2^2} \\ &\times \sum_i e_i^2 \left\{ q_i^{(1)}(x_1) G^{(2)}(x_2) \frac{(x_1 x_2 - \tau)^2 + \frac{1}{4}(x_1 x_2 + \tau + V)^2}{(x_1 x_2 - \tau + V)} \right. \\ &\left. + (1 \leftrightarrow 2, V \leftrightarrow -V) \right\}, \end{aligned} \quad (11)$$

where  $V = x_1 x_2 + \tau + x_1 \bar{x}_T e^{-y}$ ,  $\bar{x}_T$ ,  $x_2$  and  $x_{1\min}$  are defined in eqs. (10) and  $G^{(1,2)}(x)$  are the gluon densities of the incident hadrons 1 and 2.

Very recently a detailed study of the next-to-leading corrections of order  $\alpha_s^2$  at large transverse momenta has been carried out by Ellis et al. [10] for the annihilation terms in  $\pi N$  or  $\bar{p}N$  collisions. Their results can be simply expressed in terms of a correction factor  $K(p_\perp)$  to the first-order calculation, defined as

$$K(p_\perp) = \left[ \frac{1}{p_\perp} \frac{d\sigma^A}{dQ dp_\perp} \right]_{O(\alpha_s) + O(\alpha_s^2)} \Bigg/ \left[ \frac{1}{p_\perp} \frac{d\sigma^A}{dQ dp_\perp} \right]_{O(\alpha_s)}. \quad (12)$$

This correction factor turns out to be large and positive, of the order of two, and decreases as  $p_\perp$  increases. Therefore, the tail of the  $p_\perp$  distribution at high  $p_\perp$ , given by eq. (9), gets enhanced by this factor  $K(p_\perp)$ , bringing theory into better agreement with data. In the following we will assume the result (12) for fixed  $y$ .

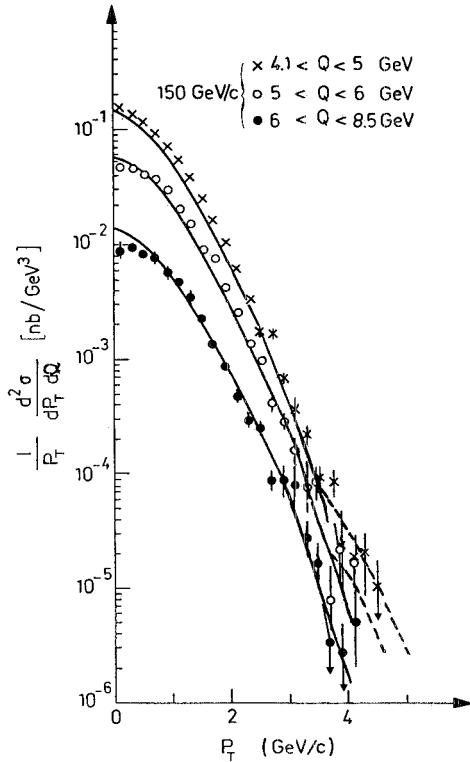


Fig. 1. The differential cross section  $(1/p_\perp) d\sigma/dp_\perp dQ$  for  $\pi^- N$  collisions at  $p_{\text{lab}} = 150$  GeV/c and  $y = 0$ . The data are from ref. [18]. The full line represents the soft contribution, eqs. (3)–(8), and the dashed line the annihilation hard term, eqs. (9)–(12).

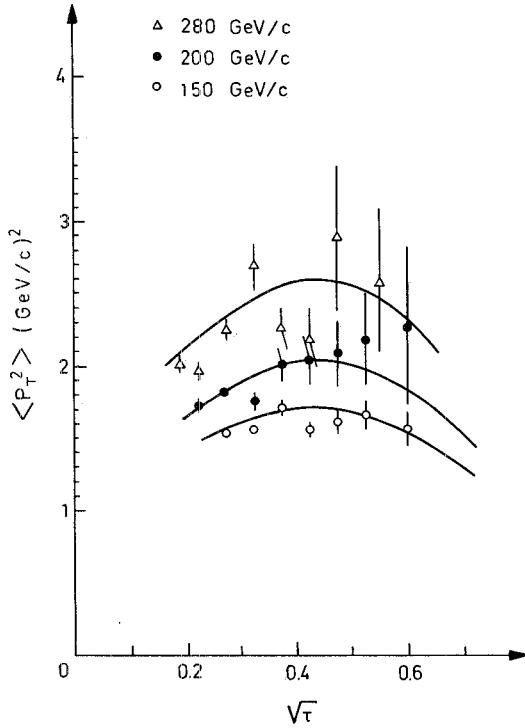


Fig. 2. The average transverse momentum squared  $\langle p_{\perp}^2 \rangle$  versus  $\sqrt{\tau}$  for  $\pi^- N$  collisions at  $p_{\text{lab}} = 150, 200$  and  $280 \text{ GeV}/c$  and  $y = 0$ . The data are from ref. [18].

So far there are no similar  $\alpha_s^2$  calculations for the Compton terms. In spite of the uncertainties in the gluon distributions, the first-order Compton contribution is important for pN collisions but is not enough to explain the data at large  $p_{\perp}$ , suggesting, therefore, the possible relevance of next-order corrections.

In comparing the experimental data with the hard distribution relative to the annihilation and Compton terms, we will show the theoretical distributions at large  $p_{\perp}$ , say  $p_{\perp} \gtrsim 2-3 \text{ GeV}$ , without considering an intrinsic transverse momentum effect. In this region, this effect should be negligible due to the rather low values of  $\langle p_{\perp}^2 \rangle_{\text{intr}}$  obtained from our analysis. Similarly, to calculate the hard component of  $\langle p_{\perp}^2 \rangle$  we will directly evaluate it from the distributions corresponding to eqs. (9), (11), (12) at high  $p_{\perp}$ .

We therefore write for the full  $\langle p_{\perp}^2 \rangle$  of the lepton pair

$$\langle p_{\perp}^2 \rangle = \langle p_{\perp}^2 \rangle_{\text{soft}} + \langle p_{\perp}^2 \rangle_{\text{intr}} + \langle p_{\perp}^2 \rangle_{\text{hard}}. \quad (13)$$

This expression will be used for comparison with the data.

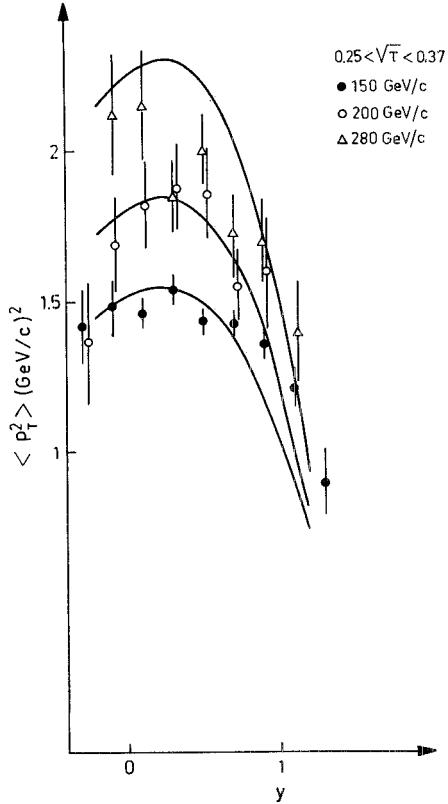


Fig. 3 The average transverse momentum squared  $\langle p_{\perp}^2 \rangle$  versus the rapidity  $y$  integrated over  $0.25 < \sqrt{s} < 0.37$  for  $\pi^- N$  collisions. The data are from ref. [22].

#### 4. Discussion and comparison with data

In the previous sections we have given all the formulae necessary for a detailed comparison with experimental data. We still have to specify the parton densities. We have used the NA3 [18] parametrization for the pion and the proton structure functions:

$$\begin{aligned}
 xV^\pi(x) &= Ax^{0.45}(1-x)^{1.04}, \\
 xS^\pi(x) &= 0.12(1-x)^{5.4}, \\
 xu^p(x) &= Bx^{0.52}(1-x)^{3.31}, \\
 xd^p(x) &= Cx^{0.52}(1-x)^{4.31}, \\
 xS^p(x) &= 0.37(1-x)^{9.4},
 \end{aligned} \tag{14}$$

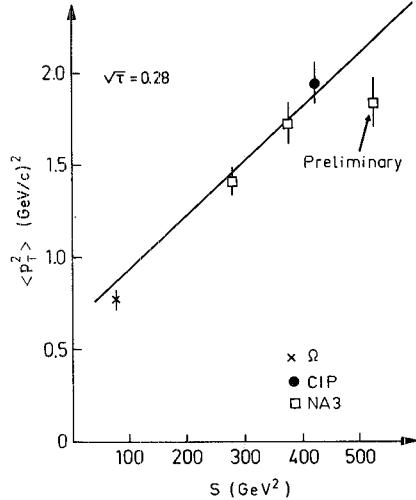


Fig. 4. The  $s$  dependence of the average transverse momentum squared  $\langle p_{\perp}^2 \rangle$ .

with  $A$ ,  $B$  and  $C$  determined from the normalization conditions  $\int V^{\pi}(x) dx = 1$ ,  $\int u^p(x) dx = 2$  and  $\int d^p(x) dx = 1$ . In the case of a platinum target, we use a mixture of 60% neutrons and 40% protons. The distributions of quarks of a given flavor in proton or neutron are related by assuming SU(2) symmetry.

For the gluon distribution, which enters in the Compton term, we have taken for the proton [19]

$$xG(x) = 2.63(1-x)^{5.9}(1+3.5x), \quad (15)$$

and for the pion [20]

$$xG(x) = 2(1-x)^3,$$

inspired by the counting rules.

Finally, we have parametrized  $\alpha(k^2)$  as

$$\alpha(k^2) = 12\pi/25 \ln\left(\frac{k^2 + \lambda^2}{\Lambda^2}\right) \quad (16)$$

in agreement with previous phenomenology at low  $k^2$  [12, 21].

Our results are then summarized in figs. 1–8 for  $\pi N$  and  $pN$  collisions and compared with experimental data [18, 22]. We have used  $\Lambda = 0.3$  GeV,  $\lambda = 0.9$  GeV and  $\langle p_{\perp}^2 \rangle_{\text{intr}} = 0.4$  GeV $^2$ . The sensitivity to variations of these parameters is discussed below. Let us briefly comment on our outcomes.

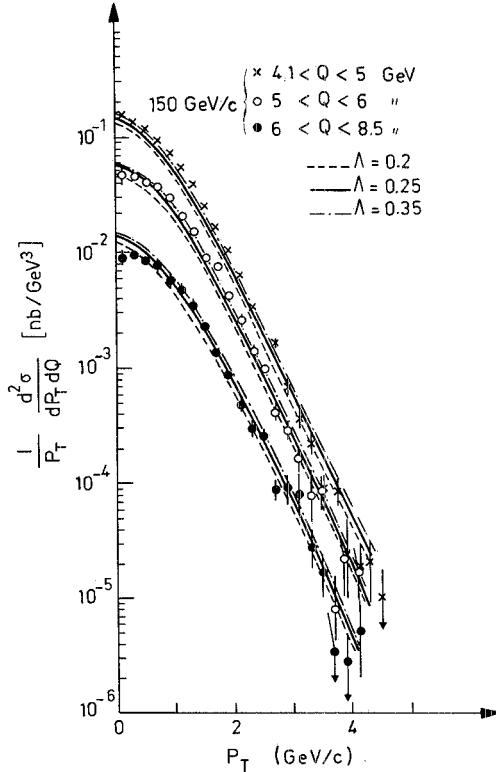


Fig. 5. The differential cross section  $(1/p_{\perp}) d\sigma/dp_{\perp} dQ$  at  $p_{\text{lab}} = 150 \text{ GeV}/c$  and  $y = 0$  for  $\pi^- N$  collisions as a function of the parameters  $\Lambda, \lambda, \langle p_{\perp}^2 \rangle_{\text{intr}}$ . Full line:  $\Lambda = 0.25 \text{ GeV}$ ,  $\lambda = 0.9 \text{ GeV}$  and  $\langle p_{\perp}^2 \rangle_{\text{intr}} = 0.4 \text{ GeV}^2$ . Dashed line:  $\Lambda = 0.2 \text{ GeV}$ ,  $\lambda = 0.9 \text{ GeV}$ , and  $\langle p_{\perp}^2 \rangle_{\text{intr}} = 0.5 \text{ GeV}^2$ . Dotted-dashed line:  $\Lambda = 0.35 \text{ GeV}$ ,  $\lambda = 1.1 \text{ GeV}$  and  $\langle p_{\perp}^2 \rangle_{\text{intr}} = 0.3 \text{ GeV}^2$ .

(i)  $\pi N$  collisions. The transverse momentum distributions are shown in fig. 1 and compared with the NA3 data [18]. The theoretical curves are calculated at  $y = 0$ . For the  $K$  factor in eq. (4) we have used the value  $K = 2.4$  given by the NA3 collaboration. The soft term is given by eq. (8), while the hard tail comes from eqs. (9), (11), (12) for  $p_{\perp} \gtrsim 3 \text{ GeV}$ .  $K(p_{\perp})$  is taken from ref. [10]. The value  $\alpha_s = \alpha(Q^2)$  is obtained through eq. (16). The Compton contribution is negligible. As clear from this figure, although the bulk of the effect comes from soft contributions, the hard terms including the  $\alpha_s^2$  corrections play an essential role for a quantitative description of the data at high  $p_{\perp}$ , say  $p_{\perp} \gtrsim 3 \text{ GeV}$ . Needless to say, a better resolution in this region would be highly desirable.

In fig. 2  $\langle p_{\perp}^2 \rangle$  is plotted versus  $\sqrt{s}$ . The theoretical curves are at  $y = 0$ . The hard term, obtained directly from the distributions of fig. 1, is  $\langle p_{\perp}^2 \rangle_{\text{hard}} \simeq 0.1 \text{ GeV}$  at  $Q \sim 4-5 \text{ GeV}$  and decreases with  $Q$  at  $s = 280 \text{ GeV}^2$ . At higher  $s$  the  $\langle p_{\perp}^2 \rangle_{\text{hard}}$  value increases at small  $Q$ . This contribution improves the agreement with respect to our

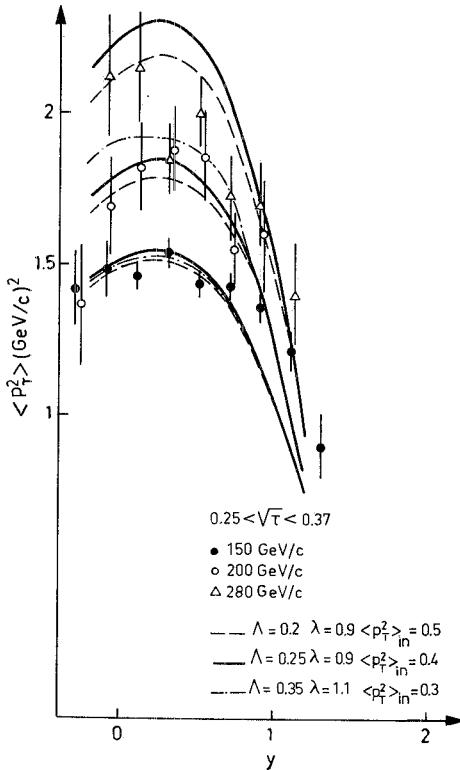


Fig. 6. The same as fig. 5 but for the average transverse momentum squared  $\langle p_{\perp}^2 \rangle$  versus  $y$ .

previous analysis [14] in the region of small  $\tau$ . The  $s$  dependence of  $\langle p_{\perp}^2 \rangle$ , which is evident from this figure, comes almost entirely from eq. (7) via  $q_{\perp \max}$ .

In fig. 3 we show the  $y$  dependence of  $\langle p_{\perp}^2 \rangle$ , integrated over a mass bin, at various energies. The data are from ref. [21]. The shape of the distribution is quite well reproduced, with the exception of too rapid a fall off near the boundary of the phase space where non-leading terms we have neglected can play a role.

In fig. 4, we plot the  $s$ -dependence of  $\langle p_{\perp}^2 \rangle$ , at  $y=0$ , for fixed  $\tau$ , in very good agreement with data.

We show in figs. 5, 6 the sensitivity of our results to a (10–20%) variation of the parameters. The  $y$  dependence of  $\langle p_{\perp}^2 \rangle$  is more sensitive to the values of  $\Lambda$  and  $\langle p_{\perp}^2 \rangle_{\text{intr}}$ , but the quality of the data is unfortunately not good enough to fix them better. From our analysis it follows that the values of  $\lambda$  and  $\Lambda$  found in ref. [21] for deep inelastic scattering can be used for  $p_{\perp}$  effects in Drell-Yan pairs.

(ii)  $pN$  collisions. We show in figs. 7 the  $p_{\perp}$  distributions at various energies for fixed  $y$ . Using the NA3 parametrizations for the structure functions the  $K$  factor is obtained by comparison with the measured [23]  $p_{\perp}$  integrated cross sections

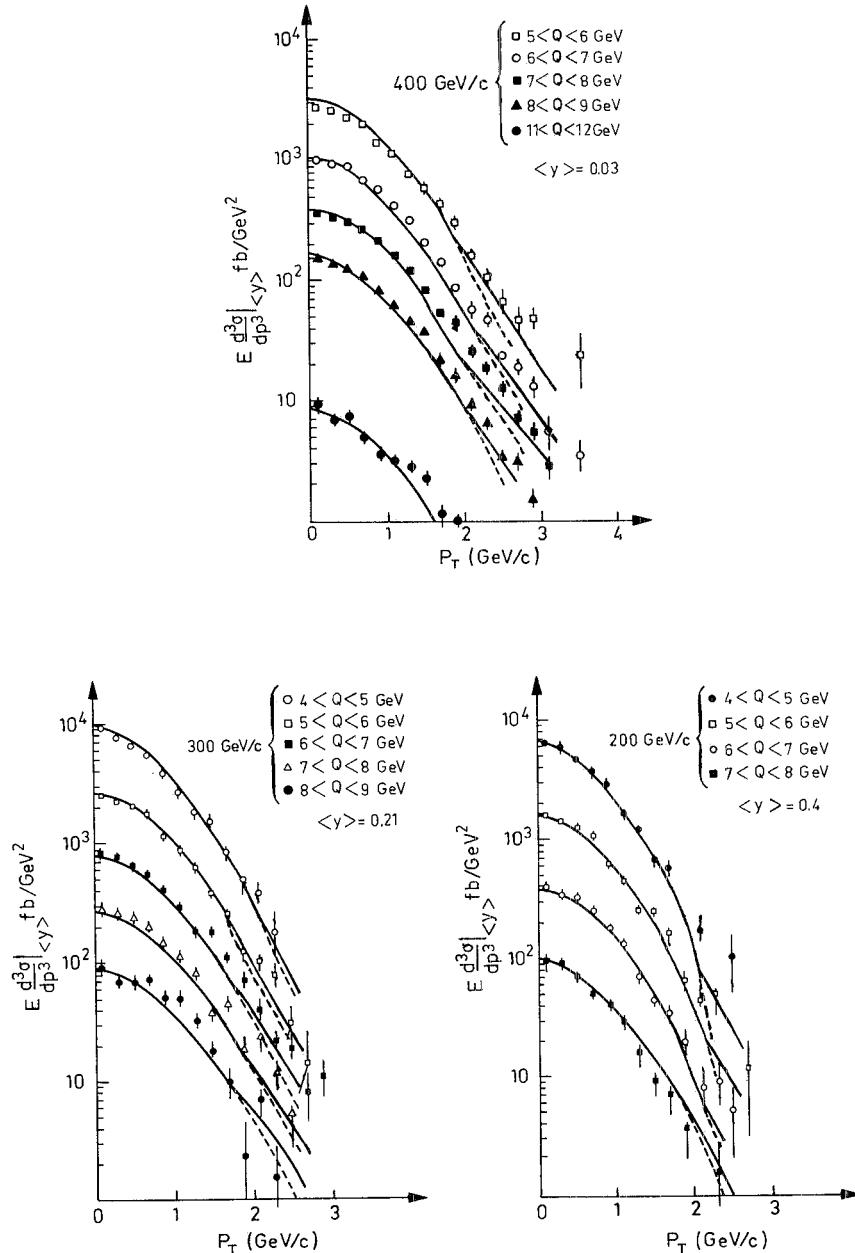


Fig. 7. The invariant cross section  $E d^3\sigma/d^3p$  for pN collisions. The data are from ref. [18]. The full line represents the total contribution: soft [eqs. (3)–(8)] + hard Compton [eq. (11)]. The dashed line represents the soft gluon contribution. The hard annihilation term is roughly 50% of the hard Compton one. (a)  $p_{\text{lab}} = 400 \text{ GeV}/c, \langle y \rangle = 0.03$ ; (b)  $p_{\text{lab}} = 300 \text{ GeV}/c, \langle y \rangle = 0.21$ ; (c)  $p_{\text{lab}} = 200 \text{ GeV}/c, \langle y \rangle = 0.4$ .

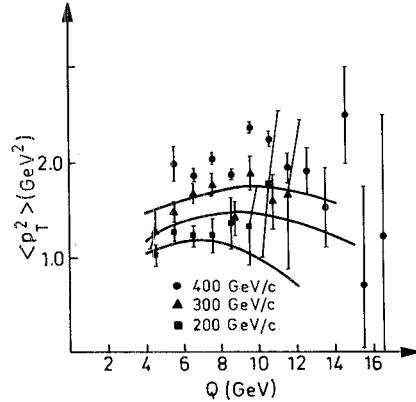


Fig. 8. The average transverse momentum squared  $\langle p_{\perp}^2 \rangle$  as a function of the lepton pair mass for  $p_{\text{lab}} = 200, 300$  and  $400 \text{ GeV}/c$  and  $\langle y \rangle = 0.4, 0.21$  and  $0.03$ . The data are from ref. [18].

$d\sigma/dQ dy$ , and takes values in the range 1.8–2.1. Then the soft corrections are absolutely normalized and in very good agreement with data. For the hard component we have proceeded as follows. The Compton term alone is not sufficient to reproduce the data for  $p_{\perp} \gtrsim 2 \text{ GeV}$  [9].

Assuming the same  $K(p_{\perp})$  factor found for  $\pi N$  scattering as a reasonable estimate of the  $\alpha_s^2$  corrections, we obtain a rather small result suggesting a sizeable  $\alpha_s^2$  correction for the Compton terms.

In fig. 8 we show the  $\langle p_{\perp}^2 \rangle$  dependence on the mass of the dilepton at various energies and  $y$ . The term  $\langle p_{\perp}^2 \rangle_{\text{hard}}$ , due to the Compton term, is of order  $0.2 \text{ GeV}^2$  at  $200 \text{ GeV}/c$  and a little higher at  $400 \text{ GeV}/c$ .

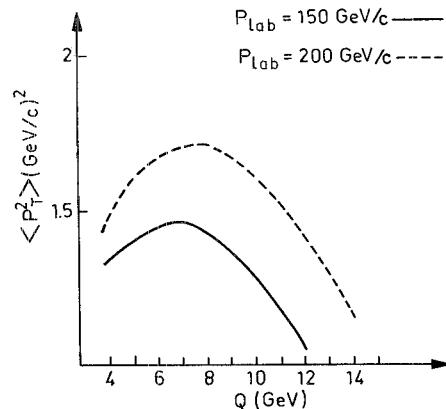


Fig. 9. The average transverse momentum squared  $\langle p_{\perp}^2 \rangle$  versus the lepton pair mass at  $y = 0$  for  $\bar{p}N$  collisions. Full line:  $p_{\text{lab}} = 150 \text{ GeV}/c$ . Dashed line:  $p_{\text{lab}} = 200 \text{ GeV}/c$ .

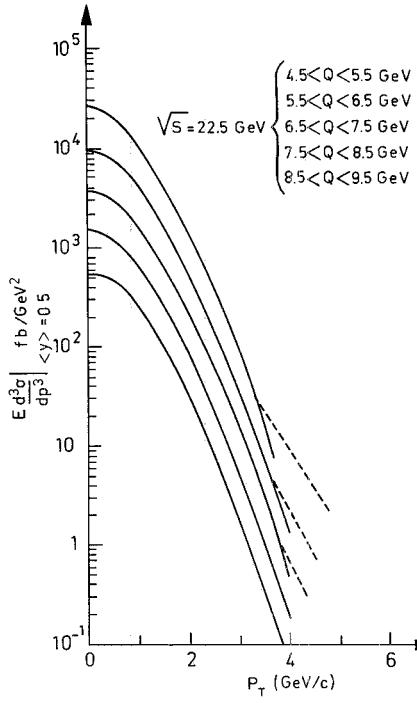


Fig. 10. The invariant cross section  $E d^3\sigma/d^3p$  at  $\sqrt{s} = 22.5$  GeV and  $y = 0.5$  for  $p\bar{p}$  collisions. Full line: the soft gluon contribution, eqs. (3)–(8). Dashed line: the hard annihilation term, eqs. (9)–(12).

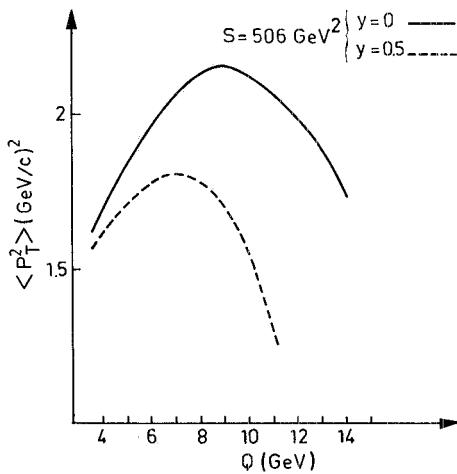


Fig. 11. The average transverse momentum squared  $\langle p_\perp^2 \rangle$  versus the lepton pair mass  $Q$  at  $\sqrt{s} = 22.5$  GeV for  $p\bar{p}$  collisions. Full line:  $y = 0$ . Dashed line:  $y = 0.5$ .

(iii)  $\bar{p}N$  collisions. In fig. 9 we give a prediction for  $\langle p_\perp^2 \rangle$  at 150 and 200 GeV/c, which can be easily tested by the NA3 collaboration. Our results seem to be in agreement with the preliminary data of ref. [22].

Finally in figs. 10 and 11 we give some predictions for  $p\bar{p}$  annihilation at collider energies which will be soon accessible [24]. We have used a  $K$  factor  $K=2.4$ , according to the NA3 results [18] and estimated the  $\alpha_s^2$  corrections to the annihilation terms as for the proton case. The transverse momentum distributions shown in fig. 10, although obviously higher than in the  $pp$  case due to valence quark-valence antiquark terms in the structure functions, present a shape similar to  $pN$  collisions. The hard annihilation term contributes for  $p_\perp \gtrsim 3$  GeV. The  $\langle p_\perp^2 \rangle$  in fig. 11 is essentially due to soft effects.

## 5. Conclusions

We have made an analysis of the transverse momentum properties of lepton pairs produced in hadronic collisions by combining the soft gluon description at small  $p_\perp$  with hard parton production at large  $p_\perp$ , including the  $\alpha_s^2$  corrections carried out recently. The bulk of the  $p_\perp$  effects measured so far is described by the soft gluon mechanism. The inclusion of appropriate kinematics in the resummed double leading log formula improves the theoretical predictions in a definite way. On the other hand, the inclusion of the  $\alpha_s^2$  corrections at large  $p_\perp$  also leads to better agreement with data for  $\pi^- N$  collisions\*.

Our analysis gives a nice description of the observed features in  $\pi N$  and  $pN$  collisions, showing a considerable success of perturbative QCD in the Drell-Yan process. Finally we have presented some predictions for  $p\bar{p}$  annihilation at collider energies.

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\* It would be highly desirable to have an estimate of the  $\alpha_s^2$  corrections to the Compton term which could be relevant in  $pN$  collisions.

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