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SPIN-ISOSPIN COLLECTIVE EXCITATIONS IN LIGHT OBLATE NUCLEI

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ABSTRACT

Spin-isospin excitations in light oblate nuclei are described as longitudinal (along the symmetry axis) and transverse oscillations in the framework of a semiclassical model. It is shown that, as a result of the competing effect of the $\sigma\tau$ -dependent and $\sigma\tau$ -independent components of the restoring force constants, these nuclei might exhibit an unsplit M2 resonance.

1 - Spin-isospin ($\sigma\tau$) or pion like excitations in nuclei have been the object of several investigations in recent years in connection with the study of precursor phenomena⁽¹⁾. The softening of some of the $\sigma\tau$ -excitation modes and the enhancement of the corresponding magnetic transition probability would in fact indicate proximity of the nucleus to a static $\sigma\tau$ -phase⁽²⁾ or pion condensation⁽³⁾.

In all those analysis the possible role of nuclear deformation has been ignored. It was pointed out however in ref. (4), to be referred to as I, that the most natural candidates for exhibiting a precursor behaviour are light oblate nuclei. This can be seen from the analogy with the $\sigma\tau$ -static phase in nuclear matter. This phase is characterized by a laminated structure due to one-dimensional crystallization along the direction of spin quantization (z-axis), which is normal to the planes alternately occupied by $\sigma_3\tau_3=1$ and $\sigma_3\tau_3=-1$ nucleons. The z-component of the average distance between these two fluids results therefore to be smaller than the transverse

one, yielding an attractive contribution from the tensor force in the Hartree approximation.

A similar configuration is realized in an oblate nucleus by a $\sigma\tau$ -displacement along the symmetry axis (longitudinal). If the OPE potential were strong enough, a static $\sigma\tau$ -phase would therefore be established. We showed in I that the actual OPE potential is not so strong, but it is attractive enough to favour zero-point longitudinal oscillations. The precritical behaviour would then be characterized by the softening of such a longitudinal oscillation mode. Such a softening has not been observed^{(5)†} in an experiment on ^{28}Si .

Having concentrated our attention on the unidimensionality of the oscillations as the main characteristic of the order, we did not investigate in I how transverse oscillations are affected by the OPE potential and we neglected the effect of deformation on the $\sigma\tau$ -independent component of the restoring force.

We show in the present note that the above deformation effect changes drastically the results of I and makes necessary the inclusion of transverse oscillations in order to give a distinctive characterization of $\sigma\tau$ excitations in deformed nuclei which is easy to test experimentally.

2 - For simplicity, we confine ourselves to ^{12}C and ^{28}Si .

The total nuclear wave function is

$$\Psi_{\nu_z \nu_T K} = \Phi_{\nu_z \nu_T K}(\vec{d}) \Lambda(\vec{d}) \quad (1)$$

where the intrinsic part $\Lambda(\vec{d})$ is a Slater determinant of displaced single-particle wave functions

$$\varphi_{n_z n_T m \sigma_3 \tau_3} = \varphi_{n_z}(z - \frac{1}{2} d_z \sigma_3 \tau_3) \varphi_{n_T m}(\vec{r}_T - \frac{1}{2} \vec{d}_T \sigma_3 \tau_3) \chi_{\sigma_3 \tau_3} \quad (2)$$

The φ_{n_z} and $\varphi_{n_T m}$ are harmonic oscillator wave functions in a cylindrical basis and $\chi_{\sigma_3 \tau_3}$ are spin-isospin wave-functions.

The shells are filled according to the following scheme

$$(\nu_z, \nu_T m) = \begin{cases} (0, 0, 0); (0, 1, \pm 1) \text{ for } ^{12}\text{C} \\ (0, 0, 0); (0, 1, \pm 1); (1, 0, 0); (0, 2, \pm 2); (2, 0, 0) \text{ for } ^{28}\text{Si}. \end{cases}$$

The collective motion is generated by the quantum fluctuations of the displacement parameter \vec{d} . In harmonic approximation, the collective Hamiltonian is

$$H = \frac{P^2}{2M} + \frac{1}{2} (C_z + K_z) d_z^2 + \frac{1}{2} (C_T + K_T) d_T^2 \quad (3)$$

where $M=Am/4$, C_z and C_T are the $\sigma\tau$ -independent restoring constants and K_z and K_T the dependent ones. These latter are determined by the equation

$$\langle \Lambda(\vec{d}) | V_{\sigma\tau} | \Lambda(\vec{d}) \rangle \sim \frac{1}{2} K_z d_z^2 + \frac{1}{2} K_T d_T^2. \quad (4)$$

$V_{\sigma\tau}$ is the $\sigma\tau$ -dependent part of the N-N interaction. This is customarily approximated by the regularized OPE potential plus a contact term which is supposed to account for the short range part of the $\sigma\tau$ -dependent interaction. This contact term parametrizes matrix elements on the Fermi surface, and such parametrization is a priori unjustified for a nonspherical Fermi surface especially in the study of anisotropic phenomena like longitudinal and transverse excitations. This point has already been emphasized in the context of static $\sigma\tau$ -

phases⁽⁶⁾. We nevertheless maintain here this parametrization to get an idea of the effect of an overall reduction of the OPE potential. Therefore $V_{\sigma\tau}$ have the expression

$$V_{\sigma\tau} = -\left(\frac{f}{m\pi}\right)^2 \varrho(p^2) \vec{\tau}_1 \cdot \vec{\tau}_2 \left[\vec{\sigma}_1 \cdot \vec{p} \vec{\sigma}_2 \cdot \vec{p} / (p^2 + m_\pi^2) - g' \vec{\sigma}_1 \cdot \vec{\sigma}_2 \right] \quad (5)$$

where

$$\frac{f^2}{4\pi} = 0.08; \quad \varrho(p^2) = \left(\frac{\Lambda^2}{\Lambda^2 + p^2}\right)^2; \quad \Lambda = 1300 \frac{\text{MeV}}{c} \quad (6)$$

The resulting expressions for K_Z and K_T are

$$K_{Z,T} = -\frac{1}{\pi^2} \left(\frac{f}{m\pi}\right)^2 \int_{-\infty}^{+\infty} dp_z \int_0^\infty dp_T p_T P_{Z,T} \left(\frac{p_z^2}{p^2 + m_\pi^2} - g' \right) \varrho(p^2) G(p_z, p_T) \quad (7)$$

In the above equation

$$P_Z = p_z^2, \quad P_T = \frac{1}{2} p_T^2,$$

$$G(p_z, p_T) = \sum_{n_z, n_1, m_1} \sum_{n_z, n_2, m_2} G_{n_z, n_1}(p_z) G_{n_z, n_2}(p_z) G_{n_1, m_1}(p_T) G_{n_2, m_2}(p_T), \quad (8)$$

$$G_{n_z}(p_z) = \langle \varphi_{n_z} | e^{izp_z} | \varphi_{n_z} \rangle, \quad G_{nm}(p_T) = \langle \varphi_{nm} | e^{i\vec{r}_T \cdot \vec{p}_T} | \varphi_{nm} \rangle. \quad (9)$$

Finally, following the prescription of the unified theory of nuclear vibrations⁽⁷⁾ we put

$$C_Z = \frac{\Lambda}{4} m \bar{\omega}_Z^2, \quad C_T = \frac{\Lambda}{4} m \bar{\omega}_T^2, \quad (10)$$

$\bar{\omega}_Z$ and $\bar{\omega}_T$ being the oscillator frequencies. The above expressions can only be indicative. In particular the underlying quasi-boson approximation is not well justified for C_Z due to the small number of single particle states available.

The first excited states are characterized by a unique quantum number ($K=0$, for $n_T=0, n_z=1$, $K=\pm 1$, for $n_T=1, n_z=0$) and by the M2 transition strength

$$B(M2; I = K = 0 \rightarrow I, K) = \frac{2}{1 + \delta_{K0}} \langle \Phi_K | \mathcal{M}_\nu (=I=2, =K) | \Phi_0 \rangle|^2, \quad (11)$$

where

$$M_{\nu}(2,K) = \frac{A}{8} \sqrt{\frac{5}{4\pi}} \sqrt{\frac{(2+K)!(2-K)!}{(1+K)!(1-K)!}} (g_p - g_n)^{K-1} d_{-K} \frac{e\hbar}{2mc} \quad (12)$$

Therefore

$$B(M2, 0 \rightarrow K) = \frac{5}{32\pi} \frac{\hbar^2}{m} A(g_p - g_n)^2 \left(\frac{1}{\hbar\omega_z} \delta_{K0} + \frac{3}{2\sqrt{2}} \frac{1}{\hbar\omega_T} \delta_{K,\pm 1} \right) \left(\frac{e\hbar}{2mc} \right)^2 \text{fm}^2. \quad (13)$$

Transitions $K=0 \rightarrow K = \pm 2$ are absent.

The above equations have been evaluated by using the free nucleon values for g_p and g_n . These values should be renormalized according to the quenching of magnetic transitions⁽⁸⁾ in order to compare with experiment.

3 - The predictions of the model are reported for 3 values of g' , i.e. $g'=0.33, 0.5, 0.7$. We have assumed $\bar{\omega}_z/\bar{\omega}_T=1.5$, which corresponds to a deformation parameter $\delta \sim -0.4$.

In order to appreciate the difference with respect to the results presented in I, we recall that in that paper the (longitudinal) restoring force constant was computed ignoring the dependence of C_z on the deformation. Furthermore K_z was evaluated for $g'=0.33$ so as to cancel the contact term in the OPE potential, and assuming a nuclear density of gaussian form. Table I shows that the removal of the latter approximation makes K_z more negative. The increase of C_z due to deformation (Table II) is however such as to overcompensate the attraction coming from K_z . As a result there is no softening of the longitudinal mode. A new characterization of the $\sigma\tau$ excitations emerges from the present results (Table II).

For all values of g' , K_T turns out to be positive. For $g'=0.33$ transverse and longitudinal excitations are almost degenerate and have comparable strength, while for $g'=0.5$ only the transverse excitation is collective. These values of g' give rise to negative values of K_z . For $g'=0.7$, K_z is instead positive and longitudinal and transverse excitations are well separated.

We can infer from the above results that for not too large values of g' only a single collective level or two levels very close in energy should appear in deformed nuclei. If g' is too large instead two well separated collective states should exist. This latter case would be the analog of the splitted E1 giant resonance in deformed nuclei.

Some experimental results are known for M2 transitions in ^{12}C and ^{28}Si . In ^{12}C there is one collective level⁽⁹⁾ at 19.3 MeV with $B(M2) \uparrow \sim 700 \mu^2 \text{fm}^2$. In ^{28}Si there is a group of levels⁽⁵⁾ centered at 14.5 MeV with total strength $B(M2) \uparrow \sim 400 \mu^2 \text{fm}^2$. These results have in common with our prediction for $g'=0.5$ the fact that the strength is concentrated at a unique value of the excitation energy in agreement with our model. The strength in ^{12}C , however, is much larger than predicted, while in ^{28}Si it is fragmented.

In order to clarify the situation it would be desirable to have a direct measurement of the $B(M2)$ in ^{12}C , whose value has been obtained through a theoretical analysis done with spherical s.p.w.f., as well as experimental results in other light oblate as well as prolate nuclei⁽¹⁰⁾.

For a more detailed comparison with experiment one should also perform a microscopic calculation, the only one which can account for possible fragmentation of the $\sigma\tau$ levels as occurs in ^{28}Si .

TABLE I - C is evaluated according to Eq. (10) with $\bar{\omega}_z = \bar{\omega}_T$, i. e., by neglecting the $C_z - C_T$ difference. The resulting value is slightly different from that of I, where $C = 41 A^{-5/3} \text{ MeV fm}^{-2}$. All other quantities are evaluated by putting $\bar{\omega}_z/\bar{\omega}_T = 1.5$, which corresponds to a deformation parameter $\delta = -0.4$. The values of B(M2) are omitted for $K/C \sim 0.1$, because in this case our collective model is not reliable.

Nucleus	g'	C MeV fm ⁻²	K_z MeV fm ⁻²	ω_z MeV	$B(M2;K=0 \rightarrow K=0)$ $\mu^2 \text{ fm}^2$	K_T MeV fm ⁻²	ω_T MeV	$B(M2;K=0 \rightarrow K=\pm 1)$ $\mu^2 \text{ fm}^2$
¹² C	0.33		-16.5	8.8	250	3.4	19.0	
	0.5	22.1	-4.5	15.6		8.4	20.5	160
	0.7		9.6	20.9	105	14.4	22.5	147
²⁸ Si	0.33		-13.6	9.6	533	5.3	14.8	
	0.5	29.3	2.0	13.6		11.6	15.6	493
	0.7		20.4	17.2	299	19.1	16.9	456

TABLE II - The same as Table I, with the exception of C_z and C_T which are evaluated according to Eq. (10) with $\bar{\omega}_z/\bar{\omega}_T = 1.5$.

Nucleus	g'	C_z MeV fm ⁻²	K_z MeV fm ⁻²	ω_z MeV	$B(M2;K=0 \rightarrow K=0)$ $\mu^2 \text{ fm}^2$	C_T MeV fm ⁻²	K_T MeV fm ⁻²	ω_T MeV	$B(M2;K=0 \rightarrow K=\pm 1)$ $\mu^2 \text{ fm}^2$
¹² C	0.33		-16.5	17.2	128		3.4	16.7	197
	0.5	37.9	-4.5	21.4		16.8	8.4	18.7	176
	0.7		9.6	25.6	86		14.4	20.8	159
²⁸ Si	0.33		-13.6	14.7	348		5.3	12.8	602
	0.5	50.3	2.0	17.6		22.3	11.6	14.2	542
	0.7		20.4	20.5	251		19.1	15.7	491

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- (10) An experiment on ²⁰Ne is now in progress. A. Richter, private communication.

† The present investigation has been performed before the publication of ref. (5). Its main results can be found in N. Lo Iudice and F. Palumbo, Frascati preprint 81/66(P), (1981).