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G. Martinelli: RECENT RESULTS FROM LATTICE QCD WITH FERMIONS

Invited talk at the "Xth Intern. Winter Meeting on Fundamental Physics" and "XIIIth Gift Intern. Seminar on Theoretical Physics" - Masella (Girona, Spain), Jan. 28 - Feb. 6, 1982.

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RECENT RESULTS FROM LATTICE QCD WITH FERMIONS

G. Martinelli
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Invited talk given at the "Xth International Winter Meeting on Fundamental Physics", and "XIIIth Gift International Seminar on Theoretical Physics", Masella (Girona, Spain), Jan. 28 - Feb. 6, 1982.

ABSTRACT

I report recent results in hadron physics obtained from lattice QCD.

1. - INTRODUCTION

In the last few months many progresses have been made in lattice gauge theories with fermions. In particular several groups computed the spectrum of the hadrons by Montecarlo simulation starting from interacting quarks and gluons on the lattice. Many questions however are still to be solved. In this talk we will give a survey of the more relevant results, discuss problems and limitations with Montecarlo techniques and indicate possible future applications.

In Section 2 several possible fermion lattice actions are presented and the problem of fermion doubling discussed; in Section 3 we describe some techniques that have been proposed to compute the fermion propagator and determinant; in Section 4 we introduce the so called quenched approximation and give details on the computation of hadron masses from appropriate correlation functions; in Section 5 we revew the results recently obtained for the dy namical symmetry breaking and for the hadronic spectrum. In the conclusion we indicate some of the possible future applications of these powerfull techniques.

SECTION 2

This section is devoted to the problems arising in putting fermions on the lattice. Let us start from the usual Dirac action for fermions interacting with gauge fields on the continuum:

$$S_C = \int d^4x \left(-\frac{1}{2}\bar{q}(x)\gamma_\mu \overleftrightarrow{D}_\mu q(x) - m_q \bar{q}(x) q(x)\right) + S_G(U) . \quad (1)$$

D_μ is the usual covariant derivative; $S_G(U)$ is the pure Yang-Mills gauge field action; summation over spinor, colour and flavour indices is understood.

The most direct (naive) way to put the theory on the lattice is the replacement (for fermion fields):

$$\partial_\mu f(x) = \frac{f(x+\hat{\mu}) - f(x)}{a} . \quad (2)$$

$\hat{\mu}$ is the unit vector in the μ direction and a is the lattice spacing ($x = na$; n is a four dimensional integer vector). We get:

$$S_N = \sum_n \left\{ -\frac{1}{2}\sum_\mu \left[\bar{\psi}(n)\gamma_\mu U_\mu(n)\psi(n+\hat{\mu}) - \bar{\psi}(n+\hat{\mu})\gamma_\mu U^+_\mu(n)\psi(n)\right] - m_q \bar{\psi}(n)\psi(n)\right\} + S_L(U) . \quad (3)$$

$a = 1$ to simplify the notation.

$S_L(U)$ is one of the possible gauge field actions on the lattice; for example, following Wilson[1] we could choose:

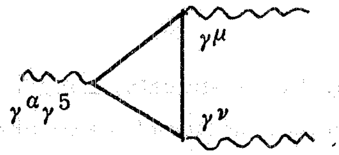
$$S_L(U) = \frac{1}{g_o^2}\sum_P t_r(U_P + U_P^+) . \quad (4)$$

U_P is the product of the links along an elementary plaquette and g_o is the lattice bare coupling constant.

The action of eq. (3) has the same (at the tree level) chiral properties of the continuum action. In fact, in the limit $m_q = 0$ it is invariant under the following global transformations:

$$\psi(n) \longrightarrow e^{i\alpha}\psi(n) , \qquad \psi(n) \longrightarrow e^{i\alpha\gamma_5}\psi(n) , \quad (5)$$

which correspond to a $U(1)_V \times U(1)_A$ symmetry group. This property contradicts Adler theorem which states that we cannot find a gauge invariant regularization which preserves the axial symmetry. We would expect the divergence of the axial current to be different from zero because of the Adler-Bell-Jackiw anomaly:



$$= \Gamma_{\alpha\mu\nu} \qquad p^\alpha \Gamma_{\alpha\mu\nu} \neq 0 \tag{6}$$

FIG. 1

Indeed Adler theorem is not violated because of fermion doubling. The action (3) contains 16 different fermions: the fermion propagator has 16 poles corresponding to different points of the Brillouin zone. The free quark propagator:

$$S(p) = \frac{1}{\sum_\mu \gamma_\mu \sin p_\mu} \tag{7}$$

has poles for $p_\mu = 0, \pi$.

The anomaly is given by the sum of the contributions of all these fermionic states:

$$p^\alpha \Gamma_{\alpha\mu\nu} = (\sum_i Q_i) I_{\mu\nu} \tag{8}$$

where Q_i are the axial charges of the looping fermions. Karsten and Smit[2] showed that the sum of the axial charges of the 16 fermions is equal to zero ($Q = +1$ for the pole $P = (0,0,0,0)$; $Q = -1$ if $P = (\pi, 0, 0, 0), \ldots$). In order to avoid this problem, Wilson[1] proposed a different action of the form:

$$S_W = \sum_n \left\{ -\bar{\psi}(n)\psi(n) + K \left[\sum_\mu (\bar{\psi}(n)(1-\gamma_\mu) U_\mu(n) \psi(n+\hat{\mu}) + \right. \right. \\ \left. \left. + \bar{\psi}(n+\hat{\mu})(1+\gamma_\mu) U^+_\mu(n) \psi(n)) \right] \right\} + S_L(U). \tag{9}$$

Eq. (9) reproduces the usual continuum action in the limit $a \to 0$. The 15 unwanted extra fermions aquire an effective mass $\sim 1/a$ for $a \to 0$ and disappear from the game. The price one has to pay is the introduction of an explicit chiral symmetry breaking term in the Lagrangian: chiral symmetry is not protected against perturbative/non perturbative corrections. Other solutions have been proposed. In the Susskind version[3] we have only (sic !) 4 fermions instead of 16; the continuum chiral symmetry is replaced by a discrete chiral symmetry. The SLAC group proposed[4] to avoid the problem of doubling by introducing a non local action which in the free case has the form (in momentum space):

$$\bar{\psi}(p)\, \not{p}\, \psi(p). \tag{10}$$

This action however gives problems because of non locality even at a perturbative level[2]. The conclusion is that is not possible to write a fermion action on the lattice which is local, has the same chiral properties of the continuum and avoids the fermion doubling[5]. We hope that, for $a \to 0$, these lattice patologies will disappear giving the correct continuum limit.

SECTION 3

In pure bosonic theories many Montecarlo methods (Metropolis, heath bath, Langevin) are aviable, unlike in the fermionic case because of the anticommuting nature of the fermionic degrees of freedom.

Let us illustrate how the Metropolis method can be applied to the computation of the expectation value of some operator in the bosonic case.

The Feynman path integral on an euclidean space time has the form:

$$\langle \hat{O}(U) \rangle = \frac{\int d[U]\, O(U) \exp - S(U)}{\int d[U] \exp - S(U)} \tag{11}$$

U are arbitrary bosonic degrees of freedom and S(U) their action. One computes the expectation value by the following algorithm:

a) Start with a trial field configuration $\{U\}$;

b) Choose a new link U'; $\{U\} \to \{U'\}$;

c) If $\exp - [S(\{U'\}) - S(\{U\})] > x$, where x is a random variable extracted with a flat probability distribution in the interval $(0,1)$, then $\{U'\}$ is accepted as new configuration.

This algorithm satisfies the detailed balance; the probability distribution of the U fields tends asymptotically to:

$$P(U)\, d(U) \to e^{-S(U)}\, dU \tag{12}$$

and the expectation value of the operator is simply its average over the field configurations:

$$\exp - S(U)\, d[U] \to \sum_{\substack{\text{field} \\ \text{configurations}}} . \tag{13}$$

It is not possible to define a probability distribution for fermions. However, because in the interesting cases, the action is quadratic in the fermion fields (see eqs. (3), (9)), these can be formally integrated out:

$$S = \sum_{i,J} \overline{\psi}_i \Delta_{iJ}(U)\, \psi_J + S_L(U) . \tag{14}$$

Then:

$$\begin{aligned} &\int d[\overline{\psi}]\; d[\psi] \exp(-S(U, \psi, \overline{\psi})) = \det[\Delta(U)] \exp(-S_L(U)) \\ &\int d[\overline{\psi}]\; d[\psi]\; \overline{\psi}_i\, \psi_J \exp(-S(U, \psi, \overline{\psi})) = \Delta^{-1}_{Ji}(U) \det(\Delta(U)) \exp(-S_L(U)) \end{aligned} \tag{15}$$

In general, the expectation value of some operator depending on U, $\psi, \overline{\psi}$ will be given by:

$$\langle \hat{O}(U, \psi, \overline{\psi}) \rangle = \frac{\int d[U] d[\psi] d[\overline{\psi}] O(U, \psi, \overline{\psi}) \exp(-S)}{d[U] d[\psi] d[\overline{\psi}] \exp(-S)} =$$

$$= \frac{\int d[U] \tilde{O}[U, \Delta^{-1}(U)] \det[\Delta(U)] \exp(-S_L(U))}{Z} . \tag{16}$$

The problem is reduced to compute the determinant and the inverse of $\Delta(U)$. If the determi_ nant has a definite sign:

$$\det[\Delta(U)] \exp(-S_L(U)) \sim \exp(-S_L(U) + \mathrm{tr} \ln|\Delta(U)|) = \exp - S_{eff}(U) . \tag{17}$$

We can apply the usual (bosonic) Montecarlo technique for the gauge fields with the effective action $S_{eff}(U) = S_L(U) - \mathrm{tr} \ln|\Delta(U)|$.

The fermionic determinant gives the modification to the Gluon Green functions due to looping quarks as shown in Fig. 2 ; for a fixed gauge field configuration $\Delta^{-1}(U)$ is the fer- mion propagator in an external field $\{U\}$ (Fig. 3).

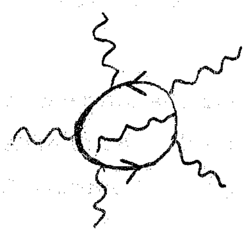


FIG. 2

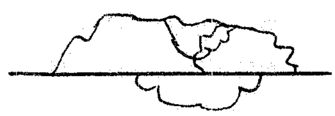


FIG. 3

The difficulty is the inversion of $\Delta(U)$ and the computation of the highly non local quantity $\det[\Delta(U)]$ (non a typical $6^3 \times 10$ lattice the fermion propagator has $\sim 1.3 \times 10^9$ degrees of freedom). Several methods have been proposed:

a) F. Fucito, E. Marinari, G. Parisi and C. Rebbi[6].

One chooses the new field configuration $\{U'\}$ so that $\delta U = U' - U$ is small.

$$S_{eff}(U') - S_{eff}(U) \simeq S_L(U') - S_L(U) - \sum_{i,J} \Delta^{-1}(U)_{iJ} \left(\frac{\delta \Delta(U)}{\delta U}\right)_{Ji} \delta U . \tag{18}$$

To compute the propagator ($\frac{\delta \Delta(U)}{\delta U}$ is trivial) we introduce the "pseudofermionic" variables as follows:

$$\Delta^{-1}_{iJ} = \frac{\int d[\overline{\Phi}] d[\Phi] \overline{\Phi}_i \Phi_J \exp(-\overline{\Phi} \Delta(U) \Phi)}{\int d[\overline{\Phi}] d[\Phi] \exp(-\overline{\Phi} \Delta(U) \Phi)} . \tag{19}$$

The bosonic field Φ carry the same quantum numbers of the corresponding quark fields. $\Delta^{-1}_{iJ}(U)$ is computed by an "internal" Montecarlo integration in the usual way. This methods could practically fail when the action has several pronounced minima because the variation of the U's must necessarily be small. The method has been successfully applied to the two dimensional Scwinger model.

b) D. J. Scalapino and R. L. Sugar(7).

In this case one computes exactly the propagator and the determinant, starting from the locality properties of $\Delta_{iJ}(U)$, using the equations:

$$R = \frac{\det\left[\Delta(U+\delta U)\right]}{\det\left[\Delta(U)\right]} = \det\left[1 + \Delta^{-1}(U)\,\delta\Delta(U)\right],$$

$$\Delta^{-1}(U+\delta U) = \Delta^{-1}(U) - \Delta^{-1}(U)\,\delta\Delta(U)\,\Delta^{-1}(U+\delta U) \tag{20}$$

where $\delta\Delta(U)$ is the variation of the $\Delta(U)$. With this method one must necessarily store all the matrix elements of the propagator. In practice this could rise unreasonable memory (or time) computer problems. It has so far been applied only to a very simple soluble model(7) (more recently it has also been applied to QCD with a lattice of very small size).

c) D. H. Weintgarten and D. N. Petcher(8).

I will not enter in all the details of their method but only describe the techniques they proposed to compute the propagator (Gauss-Seidel relaxation method). Let us write $\Delta(U) = I - B(U)$, where I is the identity matrix in position, spin, ... space. We define X to be:

$$X = \Delta^{-1}(U)\Phi . \tag{21}$$

Φ is an arbitrary fixed vector.

We write the following recursive equation:

$$X_{n+1} = B(U)X_n + \Phi . \tag{22}$$

The fixed point of eq. (22) is the solution of eq. (21). This technique has been widely used in recent Montecarlo experiments. We should add that, very recently, alternative stochastic methods have been developed(9) but they have not yet been applied to gauge theories.

SECTION 4

At present, it seems very difficult to overcome the problems connected with the computation of the fermion determinant. One possible way out is to work in the so called quenched approximation: we put $\det\left[\Delta(U)\right] = 1$, that is we completely neglect the fermion feedback on the gluons. In four dimensional QCD, with the only exception of the hopping parameter expansion (see ref. (10)), all the results for chiral symmetry breaking and for the hadron mass spectrum have been obtained in the quenched case, which corresponds to the $n_f/N_c \to 0$ limit.

This approximation is expected to work reasonably well for the following reasons :

a) It becomes exact in the limit $N_c \to \infty$; note that this is not the $N_c \to \infty$ limit because non planar gluon diagrams are not neglected.

b) Zweig rule is exactly satisfied and hadrons are made only by valence quarks and gluons; we know that this is approximately true in the real world.

c) The results of ref. (10), indicate that, at least in that approximation, the inclusion of quark loops only amounts to a small changement for the relevant physical quantities.

In any case this must be considered as a first step to develope our techniques and to understand possible sources of statistical and systematical errors. Any further development will necessarily include the computation of the determinant.

Here following we describe the techniques used to compute the chiral symmetry breaking and the hadron mass spectrum.

a) The chiral symmetry breaking.

Starting from the lattice action (3) which becomes chiral invariant in the limit $m_q \to 0$ ($a \neq 0$), at any order in perturbation theory we have :

$$\lim_{m_q \to 0} \langle \bar{\psi} \psi \rangle = 0 \tag{23}$$

because all the chiral breaking perturbative corrections must be proportional to the explicit mass term m_q.

The fact that, by Montecarlo simulation, one finds :

$$\lim_{m_q \to 0} \langle \bar{\psi} \psi \rangle \neq 0 \tag{24}$$

is a signal of dynamical chiral symmetry breaking and we expect that in the continuum limit ($a \to 0$) the usual Goldstone phenomenon will happen. For $a \to 0$, we must require the renormalization group behavior for $\langle \bar{\psi}\psi \rangle$ to be satisfied. From perturbation theory we expect (for SU(3) of colour) :

$$\langle \bar{\psi} \psi \rangle_{m_q=0} \sim \beta^{\alpha} \exp - \frac{4\pi^2}{11} \beta \qquad (\beta = 6/g_o^2) \tag{25}$$

α is a computable exponent (see ref. (11)).

In Fig. 4 we display the experimental results from ref. (11) for $\langle \bar{\psi}\psi \rangle_{m_q=0}$ as a function of $1/g_o^2$. The full line is the predicted perturbative renormalization group behavior.

After the cross over region between the strong and weak coupling regimes (analogously

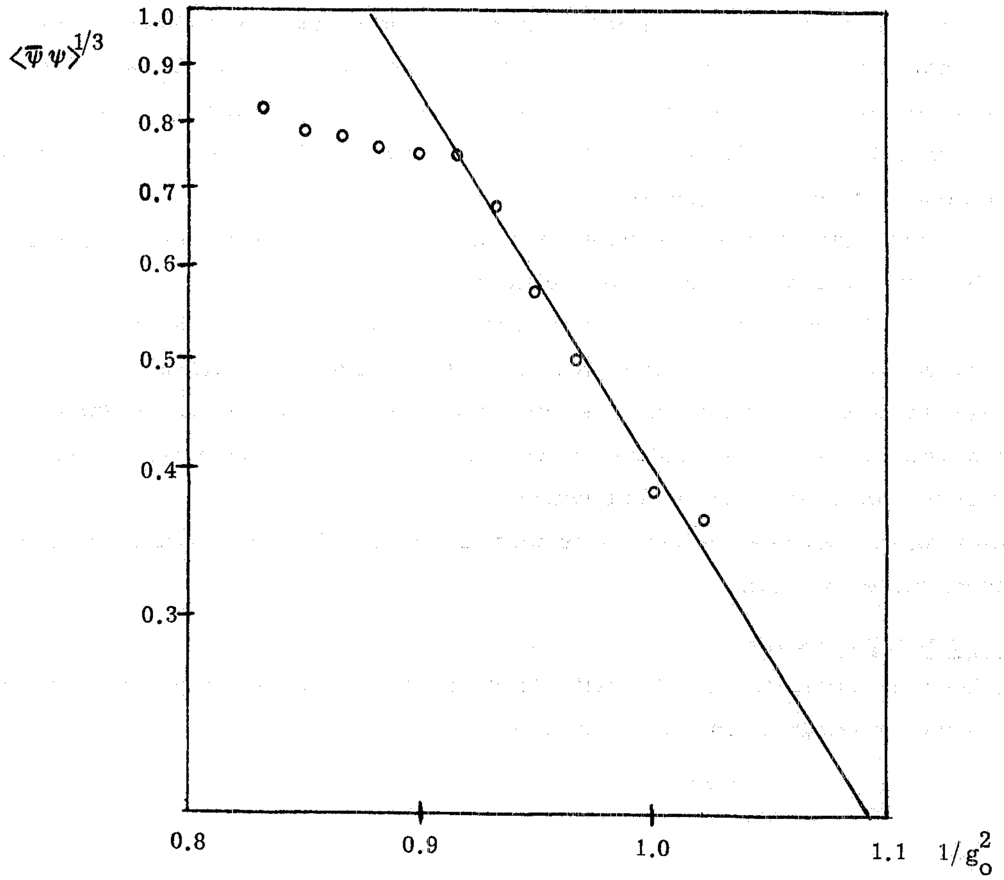


FIG. 4 - The renormalization group behavior of $\langle\bar{\psi}\psi\rangle^{1/3}$ as a function of $1/g_o^2$. The full line is the theoretical prediction. The data are from ref. (11).

to what happens for the string tension) the experimental points fit rather well the theoretical curve. The measured $\langle\psi\psi\rangle_{m_q=0}$ can be used to extract f_π starting from the renormalization group invariant relation :

$$f_\pi^2 m_\pi^2 = m_q\langle\bar{\psi}\psi\rangle \tag{26}$$

where m_π is the (measured) pion mass. The authors of ref. (11) found

$$f_\pi = 95 \pm 10 \text{ MeV} \qquad \text{for } \beta = 6 . \tag{27}$$

$\langle\bar{\psi}\psi\rangle_{m_q=0}$ has been also computed in ref. (15) for SU(2) of colour.

b) The hadron mass spectrum.

We start by defining operators which carry the same quantum numbers of the particles we want to study.

For example :

$$\pi(x) = u(x)^A_\alpha \gamma^5_{\alpha\beta} d^A_\beta(x) \qquad \underline{\text{pion}}$$

$$\varrho^\mu(x) = u(x)^A_\alpha \gamma^\mu_{\alpha\beta} d^A_\beta(x) \qquad \underline{\varrho\text{-meson}} \tag{28}$$

$$\mathbb{P}_\delta(x) = (u(x)^A_\alpha (\mathbb{C}\gamma_5)_{\alpha\beta} d^B_\beta(x))\, u^C_\delta(x)\, \epsilon_{ABC} \qquad \underline{\text{proton}}$$

$\mathbb{C}$ is the charge conjugation operator. It is straightforward to compute the correlation functions for these operators. In the case of the pion, for example:

$$G(x) = \langle \pi(x)\pi(0) \rangle = \frac{\int d[U]\, d[\psi]\, d[\bar{\psi}]\, \pi(x)\pi(0)\exp(-S)}{Z}$$

$$\frac{\int d[U]\, \mathrm{tr}\left[\gamma^5 \Delta^{-1}(U)(x,0)\gamma^5 \Delta^{-1}(U)(0,x)\right]\exp(-S_L(U))}{Z} \tag{29}$$

(remember that $\det[\Delta(U)] = 1$).

The r.h.s. of eq. (29) is diagramatically represented in Fig. 5. $\int \exp - S_L(U)\, d[U] \rightarrow \sum_{\substack{\text{link}\\ \text{configurations}}}$ in a Montecarlo simulation.

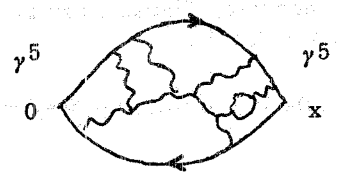


FIG. 5 - Typical diagram for the pion propagator in the quenched approximation.

If there is a single particle propagating, then we expect:

$$G(t) = \sum_{\vec{x}} G(x) \sim \exp - mt \tag{30}$$

($\sim \cosh[m(t - T/2)]$ on a periodic lattice of period T in the time direction). $\sum_{\vec{x}}$ is the sum over the spatial components and m is the mass of the particle. By fitting G(t) we measure the masses in units of the lattice spacing. In Fig. 6 we display the behavior of the masses of the particles as a function of the quark masses (these results have been taken from ref. (11)).

In order to give the masses of the particles in physical units one must fix a fundamental (strong interaction) scale and a mass parameter for each quark flavour. At fixed β we could set the scale from the mass difference between the ϱ and the π (or from Λ_{MOM} measured in deep inelastic scattering or from the Regge slope) and then assign a mass to the quarks by fixing the π, K, ... meson masses. In the Wilson formulation:

$$m^2_\varrho - m^2_\pi = \frac{1}{a^2} f(K,\beta), \qquad m^2_\pi = \frac{1}{a^2} g(K,\beta). \tag{31}$$

f and g, which govern the behavior of m^2_ϱ and m^2_π as a function of K are measured in the Montecarlo experiment. For a given value of β, K is fixed by solving eqs. (31). Once that m_ϱ and m_π are fixed from eqs. (31) all other particle masses Λ_{MOM}, the Regge slope ...

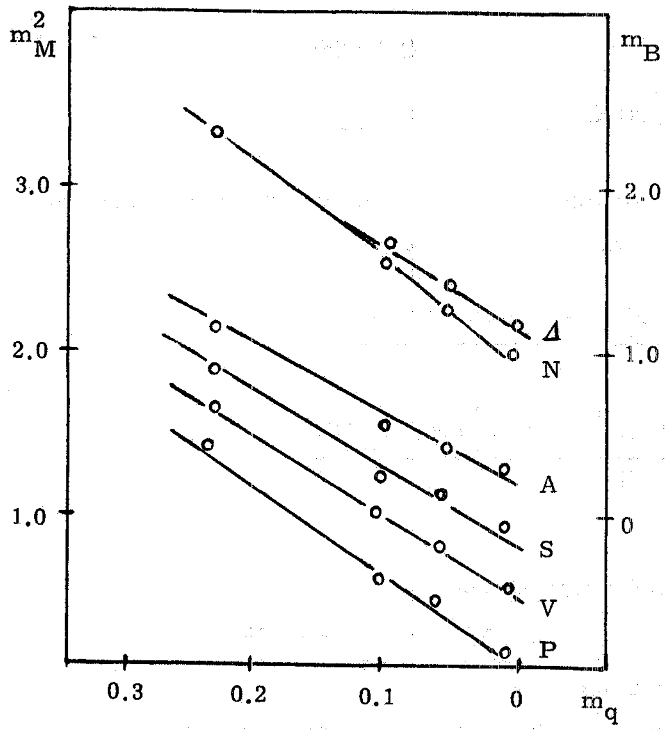


FIG. 6 - Hadron masses as functions of the quark masses in dimensionless units. For the mesons we plot the squared masses (scale on the left) and for baryons the masses (scale on the right). From below the experimental points and curves for the pseudoscalar, vector, scalar and axial mesons and the $1/2^+$ and $3/2^+$ baryons are reported. These results are taken from ref. (11).

are predictions of the theory. We expect all these quantities to scale in a predicible way with the cutoff a.

To have under control the renormalization group behavior (that is the $a \to 0$ limit) one should be able to work at several values of β. This comes out to be rather difficult because the inverse masses (i.e. the correlation lenghts) increase exponentially with β and very soon very serious finite volume problems come in. On the other hand, for too small values of β, we are far from the weak coupling regime and the lattice becomes too coorse grained to obtain sensible results. At present this problem, common to almost all lattice measurements (e.g. to the case of the string tension) certainly requires further investigations.

SECTION 5.

In this section I will briefly list and discuss the results recently obtained by several groups using the methods illustrated in Section 4:

a) E. Marinari, G. Parisi and C. Rebbi(12).

- Discrete SU(2) with the "naive" fermion action (eq. (3));
- Lattices $8^3 \times 16$ (16 in the time direction) and $8^3 \times 32$;
- Particle propagators averaged over 4 gauge configurations;
- They measured $\langle \psi \psi \rangle_{m_q=0}$ and checked the renormalization group behavior for this quantity;

- At $\beta = 2$ (corresponding to $a^{-1} = 1200$ MeV) they obtained:

$$f_{\pi} = 150 \pm 10 \text{ MeV}, \quad m_{\varrho} = 800 \pm 80 \text{ MeV}, \quad m_{\delta} = 950 \pm 100 \text{ MeV}, \quad m_q = 7 \text{ MeV};$$

- No renormalization group behavior check was made for the masses;
- The total computer time spent in this experiment was $\sim$10 h CDC 7600 CPU time.

b) D.H. Weintgarten[13]:
- Discrete SU(2) with Wilson fermions;
- Lattice size fixed in physical units for different values of β ranging from 1.05 to 2.4; the maximum size was 12^4;
- 8 gauge field configurations;
- The measured critical value of the hopping parameter K_c, corresponding to the value at which the mass of the pion becomes zero, $K_c \gtrsim 0.23$ is suspiciously near to its strong coupling value[*];
- The only quantity that was measured was the mass of the ϱ-meson. He found $m_{\varrho} = 800 \pm 100$ MeV;
- He spent $\sim$65 h CDC 7600 CPU computer time.

c) H. Hamber and G. Parisi[11]:
- SU(3) with "naive" and Wilson fermions;
- $6^3 \times 10$ lattice;
- The number of gauge field configurations used to compute the particle propagators was different (6-25) for different values of the quark masses;
- They measured $\langle \bar{\psi} \psi \rangle_{m_q=0}$ (for the "naive" action) and checked its renormalization group behavior; the results are in agreement with theoretical expectations;
- At $\beta = 6$ ($a^{-1} = 1120$ MeV) with Wilson fermions they obtained:

$$m_{\varrho} = 800 \pm 100 \text{ MeV}, \quad m_{\delta} = 1000 \pm 100 \text{ MeV}, \quad m_{A1} = 1200 \pm 100 \text{ MeV},$$

$$m_P = 950 \pm 100 \text{ MeV}, \quad m_{\Delta} = 1300 \pm 100 \text{ MeV}, \quad m_{u,d} \sim 3\text{-}5 \text{ MeV}.$$

- They spent $\sim$25 h CDC 7600 CPU computer time.

Besides these published papers another Montecarlo experiment has been initiated at CERN and Rome[14]. We are working with parameter similar to those of ref. (11).
- SU(3) with Wilson fermions; $\beta = 6$;
- $5^3 \times 10$ lattice;
- 32 gauge field configurations for each value of the quark masses at which we want to measure hadron masses.

(*) - After the end of this Meeting I heard that there is a revised version of the paper to be published on Physics Letters B.

The analysis has not been completed, however with the experience allready gained, we believe that, at least for Wilson fermions, there was an underextimation of statistical errors, probably due to a strong correlation among different gauge field configurations, in previous papers. A more carefull analysis of the systematical errors (beside the usual finite volume effects) is also needed because it appeared rather hard, with actual lattice sizes, to isolate in the propagator the low lying states poles from higher mass excitations with the same propagating quantum numbers simultaneously.

A completely different strategy based on the hopping parameter expansion has been carried out by the authors of ref. (10). At present their result show that this method works reasonably well for the meson spectrum but can have serious problem in the baryon sector.

CONCLUSION

Many important and exciting progresses in computing hadron spectroscopy from lattice QCD have been done in the last few months. A better understanding of these results from Montecarlo simulation is needed. I believe that in a short time it will be possible to obtain many physical predictions : among the others G_A/G_V and the anomalous magnetic moment. These techniques will be hopefully applied to new theories which may require the knowledge of non perturbative dynamics like supersymmetry.

ACKNOWLEDGMENTS

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