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G. Martinelli: HADRON SPECTROSCOPY AND THE COMPUTATION OF THE PROTON AND NEUTRON ANOMALOUS MAGNETIC MOMENTS IN LATTICE QCD

Invited talk at the "VI<sup>th</sup> Workshop on Current Problems in High Energy Particle Theory",  
Firenze, June 2-4, 1982.

G. Martinelli: HADRON SPECTROSCOPY AND THE COMPUTATION OF THE PROTON AND NEUTRON ANOMALOUS MAGNETIC MOMENTS IN LATTICE QCD.

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## 1. - INTRODUCTION

We report the results recently obtained by Montecarlo simulation by a CERN-Rome collaboration<sup>(1, 2, 3)</sup> on hadron spectroscopy and on the proton and neutron anomalous magnetic moments.

In Section 2 we give all the relevant informations on our Montecarlo experiments; in Section 3 we discuss in detail possible sources of statistical and systematic errors; having in mind these problems, in Section 4 we describe the computation of the anomalous magnetic moments.

Other results in lattice QCD with fermions can be found in refs. (4).

## SECTION 2

We start by describing our "experimental apparatus", that is the parameters chosen for this Montecarlo experiment. We worked with SU(3) coloured gluons and quarks, the fermion were put à la Wilson<sup>(\*)</sup> on a  $5^3 \times 10$  (10 in the time direction) lattice with periodic boundary conditions; the fermionic determinant was taken equal to 1, corresponding to the so called "quenched" approximation and the quark propagator was computed using the Gauss-Seidel iterative equation<sup>(5)</sup>. All the measurements were taken only at one value of the lattice

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(\*) - For a discussion of different lattice fermion actions see, for example, the talk by P. Hasenfratz at this Workshop.

coupling constant,  $g_0 = 1$ , in order to have a reasonably large statistics. In this way however we could not check the renormalization group behavior for varying  $g_0$ . We measured meson and baryon masses from 32 gauge field configurations at each of the values of the quark masses we considered, corresponding to values for the hopping parameter  $K = 0.130, 0.145, 0.1475, 0.150$  and  $0.1525^{(x)}$ . These 32 gauge field configurations were generated by usual Montecarlo techniques (Metropolis algorithm) after an initial 500 sweeps thermalization. Two successive configurations were always separated by 100 Montecarlo sweeps. The experiment took  $\sim 90$  h CDC 7600 CPU time.

To compute the masses we defined operators carrying the same quantum numbers of the particles which we want to study. For example for the pseudoscalar and vector mesons and for the spin  $1/2^+$  and  $3/2^+$  baryons we used<sup>(6)(o)</sup>:

$$\begin{aligned} \pi^+(x) &= \bar{u}^A(x) \gamma_5 d^A(x), \\ \rho_\mu^+(x) &= \bar{u}^A(x) \gamma^\mu d^A(x), \\ P_\delta(x) &= \left[ u^A(x) \mathbf{C} \gamma_5 d^B(x) \right] u_\delta^C(x) \in_{ABC}, \\ \Delta_{\mu,\delta}^{++}(x) &= \left[ u^A(x) \mathbf{C} \gamma_\mu u^B(x) \right] u_\delta^C(x) \in_{ABC}. \end{aligned} \quad (1)$$

$\mathbf{C}$  is the charge conjugation operator. Then we measured, averaging over the link configurations, the correlation functions of these operators. For example the pion correlation function:

$$G(x) = \frac{\sum_{\text{link configurations } \{U\}} \text{tr} \left[ G^u(0, x, \{U\}) \gamma_5 G^d(x, 0, \{U\}) \gamma_5 \right]}{\sum_{\text{link configurations } \{U\}} 1}. \quad (2)$$

Remember that the fermion determinant is put equal to 1.  $G^{u,d}(0, x, \{U\})$  is the up (down) quark propagator between the lattice points  $0, x$  in presence of an external gauge field  $\{U\}$ . The r.h.s. of eq. (2) is diagrammatically represented in Fig. 1.

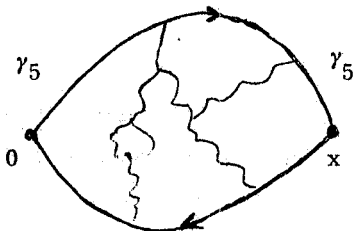


FIG. 1 - Typical meson (pion) diagram in the "quenched" approximation.

- (x) - The data at  $K = 0.130$ , being to far from the physically interesting region, will not be considered in the following.
- (o) - Note that the vector meson and spin  $3/2^+$  operators have too many components and a projection operator on the appropriate states must be applied.

We put the spatial components of the momentum equal to zero in Fourier space by summing over the spatial coordinates  $\vec{x}$ :

$$G(t) = \sum_{\vec{x}} G(\vec{x}, t) . \quad (3)$$

On general grounds, for periodic boundary conditions, we expect that if there is a pole corresponding to one meson/baryon state the propagator will have the form:

$$\begin{aligned} G_M(t) &\sim \cosh \left[ m(t - T/2) \right] , \\ G_B(t) &\sim (1 + \gamma_0) \left\{ C_+ \exp \left[ -m_+ t \right] + C_- \exp \left[ -m_-(T - t) \right] \right\} + \\ &\quad + (1 - \gamma_0) \left\{ C_+ \exp \left[ -m_+(T - t) \right] + C_- \exp \left[ -m_- t \right] \right\} \end{aligned} \quad (4)$$

$T$  is the period in the time direction,  $m$  is the meson mass and  $m_{\pm}$  are the masses of opposite parity baryons: with periodic boundary conditions, each time one has a baryon (let us say the proton) propagating forward in time, one has simultaneously an opposite parity state (the  $N^*(1535)$ ) propagating in the backward direction<sup>(o)</sup>.

In real cases many other higher mass states can propagate in the same channel and it is rather difficult, because of the smallness of the lattice in the time direction, to isolate the low lying states to which we are interested. In order to give an estimate of the effect of these states one can parametrize the meson propagator in the following way:

$$G_M(t) = a_1 \cosh \left[ m_1(t - T/2) \right] + a_2 \cos \left[ m_2(t - T/2) \right] \quad m_2 > m_1 \quad (5)$$

and analogously for the baryon propagator. One should not necessarily give to  $m_2$  the meaning of the mass of a real particle. To show how eqs. (4) and (5) work, in Fig. 2 we display the pion-like correlation function, averaged over all 32 configurations at  $K = 0.1475$ . The crosses represent the experimental points; the full line is the fit with the first of eqs. (4) over the five central points. The dashed line is the fit from eq. (5) over all the points. It is clear from the figure that it is not possible to fit the propagator with the contribution of a single particle state. For mesons, the systematic error induced by the presence of higher excited states was estimated to  $\sim 12\%$ . These effects give always an over estimate of the measured value of the mass.

For baryons the situation is worse: we have at least two low lying opposite parity states propagating and it is rather difficult to separate them in the central region,  $t \sim T/2$ , where the baryon pole could be more easily separated from higher excitations.

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(o) - This can be avoided by using free boundary conditions.

Moreover the mass difference of the first excitation from the lowest mass baryon is rather small (contrary to the meson case) and only far from the origin they can be clearly separated. With the limitations in size imposed to our lattice in the time direction we never see a clear signal of the propagation of a single particle. This is shown in Fig. 3 where the

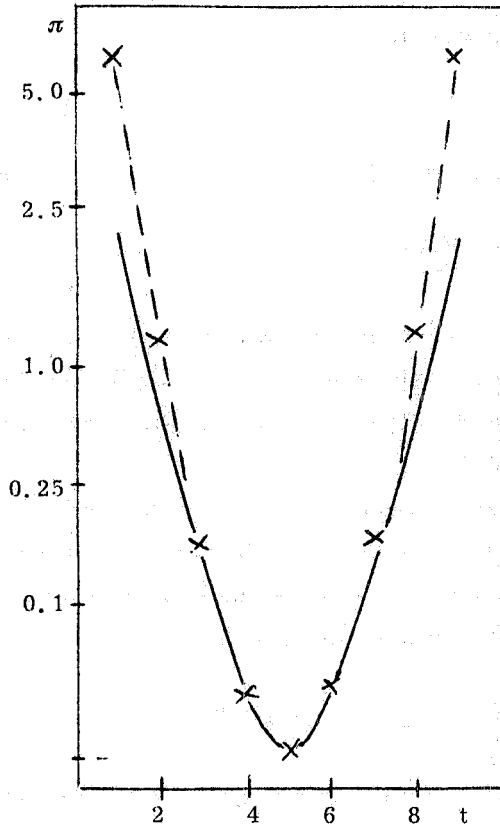


FIG. 2 - The pion propagator at  $K=0.1475$  as a function of  $t$ . The crosses are the experimental points. The full and dashed lines are obtained from the first of eqs. (4) (only the five central points) and eq. (5) respectively.

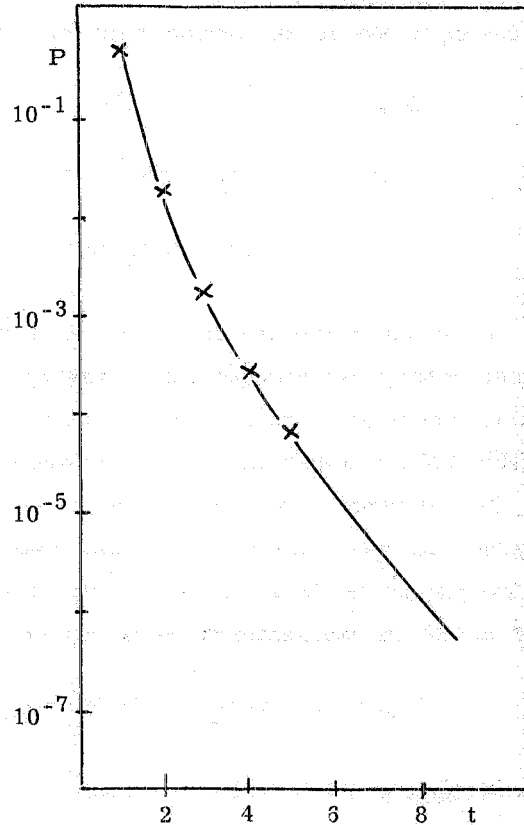


FIG. 3 - The proton propagator for  $t \leq 5$  at  $K=0.1475$ ; the full line is the fit using the second of eqs. (4) and including also the effects of higher mass states.

experimental points (crosses) for the proton-like correlation function are displayed for  $t = 1 \rightarrow 5$ . The full curve is the fit with the second of eqs. (4) including extra terms coming from excited states. We see that the proton pole, corresponding to a straight line behavior of the propagator on a logarithmic scale, stems out only for  $t \geq 4-5$  where, for  $T=10$ , the opposite parity states start to propagate. For baryons, we estimated the systematic error  $\sim 20\%$ . It should also be noticed that for baryons we have few points and many parameters whose fluctuations give a higher statistical error than in the meson case, being the covariance matrix rather flat.

SECTION 3

A surprising result we found in our experiment was the strong correlation ( $\sim 400-500$  Montecarlo sweeps) among different link configurations as shown in Fig. 4. The origin of



FIG. 4 - The squared mass of the  $\rho$ -meson time the lattice spacing  $a$  at  $K=0.1475$  as a function of the total number of Montecarlo sweeps. Analogous curves could be drawn for the other particles.

this correlation is not presently understood<sup>(x)</sup>. It is clear from the figure that we do not have really 32 independent measurements. In order to give an estimate of the statistical errors we divided the full set of configurations in different clusters (2, 3, 4), averaged the particle propagators and measured the particle masses using eqs. (4) and (5) for each cluster and finally computed the mass and dispersion by averaging over the clusters. The results are displayed in Fig. 5. The full lines are given by a linear fit over the points  $K=0.145, 0.1475, 0.150$  of the form :

$$\begin{aligned}
 m_M^2 &= a_M \frac{1}{K} + b_M && \text{for mesons,} \\
 m_B &= a_B \frac{1}{K} + b_B && \text{for baryons.}
 \end{aligned}
 \tag{6}$$

One observes that all the points, but those at  $K = 0.1525$ , are well fitted by eqs. (6). The points for  $K \geq 0.1525$  are affected by strong finite volume effects coming from the fact that the pion

(x) - There is a suggestion by P. Hasenfratz that it could be due to finite temperature effects.

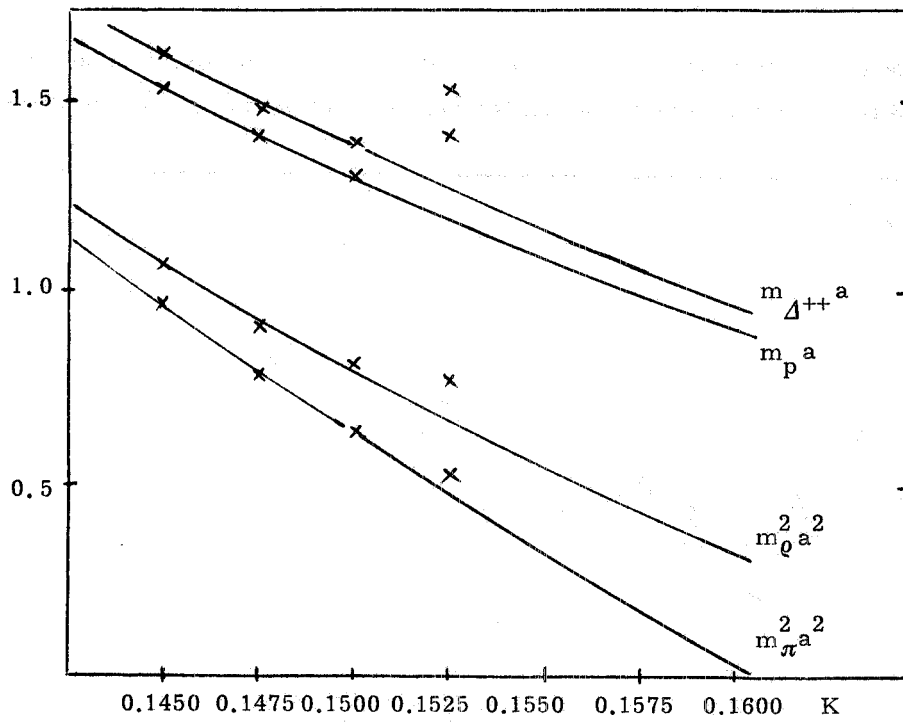


FIG. 5 - Hadron masses ( $\pi$ ,  $\rho$ , p and  $\Delta^{++}$ ) as a function of K. The crosses are the experimental points. The linear fits (see text) are plotted as full lines.

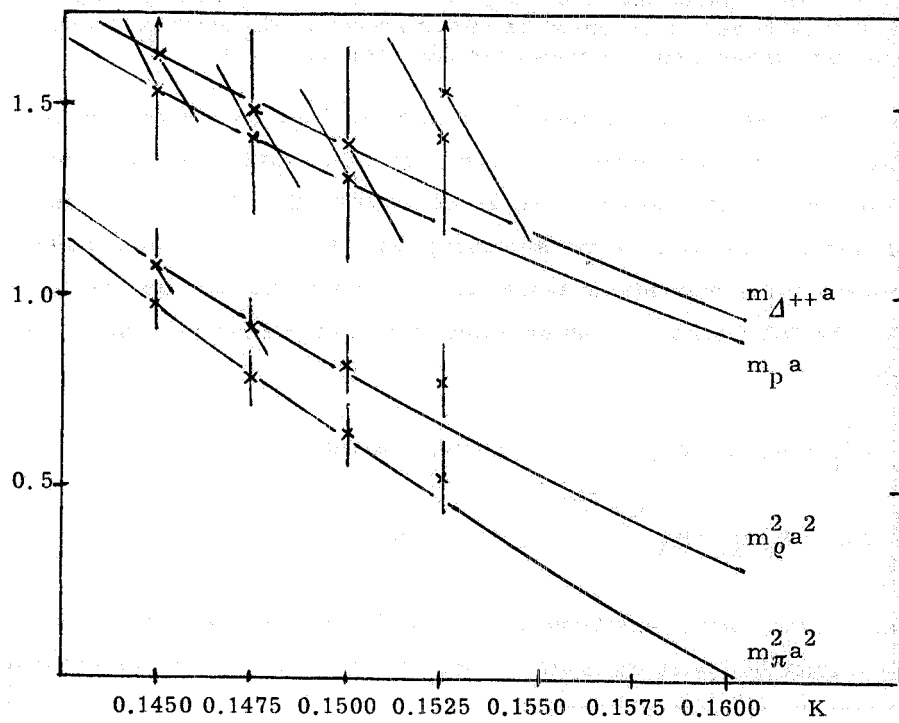


FIG. 6 - Same as in Fig. 5 but with statistical errors on the experimental points.

is becoming massless for increasing  $K$ . We experimentally found sensible results only when  $1/m_\pi = 1/m_\pi \leq Na/3$ , where  $Na$  is the lattice size ( $a$  is the lattice spacing)<sup>(o)</sup>. Note that for  $K \geq 0.1525$  also the Gauss-Seidel method to compute the quark propagator starts to fail. The points at 0.1525 will not be used in the following.

With the statistical errors one gets at  $K = 0.145-0.150$  (see Fig. 6), by extrapolating to  $K \sim 0.160$  which corresponds to the physically interesting region, one would obtain errors greater  $\sim 100\%$  for the masses. There is however a solution: if, cluster by cluster one first extrapolates and then averages over the clusters, the statistical error is enormously reduced. One gets an error  $\sim 10\%$  for meson and  $\sim 30-40\%$  for baryon, the reason being that, using the same configurations at different  $K$ 's, there are coherent fluctuations of the values of the masses with an (almost) fixed slope in  $K$ , as shown in Fig. 7.

It is crucial to have reasonable statistical errors to work with the same configurations at different  $K$ .

It is some time convenient for quantities which are experimentally known to be only slightly dependent on quark masses, to measure them directly at the accessible  $K$ 's: this method has been applied as an alternative way of measuring the masses for  $m_{\Delta^{++}}/m_\rho$ ,  $m_p/m_\rho$ ,  $m_\Delta^2 - m_p^2$ . We expect that results obtained by different averages, will be, inside statistical errors compatible.

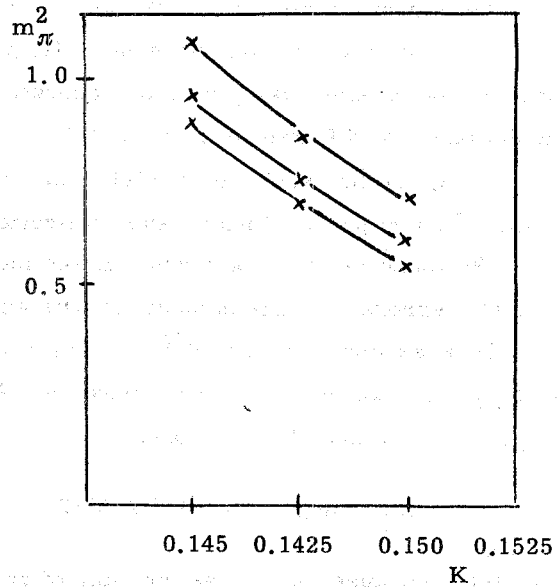


FIG. 7 - The mass of the pion for 3 different clusters is plotted against  $K$ . The lines are drawn to guide the eyes. We see that the slope in  $1/K$  is almost the same for all the clusters.

To set the strong interaction scale and the quark masses we fixed  $m_\rho^2 - m_\pi^2$  and  $m_\pi^2$ ,  $m_K^2$  ... respectively. Our final predictions are<sup>(1)</sup>:

$$\begin{aligned}
 \Lambda_{\overline{MS}} &= 75 \pm 8 \text{ MeV} \\
 m_p &= 1.27 \pm 0.44 \text{ GeV} & 1.14 \pm 0.18 \text{ GeV} \\
 m_{\Delta^{++}} &= 1.37 \pm 0.60 \text{ GeV} & 1.21 \pm 0.19 \text{ GeV} \\
 m_\rho &= 1.68 \pm 0.50 \text{ GeV} & 1.58 \pm 0.24 \text{ GeV} \\
 m_{K^*} &= 0.89 \pm 0.07 \text{ GeV}
 \end{aligned} \tag{7}$$

(o) - This seems to be confirmed by preliminary results at the same  $g_0$  but with a different lattice size ( $8^3 \times 16$ ) from a work in progress at Edinburgh.



$$\begin{aligned}
 m_{\Phi} &= 0.99 \pm 0.05 \text{ GeV} \\
 \frac{m_{N^*}}{m_p} &= 1.65 \quad (\text{exp } 1.65) \\
 \frac{m_{\Delta(3/2^-)}}{m_{\Delta(3/2^+)}} &= 1.57 \quad (\text{exp } 1.36) \quad (7) \\
 m_{\Delta} - m_p &= 0.22 \pm 0.09 \text{ GeV} \\
 \hat{m}_u - \hat{m}_d &= 7.3 \pm 1.0 \text{ MeV} \\
 \hat{m}_s &= 188 \pm 11 \text{ MeV}
 \end{aligned}$$

We measured also the  $\delta$ , B,  $A_1$  ( $L=1$ ) masses. However we were unable to give any prediction for these mesons because of large statistical fluctuations. Perhaps an improvement can be obtained using extended operators (i. e. operators which create the quark and the antiquark at different lattice sites)<sup>(x)</sup>

In eq. (7) the results on the left side are obtained by using the second method described above.  $\hat{m}_q$  are the continuum renormalization group invariant quark masses<sup>(7)</sup>.

We found rather good results in the meson sector and, within larger statistical and systematic errors, results compatible with experimental values for baryons.

In a second experiment<sup>(2)</sup> we tried to estimate mass terms coming from the  $SU(3)$  flavour explicit symmetry breaking. Among the others we measured the mass splitting between  $\Sigma$  and  $\Lambda^0$ . We found:

$$m_{\Sigma} - m_{\Lambda^0} \approx 18 \pm 9 \text{ MeV} \quad (\text{exp } \sim 77). \quad (8)$$

Note that, because the coarse graining of our lattice, spin-spin forces, responsible for the  $\Delta^{++}$ -p and  $\Sigma$ - $\Lambda^0$  mass splittings, although of the correct sign, are systematically underestimated.

In conclusion strong correlations among different configurations, difficulties in isolating low lying states and necessity of taking measurements far from the physically interesting ( $m_{\pi} \sim 0$ ) region all originates from the small size of available lattices.

#### SECTION 4

I will report in this section the results recently obtained in ref. (3) for the proton and neutron anomalous magnetic moments. The experimental conditions are those described in Section 2.

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(x) - See the talk by Z. Kuntz at this Workshop.

To measure the magnetic moment we put an uniform time-independent magnetic field on the lattice and measured the proton/neutron mass in presence of this magnetic field. If the field is weak enough the mass of a Dirac particle in presence of an external magnetic field  $H$  is given by:

$$m_n = m_0 + \frac{e|\vec{H}|}{m_0} \left(n + \frac{1}{2}\right) + \vec{\mu}\vec{H}. \quad (9)$$

$m_0$  is the mass in absence of the field;  $e$  is its electric charge;  $n$  is an integer labelling different Landau levels and  $\vec{\mu}$  is the magnetic moment. The measurement of the magnetic moment is thus reduced to an usual mass measurement as described in Section 2.

To put the magnetic field on the lattice one may replace the colour links  $U_\mu(x)$  by:

$$\begin{aligned} U_\mu(x) &\rightarrow U_\mu(x) \times U_\mu^{\text{ext}}(x) \\ U_{x,y,z}(x) &= 1 \quad U_y(x) = e^{iax}. \end{aligned} \quad (10)$$

This corresponds to an uniform magnetic field directed along the  $z$ -axes ( $\vec{H} = H\hat{z}$ ) with strenght  $eH = a/a^2$ . With periodic boundary conditions ( $F(x+N_x, y\hat{\mu}_{x,y}) = F(x)$ ; where  $\hat{\mu}_{x,y}$  and  $N_x = N_y = N$  are the unit vectors and periods in the  $x$ - $y$  directions) the minimum possible magnetic field is:

$$H = \frac{1}{e_f} \frac{2\pi}{(N+1)^2 a^2}. \quad (11)$$

$e_f$  is the electric charge of the quark of flavour  $f$  coupled to the external field. This corresponds with our lattice to a shift in the mass of the baryon:

$$\delta m = \mu H \sim 0.5 - 0.6 \text{ GeV} \quad (12)$$

and eq. (9), valid for weak fields is no more valid.

To be able to switch continously the strenght of  $H$ , we broke the periodicity of the lattice in the  $x$ -direction by putting:

$$U_x(N_x) = 0. \quad (13)$$

The finite volume effects on the baryon masses induced by these boundary conditions are expected to be small ( $\sim 10\%$ ) and even smaller for the computation of the giromagnetic factor.

We looked to the region were the baryon masses are linear in  $H$ . Let us define:

$$R(t) = \left. \frac{G(\uparrow) - G(\downarrow)}{G(\uparrow) + G(\downarrow)} \right|_t \quad (14)$$

$G(\uparrow, \downarrow)$  is the spin-up, spin-down propagator at distance  $t$  as defined in eq. (3). For small  $H$  we expect :

$$\bar{R}(t) = R(t) - R(t-1) \approx \mu H. \quad (15)$$

In Fig. 8  $R(t)$  is displayed for a particular link configuration as a function of  $H$ . We chose  $H = 0.075$ , well inside the linear region to measure the magnetic moment. Because of the limited precision (to save computer time) in computing the quark propagator ( $\sim 10^{-2}$ ) we did not take a smaller magnetic field.

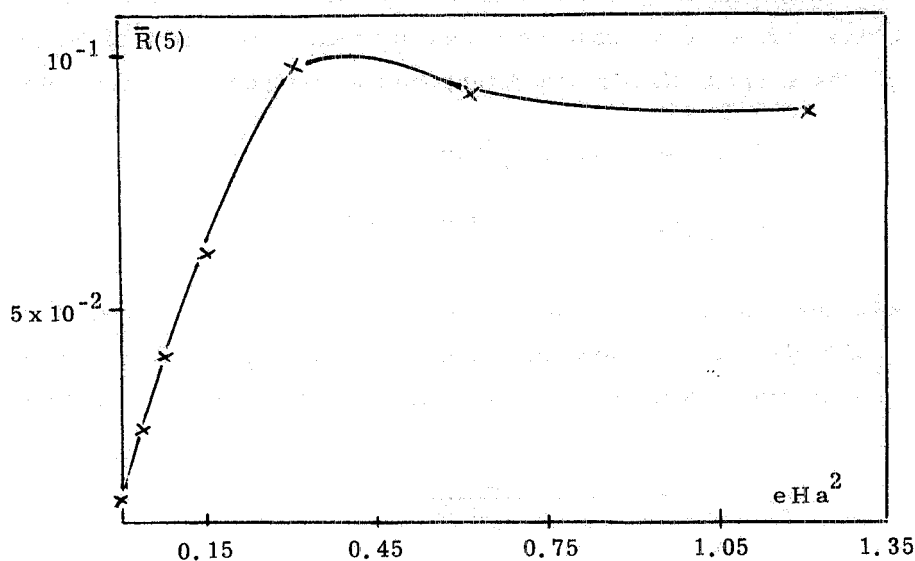


FIG. 8 -  $\bar{R}(5)$  is plotted against the magnetic field  $H$ . For large  $H$  one expects  $\bar{R}(5)$  to go as the difference of two hyperbolic tangents. Note that because of the asymmetry for only one configuration with respect to a  $z$ -axis reflection,  $\bar{R}(5)$  is not vanishing when  $H = 0$ .

We measured the proton and neutron giromagnetic factors defined as :

$$g_{P,N} = \frac{2 m_{P,N}}{e} \mu_{P,N} \quad (16)$$

$m_{P,N}$  is the measured baryon mass and  $\mu_{P,N}$  is the magnetic moment measured using eq. (15). At  $t=5$  we obtained :

$K$	$g_P$	$g_N$
0.145	$2.89 \pm 0.29$	$-1.87 \pm 0.24$
0.1475	$3.09 \pm 0.53$	$-2.02 \pm 0.42$
0.150	$3.33 \pm 0.93$	$-2.21 \pm 0.68$

(17)

From the discussion of section 2 we know that the approximation of considering only one particle propagating in some channel is rather rough. To see how the results (17) are affected by the presence of higher excited states we measured  $g_{P,N}$  at distance  $t=4$ . The results change of about 5%.

Because of the rapidly increasing statistical errors with  $K$  it is not possible to evaluate reasonably the  $K$ -dependence for  $g_{P,N}$  from our data. This dependence appear to be rather small with respect to what naively expected on the basis of the quark model ( $g_{P,N} \sim m_{P,N}/m_q$ ) in the range of quark masses we considered.

Averaging the values of  $g_{P,N}$  over  $K$  weighted with their statistical errors we find:

$$\begin{aligned} g_P &= 2.96 \pm 0.58 && (\text{exp} \sim 2.79) \\ g_N &= -1.93 \pm 0.45 && (\text{exp} \sim -1.90) \\ |g_P/g_N| &= 1.60 \pm 0.15 && (\text{exp} \sim 1.47) \end{aligned} \tag{18}$$

in remarkable agreement with experimental values.

The technique used in the computation of the anomalous magnetic moment can be easily extended to the computation of other form factors.

For completeness we report the results obtained in ref. (8) for  $G_A/G_V$ :

$$\begin{aligned} g_F &= 0.52 \pm 0.05 && (\text{exp} \sim 0.44) \\ g_F - g_D &= -0.32 \pm 0.03 && (\text{exp} \sim 0.37) \\ g_D/g_F &= 1.62 && (\text{exp} \sim 1.84) . \end{aligned} \tag{19}$$

#### ACKNOWLEDGMENTS

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