ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-82/38

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Estratto da:
Phys. Letters 113B, 113 (1982)

Servizio Documentazione
dei Laboratori Nazionali di Frascati

SINGLE PARTICLE WAVE FUNCTIONS OF $\sigma-\tau$ PHASES IN NUCLEAR MATTER

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Received 29 January 1982

$\sigma-\tau$ phases in nuclear matter are characterized by one-dimensional crystallization. Taking this into account in the choice of spwf results in an energy gain of a few MeV per nucleon at normal density.

There have been quite recently two new attempts [‡1] to relate $\sigma-\tau$ phases to the phenomenology of nuclei. First a non-static realization of $\sigma-\tau$ order has been proposed [2], whose signature is the existence of an unsplit isovector M2 resonance [3] in light deformed nuclei. Preliminary results in ^{12}C and ^{28}Si seem to support this prediction [4].

The second attempt [5] is to explain the cross section of anomalous nuclei which exceeds the geometrical bound [6] by a metastable $\sigma-\tau$ phase. This last attempt is merely speculative due to the practical impossibility, at present, of a reliable calculation.

A step towards such a calculation is the assessment of the value of the critical density for $\sigma-\tau$ phases in nuclear matter. There is still considerable uncertainty on this point. In the context of Migdal's approach [7] it is generally accepted that the critical density is at least twice the normal density. It has been shown, however, that the RPA gives a non-convergent result [8], due to a mechanism already known in the theory of effective interactions [9]. Moreover, it has been emphasized [10] that the results obtained in the framework of Migdal's approach are relevant only to a second-order phase transition.

In the potential approach [11,12] the phase transition was actually found to be of first order. At the critical density, in fact, the Fermi surface changes abruptly from a spherical to a cylindrical shape and nuclear matter undergoes a one-dimensional crystallization.

Evaluation of the critical density in this case is much more difficult because it is necessary to compare the energy of the normal phase to that of the $\sigma-\tau$ phase. But in this phase the short-range correlations (src) must be anisotropic and $\sigma-\tau$ dependent.

A first attempt to introduce src with a realistic potential has been made by Sandler and Clark [13]. These authors found that there is no phase transition at all. Their result, however, cannot be considered as conclusive because:

(i) They use a spherical Fermi surface rather than a cylindrical one.

(ii) Their src are isotropic and $\sigma-\tau$ independent.

(iii) Their sp wave functions are not the best ones as explained below.

(iv) There are no three-body potentials corresponding to the Δ-resonance in Migdal's approach.

Benhar has overcome point (i) in a calculation on neutron matter [14] and is presently doing [15] a calculation for nuclear matter with a cylindrical Fermi

‡1 For the previous ones see ref. [1]

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surface using the hypernetted chain method.

In refs. [13–15] the formalism of ref. [12] is used, where the spwf are not the best ones, as observed by Tamagaki and collaborators [16]. The purpose of this note is to modify the formalism of ref. [12] in order to take into account Tamagaki's observation and to perform again the numerical calculations in order to show its effect. Taking into account this modification will also allow a comparison between the results of the forthcoming papers which introduce Jastrow correlation functions in ref. [12] and those of the Japanese school [17,18] which use a Brueckner approach.

In ref. [12] a calculation of the energy per particle in nuclear matter has been done with the OBEP of the Ueda and Green School. The variational parameters are the Fermi momentum in the z direction k_{zF}, and a parameter α which determines the amplitude of density fluctuations. Src have not been included, so that the energy per particle turned out to be positive, but local minima have been found already at normal density.

In the following we use the notations of ref. [12] with minor obvious changes. The orbital spwf used in that work for phase 6 are

$$\psi_{\boldsymbol{k}_T,k_z,\sigma_3,\tau_3} = L^{-3/2}\exp(i\boldsymbol{k}_T\cdot\boldsymbol{r}_T)\exp(ik_z z)\chi_{\sigma_3\tau_3}(z), \tag{1}$$

$$\chi_{\sigma_3\tau_3}(z) = \chi(z - \tfrac{1}{4}l\,\sigma_3\tau_3 - \tfrac{1}{4}l), \quad l = \pi/k_{zF}, \tag{2}$$

$\chi(z)$ being a periodic function with period l and normalization

$$\int_{-l/2}^{l/2} dz|\chi|^2 = l. \tag{3}$$

With wf (1) the average kinetic energy per nucleon is

$$\bar{T}/A = \tfrac{3}{5}(\hbar^2k_F^2/2m)\tfrac{5}{9}k_F/k_{zF} + t + \tfrac{1}{3}\hbar^2k_{zF}^2/2m, \tag{4}$$

with

$$t = -\int_{-l/2}^{l/2} dz\, l^{-1}\chi^*(\hbar^2\Delta/2m)\chi. \tag{5}$$

As already said, in the original work [11] a complete crystallization along the z axis was found. This result was confirmed in ref. [12] where, however, due to the limited flexibility of the trial wave functions used, the localization was well-defined but not complete (the wf χ was very small but not vanishing on the boundary of the unit cell). As observed by Tamagaki and coworkers [16], once the localization is well-defined, the term e^{ik_zz} in eq. (1) can be dropped with the consequent elimination of the last term in eq. (4). This gives rise to a considerable gain in kinetic energy. For instance, in the calculation of Sandler and Clark [13], this term takes a value of about 13 MeV for balanced nuclear matter at normal density and complete localization (their parameter $\alpha = 1$).

In order to get rid of the superfluous factor e^{ik_zz} (in the presence of complete localization), let us define the wf

$$\varphi_n(z) = \chi(z)\,\theta(\tfrac{1}{2}l - z - nl)\theta(\tfrac{1}{2}l + z + nl), \tag{6}$$

where $\theta(z)$ is the step function. According to Tamagaki [16] the wf (1) can now be replaced by

$$\psi_{\boldsymbol{k}_T,n,\sigma_3,\tau_3} = L^{-1}\exp(i\boldsymbol{k}_T\cdot\boldsymbol{r}_T)\,l^{-1/2}\varphi_{n\sigma_3\tau_3}(z), \tag{7}$$

$$\varphi_{n\sigma_3\tau_3}(z) = \varphi_n(z - \tfrac{1}{4}l\,\sigma_3\tau_3 - \tfrac{1}{4}l). \tag{8}$$

It is more convenient for us to use the Block wave function obtained from (7) by a unitary transformation

$$\psi'_{\boldsymbol{k}_T,k_z,\sigma_3,\tau_3} = L^{-3/2}\exp(i\boldsymbol{k}_T\cdot\boldsymbol{r}_T) \times \sum_{n=-L/2l}^{L/2l}\exp(ink_zl)\,\varphi_{n\sigma_3\tau_3}(z). \tag{9}$$

The one-body density matrix is now

$$F_{\sigma_3\tau_3}(\boldsymbol{r}_1,\boldsymbol{r}_2) = \frac{1}{(2\pi)^2}\int d\boldsymbol{k}_T\exp[i\boldsymbol{k}_T(\boldsymbol{r}_{2T} - \boldsymbol{r}_{1T})] \times \frac{1}{l}\sum_n \varphi_{n\sigma_3\tau_3}(z_2)\varphi^*_{n\sigma_3\tau_3}(z_1), \tag{10}$$

to be compared with eq. (2.13) of ref. [12] which, rewritten in terms of ψ', is

$$F_{\sigma_3\tau_3}(\boldsymbol{r}_1,\boldsymbol{r}_2) = \frac{1}{(2\pi)^2}\int d\boldsymbol{k}_T\int dk_z\exp[i\boldsymbol{k}_T(\boldsymbol{r}_{2T} - \boldsymbol{r}_{1T})] \times \frac{1}{l}\exp[ik_z(z_2 - z_1)]\sum_{nn'}\varphi_{n\sigma_3\tau_3}(z_2)\varphi^*_{n'\sigma_3\tau_3}(z_1). \tag{11}$$

The direct part of the velocity-independent potential energy depends only on $F_{\sigma_3\tau_3}(\boldsymbol{r}_1,\boldsymbol{r}_2)$ and therefore does not vary, while the modification in the ex-

Table 1
ΔV, ΔT and ΔE are the average potential, kinetic and total energy per particle of the $\sigma-\tau$ phase with respect to the normal one. The new values of these quantities are reported in columns 5, 6 and 7, while the old ones are given in columns 10, 11 and 12.

k_F	phase	k_{zF} (fm^{-1})	α^{-1} (fm)	ΔV (MeV)	ΔT (MeV)	ΔE (MeV)	k_{zF} (fm^{-1})	α^{-1} (fm)	ΔV (MeV)	ΔT (MeV)	ΔE (MeV)
1.4	1	0.65	0.75	−32.47	23.49	−8.99	0.60	0.70	−36.44	30.92	−5.5
	6	0.85	0.80	−17.08	14.20	−2.89	0.90	0.90	−9.38	10.81	1.42
1.8	1	0.70	0.50	−91.04	58.99	−32.05	0.70	0.50	−88.54	62.09	−26.4
	6	1.10	0.63	−34.95	23.20	−11.75	1.00	0.70	−27.04	25.29	−1.7

change term is obtained by replacing the Pauli correlation function $J(k_T r_T) j_0(k_z z)$ by $J(k_T r_T)$. The changes in the velocity-dependent potentials are not reported here for the sake of brevity, but are included in the numerical computation.

Since our trial wave function, χ, although well-localized, does not vanish on the boundary of the unit cell, we modify it by replacing in eq. (6) the step function θ by

$$j(z) = 1, \quad \text{for } z > a,$$

$$= \tfrac{1}{2}\{1 + \cos[\pi(z-a)/a]\}, \quad \text{for } 0 \leqslant z \leqslant a,$$

$$= 0, \quad \text{for } z < 0. \tag{12}$$

This introduces the new variational parameter a. For the sake of simplicity, however, we have not varied a, but we have performed the variational calculation by requiring that for all k_{zF} and α

$$\int \mathrm{d}z\, |\chi(z)|^2 [\theta(\tfrac{1}{2}l - z)\theta(\tfrac{1}{2}l + z) - s(\tfrac{1}{2}l - z)\, s(\tfrac{1}{2}l + z)]$$

$$\leqslant 5 \times 10^{-3}. \tag{13}$$

The above condition has been chosen in order to allow some numerical approximations.

The calculations have been done for the phases 1 and 6 with the UG1 potential. The results, reported in table 1, along with the old ones show a sizeable energy gain, which becomes more pronounced the higher the density. In particular, for the phase 6 at normal density the local minimum becomes an absolute minimum.

These results show the importance of taking into account Tamagaki's observation. Since this does not change what is left of the kinetic energy after the suppression of the last term in eq. (4), does not change the velocity-independent potential term and changes very little the exchange potential term, inclusion of src cannot substantially alter our conclusion.

One of us (F.P.) is grateful to T.E.O. Ericson for a critical reading of the manuscript.

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