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A SIMPLE EXPRESSION FOR PLANAR FIELD THEORIES

Giorgio PARISI
Laboratori Nazionali INFN, Frascati, Italy
and II Università di Roma, Tor Vergata, Rome, Italy

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We argue that in a U(N) invariant theory in presence of a random background gauge field, the quenched expectation values of U(N) invariant objects are volume independent in the limit $N \to \infty$.

In a recent paper [1] it has been claimed that the partition function of a lattice U(N) gauge theory can be computed in the infinite-volume limit restricting the functional integral to only one site. In a more recent paper [2] it has been pointed out that the results of ref. [1] are not valid in the small-coupling region, but a modified form of the one-site model is proposed to coincide with the infinite-volume limit.

In this note the following general conjecture is proposed: let us consider a d dimensional field theory where the fields ϕ_i and M_{ij} belong to the fundamental and to the adjoint representation of U(N), respectively; if we consider a finite-size box of length L, we have to prescribe the boundary conditions; a generalization of the periodic boundary conditions is given by:

$$\phi_i(x + La_\nu) = \exp(\mathrm{i}\theta_i^\nu)\phi_i(x),$$

$$M_{ij}(x + La_\nu) = \exp(\mathrm{i}\theta_i^\nu + \theta_k^\nu)M_{ij}(x), \qquad (1)$$

a_ν being the vector in the ν direction.

A non-vanishing value of the θ's is equivalent to a background gauge field of $\mathrm{U}(1)^N$ type which is locally (not globally) a pure gauge.

The vacuum energy is obviously θ dependent:

$$E_\mathrm{L}(\theta) = -L^{-d} \times \ln\left[\int \mathrm{d}[\phi]\, \mathrm{d}[M]\, \exp\left(-\int_L \mathrm{d}^d x\, S(\phi, M)\right)\right]. \qquad (2)$$

The conjecture is that the quenched average [3]

$$E_\mathrm{L}^\mathrm{Q} = \int \mathrm{d}\theta_i^\nu\, E_\mathrm{L}(\theta) \qquad (3)$$

is L independent in the limit $N \to \infty$ both for the lattice and the continuum field theory. In particular on the lattice we can take $L = 1$ and reduce the theory to only one point.

This conjecture contains the result of ref. [2] as a particular case; it can be easily verified in the case of a scalar field theory.

In the simplest case the quenched one-link action is given by:

$$S = \tfrac{1}{2}|M_{ik}|^2\left(d - \sum_{\nu=1}^{d} \cos(\theta_i^\nu + \theta_k^\nu)\right) + \tfrac{1}{2}m^2\, \mathrm{tr}(MM^+) + (g/4N)\, \mathrm{tr}(MM^+MM^+). \qquad (4)$$

Let us concentrate our attention for definiteness on the propagator defined by

$$G(x) = N^{-1}\langle \mathrm{tr}\, M(x)\, \mathrm{tr}\, M^+(0)\rangle \qquad (5)$$

in the infinite-volume limit.

We want to prove that

$$G(x) = \frac{1}{N}\int \sum_{i,k} \langle M_{ii} M_{kk}^+\rangle_\theta\, \exp(\mathrm{i}2x_\nu\theta_i^\nu)\, \mathrm{d}\theta_i^\nu, \qquad (6)$$

which implies as a particular case that the one-site quenched average and the infinite-volume expectation values of $\mathrm{tr}\, M\, \mathrm{tr}\, M^+$ are equal.

In the free case the result is immediate; indeed:

sical Poisson brackets with commutators and by employing the concept of the wave section [10]. This is straightforward and is therefore omitted.

More detailed and extended discussions including quantum-mechanical calculations will be published elsewhere [11].

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