To be submitted to
Physics Letters

ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-82/33(P)
13 Maggio 1982

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COMPUTATION OF THE RELATION BETWEEN THE QUARK MASSES IN LATTICE GAUGE THEORIES AND ON THE CONTINUUM.

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COMPUTATION OF THE RELATION BETWEEN THE QUARK MASSES IN LATTICE GAUGE THEORIES AND ON THE CONTINUUM

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ABSTRACT

We compute the relation between the quark masses on the lattice and on the continuum for a generic fermion lattice action.

Very recently many promising results have been obtained for the hadron mass spectrum by Montecarlo simulations on a lattice(1-6). Among the others the values of the masses for the up, down, strange and charmed quarks have been evaluated.

On the other hand several extimates of the current quark masses have been given also using the Wilson operator expansion for the correlation functions of axial currents(8-10)(*). In perturbative QCD it is possible to define a renormalization group invariant mass starting from the effective quark mass at a certain scale. The definition of this quantity however de-

(*) - The masses of the quarks are interesting also for their connection with the spontaneous symmetry breaking of grand unified theories.

pends on the renormalization scheme. In this paper we compute the complete one loop corrections to the quark propagator on the lattice and the relation between the mass on the lattice and on the continuum : in this way it is possible to compare the results from lattice Montecarlo simulations with those from current algebra.

In analogy with the case of the coupling constant(11-14) we can introduce a scale Λ_m^{LATT} in the equation which governs the renormalization group behavior of the bare quark mass $m_f(a)$ on the lattice :

$$m_f(a) = \frac{\hat{m}_f}{\left[\frac{b_o}{N} \ln\left(\frac{1}{a\Lambda_m^{LATT}}\right)\right]^{\gamma_m}} \tag{1}$$

a is the lattice spacing and N the number of colours, γ_m is the renormalization group exponent :

$$\gamma_m = \frac{3}{2\pi b_o}\left(\frac{N^2-1}{2N}\right), \qquad b_o = \frac{11N}{6\pi}. \tag{2}$$

In eq. (1) b_o/N is an arbitrary multiplicative constant which defines the regularization scheme independent quantity $\hat{m}_f$. On the continuum the Λ_m parameter in some regularization scheme (for example the MS scheme) is defined by an equation for the quark mass at a scale μ similar to eq. (1) :

$$m_f(\mu) = \frac{\hat{m}_f}{\left[\frac{b_o}{N} \ln\left(\frac{\mu}{\Lambda_m}\right)\right]^{\gamma_m}} . \tag{3}$$

The relation between $m_f(a)$ and $m_f(\mu)$ is a computable quantity of the form :

$$m_f(\mu) = m_f(a)\left[1 - \frac{6g_o^2}{16\pi^2}\left(\frac{N^2-1}{2N}\right)(\ln a\mu - \ln C_m)\right] \tag{4}$$

where g_o is the lattice coupling constant. We have :

$$C_m = \frac{\Lambda_m}{\Lambda_m^{LATT}} . \tag{5}$$

From the knowledge of C_m we can relate $m_f(a)$ with the continuum quark mass. On the continuum the renormalization group invariant mass is usually defined as(15) :

$$\tilde{m}_f = \left(\frac{b_o}{N} \ln \frac{\mu}{\Lambda}\right)^{\gamma_m} m_f(\mu) \tag{6}$$

where Λ is the continuum coupling constant Λ-parameter in some renormalization scheme. From eqs. (3), (4), (6) one finds :

$$\tilde{m}_f = \left(\frac{b_o}{N} \ln \frac{C_m}{a\Lambda}\right)^{\gamma_m} m_f(a) . \tag{7}$$

$\tilde{m}_f$ is the quantity to be compared with current algebra predictions[(15)(+)]. C_m is computed from the $O(g_o^2)$ corrections to the fermion selfenergy.

Following ref. (16) we write the fermion lattice action on which C_m depends as:

$$S_\psi = \sum_x \left\{ - \sum_\mu \left[\frac{1}{2a} \left[\bar{\psi}(x)(r - \gamma_\mu) U_\mu(x) \psi(x+\hat{\mu}) + \right. \right. \right.$$
$$\left. \left. \left. + \bar{\psi}(x+\hat{\mu})(r+\gamma_\mu) U_\mu^+(x) \psi(x) \right] \right] + (m_f(a) + \frac{4r}{a}) \bar{\psi}(x) \psi(x) \right\} . \qquad (8)$$

The parameter r has been introduced to give a mass of order of the cutoff ($\sim \frac{1}{a}$) to the extra fermions which are present at $r = 0$ on the lattice; for $r = 1$ we have the usual Wilson action.

The general expression of the $O(g_o^2)$ fermion self energy for $a \to 0$ has the form[(o)]:

$$\Sigma^{LATT}(q) = \frac{g_o^2}{16\pi^2} \left(\frac{N^2 - 1}{2N} \right) \left[\frac{r}{a} \Sigma_o^{LATT}(r^2) + i\not{q}\, \Sigma_1^{LATT}(r^2, (ma)^2, (qa)^2) + \right.$$
$$\left. + m\, \Sigma_2^{LATT}(r^2, (ma)^2, (qa)^2) \right] . \qquad (9)$$

FIG. 1 - Feynman diagrams relevant to the computation of $O(g_o^2)$ corrections to the fermion self-energy on the lattice.

The linear divergence $\sim \frac{r}{a}$ in eq. (9) disappear by requiring the mass of the quark to be zero when the pion becomes massless[(16)]. $\Sigma_o(r)$ only gives a shift of the critical value of the Wilson hopping parameter ($K_c = \frac{1}{8}$ at tree level) in perturbation theory. We have computed Σ^{LATT} from the diagrams of Fig. 1 and we obtained:

$$\Sigma_o^{LATT} = - \frac{1}{(2\pi)^2} \int_{-\pi}^{\pi} d^4 p_\mu \left[\frac{2(1+r^2) \sum_v \cos^2(p_v/2)}{\Delta_2} - \frac{8r^2}{\Delta_2} - \right.$$
$$\left. - \frac{\sum_v \sin^2(p_v)}{\Delta_1 \Delta_2} + \frac{2}{\Delta_1} \right] \qquad (10)$$

(+) - Note that since we are truncating the perturbative series the effective values of $\tilde{m}_f$ and $\hat{m}_f$ of eq. (3) are different.

(o) - We adopt the same representation for Dirac matrices as in Ref. (13).

$$\Sigma_1^{LATT} = 2\int_0^1 dx\, x \ln\left[(1-x)(q^2x+m^2)a^2\right] + \tilde{\Sigma}_1^{LATT} =$$

$$= 2\int_0^1 dx\, x \ln\left[(1-x)(xq^2+m^2)a^2\right] + \gamma_E - F_{0001} -$$

$$- \frac{1}{(2\pi)^2}\int_{-\pi}^{\pi} d^4p_\mu \left\{ \frac{\frac{1}{4}(\sum_v \sin^2(p_v))\left[\sum_s \sin^4(p_s/2) - r^2(\sum_s \sin^2(p_s/2))^2\right]}{\Delta_1^3\Delta_2} + \right.$$

$$\left. + \frac{\frac{1}{4}\sum_v \sin^2(p_v)\sin^2(p_v/2)}{\Delta_1^2\Delta_2} - \frac{r^2}{2}\frac{\sum_v \cos(p_v)}{\Delta_2} - \frac{2}{\Delta_1}\right\} \qquad (10)$$

$$\Sigma_2^{LATT} = 4\int_0^1 dx \ln\left[(1-x)(q^2x+m^2)a^2\right] + \tilde{\Sigma}_2^{LATT} =$$

$$= 4\int_0^1 dx \ln\left[(1-x)(q^2x+m^2)a^2\right] + 4\gamma_E - 4F_{0000} -$$

$$- \frac{1}{(2\pi)^2}\int_{-\pi}^{\pi} d^4p_\mu \left\{ \frac{4\left[\sum_s \sin^4(p_s/2) - r^2(\sum_s \sin^2 p_s/2))^2\right]}{\Delta_1^2\Delta_2} - \frac{(1+r^2)}{\Delta_2}\right\}.$$

F_{0000} and F_{0001} are the numerical constant defined in ref. (14) ; $F_{0000} \sim 4.37$ and $F_{0001} \sim 1.41$. γ_E is the Eulero-Mascheroni constant

$$\Delta_1 = \sum_\mu \sin^2(p_\mu/2) \;;\quad \Delta_2 = \sum_\mu \sin^2(p_\mu) + 4r^2(\sum_\mu \sin^2(p_\mu/2))^2 . \qquad (11)$$

We can repeat the same computation on the continuum using dimensional regularization. In the minimal subtraction scheme (MS) we find :

$$\Sigma_1 = 2\int_0^1 dx\, x \ln\left[(1-x)(q^2x+m^2)/\mu^2\right] + \tilde{\Sigma}_1 =$$

$$= 2\int_0^1 dx\, x \ln\left[(1-x)(q^2x+m^2)/\mu^2\right] + \gamma_E - \ln(4\pi) + 1 \qquad (12)$$

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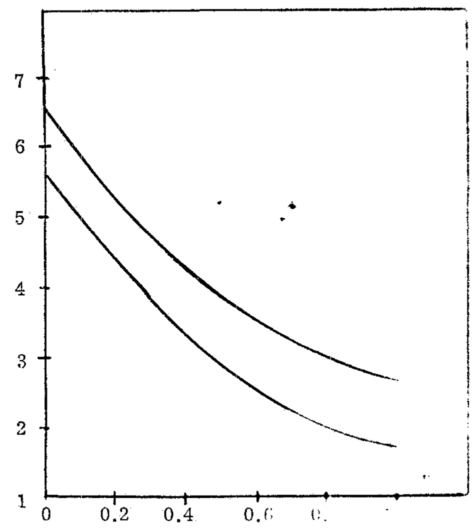
The last term "$-\frac{2}{\Delta_1}$" in the second of eqs. (10) (Σ_1^{LATT})

must be changed to "$-\frac{1}{2\Delta_1}$".

Consequently Fig. 2 is changed to:



Eqs. (15), (16) and (17) change as follows:

$$\tilde{m}_f = m_f(a)\left[\frac{2\pi}{9}\left(6 \begin{matrix} -1.99 + 1.38 \\ -2.81 + 2.20 \end{matrix}\right)\right]^{4/11} \sim \left(\begin{matrix} 1.62 \\ 1.62 \end{matrix}\right) m_f(a) \qquad (15)$$

$$\tilde{m}_u = \tilde{m}_d = 7.3 \pm 1.0 \text{ MeV}, \qquad \tilde{m}_s = 188 \pm 11 \text{ MeV}. \qquad (16)$$

$$\tilde{m}_f = m_f(a), \qquad m_c(a) = 930 \pm 70 \text{ MeV}, \qquad \tilde{m}_c = 1500 \pm 110 \text{ MeV}. \qquad (17)$$

$$\Sigma_2 = 4\int_0^1 dx \ln\left[(1-x)(q^2x+m^2)/\mu^2\right] + \tilde{\Sigma}_2 =$$

$$= 4\int_0^1 dx\, x \ln\left[(1-x)(q^2x+m^2)/\mu^2\right] + 4\gamma_E - 4\ln(4\pi) + 2 \,. \tag{12}$$

The result in the $\overline{MS}$ scheme is obtained by dropping the terms proportional to $\gamma_E - \ln(4\pi)$. C_m is related to the Σ's through the relation:

$$C_m = \exp\left[\frac{(\tilde{\Sigma}_1^{LATT} - \tilde{\Sigma}_2^{LATT}) - (\tilde{\Sigma}_1 - \tilde{\Sigma}_2)}{6}\right] \tag{13}$$

C_m has been numerically evaluated using eqs. (10), (12) with an accuracy of less that 1% (by Montecarlo integration) for several values of r ranging from zero to one. We report in Fig.2 $\ln(C_m)$ as a function of r in the $\overline{MS}$ and MS schemes.

Let us write eq. (7) in the lattice formalism by expressing $\tilde{m}_f$ as a function of $\beta = \frac{2N}{g_o^2}$:

$$\tilde{m}_f = m_f(a)\left[\frac{2\pi}{N^2}\left(\beta + \frac{Nb_o}{2\pi}\ln\left(\frac{\Lambda^{LATT}}{\Lambda}\right) + \frac{Nb_o}{2\pi}\ln C_m\right)\right]^{\gamma_m} \tag{14}$$

Λ^{LATT} is the coupling constant scale on the lattice. The term depending on the ratio between the Λ's is simply due to the difference between the lattice and the continuum coupling constants. Note that $\ln(C_m)$ gives a correction of opposite sign to what naively expected from the ratio between the coupling constants. We use eq. (14) together with the results reported in refs. (6, 7) to obtain the renormalization group invariant masses in the continuum starting from Montecarlo results.

For the up, down and strange quarks we take the results from ref. (6) with $N = 3$, $r = 1$, $\beta = 6$:

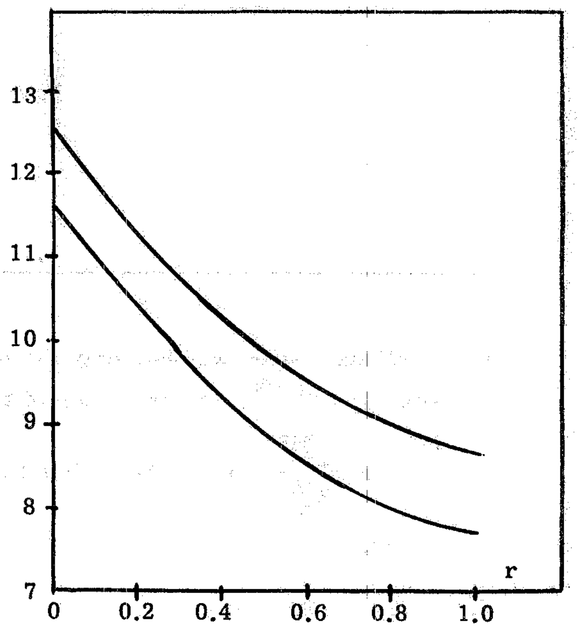


FIG. 2 - $S_m = \ln C_m$ as a function of r. The upper and lower lines refer to S_m from the comparison between the lattice and the $\overline{MS}$ scheme and the MS scheme respectively.

$$\tilde{m}_f = m_f(a)\left[\frac{2\pi}{9}\left(6 \begin{matrix} -1.99 + 6.51 \\ -2.81 + 7.32 \end{matrix}\right)\right]^{4/11} \sim \begin{pmatrix} 2.06 \\ 2.06 \end{pmatrix} m_f(a) \qquad (15)$$

-1.99 and -2.81 are $\frac{3b_o}{2\pi} \ln\left(\frac{\Lambda^{LATT}}{\Lambda_{MS, \overline{MS}}}\right)\Big|_{n_f = 0}$ respectively (*),

Using $m_u(a) = m_d(a) = 4.5 \pm 0.6$ MeV, $m_s(a) = 116 \pm 7$ MeV ($a^{-1} = 1420$ MeV; $\Lambda_{\overline{MS}} = 76$ MeV) we obtain:

$$\tilde{m}_u = \tilde{m}_d = 9.3 \pm 1.2 \text{ MeV}, \qquad \tilde{m}_s = 239 \pm 14 \text{ MeV}. \qquad (16)$$

For the charmed quark, taking the results of ref. (7) ($N = 3$, $r = 1$, $\beta = 6.4$):

$$\tilde{m}_f = m_f(a), \qquad m_c(a) = 930 \pm 70 \text{ MeV}, \qquad \tilde{m}_c = 1940 \pm 150 \text{ MeV}. \qquad (17)$$

These results, obtained from Montecarlo experiments are in fairly good agreement with current algebra results. For example from ref. (17) one finds:

$$\frac{\tilde{m}_u + \tilde{m}_d}{2} = 10 \pm 3 \text{ MeV}, \qquad \tilde{m}_s = 260 \pm 80 \text{ MeV}, \qquad \Lambda_{\overline{MS}} = 100 \text{ MeV}.$$

One of us (G.M.) thanks P. Hasenfratz and G. Parisi for usefull discussions.

(*) - In all this paper we gave only the leading logs formulae for the masses (as for example in eqs. (1), (3))[18]. The inclusion of the next to leading corrections amounts to add a term $\frac{Nb_o}{2\pi} \ln\left(\frac{\Lambda^{MS}}{\Lambda_m^{MS}}\right)$ in eq. (14). This term turns out to be numerically irrelevant in practical cases.

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