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THE MANY-BODY CONTENT OF QUANTUM GAUGE THEORIES AND SPONTANEOUS SYMMETRY BREAKING

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The many-body content of quantum field theories is studied by performing the limit velocity of light $\to \infty$. It is found that the limit of the Goldstone model is the Bogoliubov model of superfluidity, and the limit of the vacuum of spontaneously broken abelian theories is a plasma whose excitations are the limit of the massive gauge bosons. The method appears suitable to study dynamical supersymmetry breaking and colour confinement.

1. Introduction

All the ideas concerning spontaneous symmetry breaking [1,2] and mass generation [3,4] have been imported in quantum field theory from the theory of many-body systems, with the exception of the Schwinger [5] conjecture. According to this conjecture, local gauge invariance does not necessarily imply masslessness for the gauge bosons. Even for this, however, as noted by Anderson [3], the physics of many-body systems provides an example with the plasma excitations which are the analogue of the massive photons conjectured by Schwinger.

What has not been noted is that plasma excitations *are* the massive photons and the plasma *is* the vacuum in the non-relativistic limit of the theory considered by Schwinger.

Here is an example of a general situation. Relativistic theories which exhibit spontaneous symmetry breaking have a non-trivial many-body content. Studying this content is the purpose of the present paper. The way to do it is to derive a non-relativistic approximation for these theories.

A number of non-relativistic field theories are already available. Lévy-Leblond [6,7] has constructed a galilean version of the scalar-spinor interaction, the Lee model and electromagnetism. A different form of galilean electromagnetism, extended to non-abelian gauge theories, has been found by the present author [8], and a still different form by Kapuscik [9]. Also supersymmetry [10] has been combined [11,12] with galilean invariance.

Most of these models have been constructed to learn about relativistic theories. The work of ref. [8], for instance, aimed at the study of colour confinement, while that of ref. [12] was a test of the conjecture [13] that fermions and bosons should condense at the time in supersymmetry. These are actually additional motivations for the present work, along with an interest in relativistic effects in many-body systems like nuclear matter.

Little effort, however, has been made, in general, to relate in a precise way non-relativistic theories to relativistic ones, so that one could study a non-relativistic model which cannot be obtained as approximation to a relativistic one. Such a possibility is best exemplified by the mentioned gauge theories. Only two of them have been obtained from Maxwell theory by the limit of the velocity of light $c \to \infty$. In one of the two, however, local gauge invariance is lost [7]. The other one [8], which can be obtained [14] by preserving local gauge invariance, has the same infrared behaviour as relativistic QED [8].

We therefore will derive the non-relativistic approximation to quantum gauge theories by performing the limit $c \to \infty$ in such a way as to conserve all the symmetries.

The idea of a $1/c$ expansion is not a new one★. What we think is new is the requirement of conserving all the symmetries in the expansion. In the standard non-relativistic reduction of the Dirac equation, for instance, charge symmetry is lost.

The limit $c \to \infty$ will be performed within the path-integral formulation of quantum field theory. This makes the procedure the simplest one. Intermediate steps, like changing the metric of the Hilbert space [16] in the hamiltonian formulation, are replaced in this way by simple functional transformations on the fields. Such transformations are c-dependent, and give rise to terms in the lagrangian which diverge with c, terms independent of c, and terms or order $1/c$. The galilean lagrangian can contain c only in the mass terms which are in the centre of the Galilei algebra. Therefore the functional transformations must be performed in such a way that the terms divergent with c either cancel out or give rise to mass terms and Galilei-invariant constraints, while the c-independent terms must be Galilei invariant. Both lagrangian and constraints must be invariant with respect to the symmetries of the relativistic lagrangian.

The obvious worry in this strategy is that some infrared feature peculiar to the relativistic theory could be lost in the limit. We will see that this is not the case in theories whose low-energy properties are known, and that actually the non-relativistic approximation can be quantitatively good.

The limit will be performed for scalars, spinors and gauge fields in sects. 2, 3 and 4. The limit for gauge fields has already been done [14] and is reported here for completeness. In sect. 5 we discuss spontaneous symmetry breaking and mass

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★ A partial list of references can be found in [15].

generation. We show that the limit of the Goldstone model is the Bogoliubov model of superfluidity★ for a charged gas. The peculiar excitations of this model, which are linear in the momentum at small momenta, correspond to the Goldstone boson. We discuss mass generation in this model and in the Schwinger mechanism [5]. This latter will be the subject of a future publication [18] but we anticipate some results showing that massive photons become, in the limit the transverse collective excitations of a plasma spontaneously created. This is, therefore, not a mere analogue, but just the many-body content of the Schwinger mechanism.

In these sections only abelian theories are considered. The non-abelian ones are discussed in sect. 6 and our conclusions are presented in sect. 7.

Throughout the paper, we put $\hbar = 1$.

2. The limit for scalars

The lagrangian density for a complex scalar field in the presence of an electromagnetic field is

$$\mathcal{L}_\mathrm{S} = -\frac{1}{c^2}|\mathcal{D}_t\Phi|^2 - |\mathcal{D}_k\Phi|^2 - \sigma\mu^2 c^2|\Phi|^2 - \lambda|\Phi|^4, \tag{2.1}$$

where

$$\mathcal{D}_t = \partial_t + ieV, \qquad \mathcal{D}_k = \partial_k - ieA_k. \tag{2.2}$$

The quartic self-interaction is necessary for the theory to be renormalizable. For $\sigma = 1$ we have the normal scalar electrodynamics, while for $\sigma = -1$ we have the Higgs model [4]. The electromagnetic potentials are defined as in sect. 4. $\mathcal{L}_\mathrm{S}$ is invariant under the gauge transformations

$$\Phi \to e^{ie\Lambda}\Phi, \tag{2.3}$$

$$V \to V - \partial_t\Lambda, \qquad A_k \to A_k + \partial_k\Lambda. \tag{2.4}$$

The quantum theory is described by the generating functional

$$W = \int [d\Phi][d\Phi^*]\exp\left(i\int dt\,d^3x\,\mathcal{L}_\mathrm{S}\right). \tag{2.5}$$

If we allow $c \to \infty$, other things being finite, we are left with a lagrangian without time derivatives, as appropriate for massless galilean particles [6]. In order to obtain a galilean lagrangian for massive particles it is necessary to make a functional change of variables.

First we observe that the galilean lagrangian contains only first-order time derivatives. We therefore get rid of the second-order time derivatives by introducing

★ See any textbook on many-body physics, for instance ref. [17].

extra integrations in the generating functional

$$W = \int [\mathrm{d}\Phi][\mathrm{d}\Phi^*][\mathrm{d}\theta][\mathrm{d}\theta^*] \exp\left(i\int \mathrm{d}t\,\mathrm{d}^3x\, \mathcal{L}'_S \right),$$

$$\mathcal{L}'_S = -\mu\left\{ \theta^* i\mathcal{D}_t \Phi + \Phi^* i\mathcal{D}_t \theta + \sigma\mu c^2 \left(|\Phi|^2 + |\theta|^2 \right) + \frac{1}{\mu}|\mathcal{D}_k \Phi|^2 + \frac{\lambda}{\mu}|\Phi|^4 \right\}.$$

Performing the gaussian integral over θ and θ^* we would go back to $\mathcal{L}_S$.

Now we must perform a further transformation in order to diagonalize the kinetic terms. We try

$$\Phi = \frac{1}{\sqrt{2\mu}}\left(1 + \sigma\frac{O}{4\mu^2 c^2} \right)(\phi_1 + \phi_2^*),$$

$$\Phi^* = \frac{1}{\sqrt{2\mu}}\left(1 + \sigma\frac{O^*}{4\mu^2 c^2} \right)(\phi_1^* + \phi_2),$$

$$\theta = \frac{1}{\sqrt{2\mu}}\left(1 - \sigma\frac{O}{4\mu^2 c^2} \right)(-\phi_1 + \phi_2^*),$$

$$\theta^* = \frac{1}{\sqrt{2\mu}}\left(1 - \sigma\frac{O^*}{4\mu^2 c^2} \right)(-\phi_1^* + \phi_2), \tag{2.6}$$

where O is an operator to be determined.

In the limit $c \to \infty$ we will get terms involving O only from $\sigma\mu c^2$ ($|\Phi|^2 + |\theta|^2$). By choosing $O = \mathcal{D}_k \mathcal{D}_k = \mathcal{D}^2$, the kinetic terms become diagonal. Moreover, the unique term involving c is $\sigma\mu c^2 \nu_S$, with

$$\nu_S = \phi_1^* \phi_1 + \phi_2^* \phi_2, \tag{2.7}$$

which is allowed because it is a mass term. From $|\Phi|^4$, however, we get contributions involving the combination $\phi_1\phi_2 + \phi_1^*\phi_2^*$ which is not Galilei-invariant. We can get rid of them by choosing

$$O = \mathcal{D}^2 - \frac{\lambda}{2\mu}(2\nu_S + \phi_1\phi_2 + \phi_1^*\phi_2^*), \tag{2.8}$$

so that

$$\mathcal{L}_S \xrightarrow[c \to \infty]{} \mathcal{L}_S^{(0)} = \phi_1^* i\mathcal{D}_t \phi_1 + \phi_2^* i\mathcal{D}_t^* \phi_2 + \phi_1^* \frac{\mathcal{D}^2}{2\mu}\phi_1$$

$$+ \phi_2^* \frac{\mathcal{D}^{*2}}{2\mu}\phi_2 - \sigma\mu c^2 \nu_S - \frac{\lambda}{4\mu^2}\nu_S^2 - \frac{\lambda}{2\mu^2}\phi_1^* \phi_1 \phi_2^* \phi_2. \tag{2.9}$$

Since the jacobian of the transformation is a constant for $c \to \infty$

$$W \to W^{(0)} = \int [d\phi_1][d\phi_1^*][d\phi_2][d\phi_2^*] \exp\left(i\int dt\, d^3x\, \mathcal{L}_S^{(0)} \right). \tag{2.10}$$

$\mathcal{L}_S^{(0)}$ describes spinless particles of opposite charge with a contact self-repulsion and a chemical potential $\sigma\mu c^2$. It is invariant under the gauge transformation (2.4) and

$$\phi_1 \to e^{ie\Lambda}\phi_1, \qquad \phi_2 \to e^{-ie\Lambda}\phi_2. \tag{2.11}$$

Using the gauge condition $\partial_k A_k = 0$ (see sect. 4) the electromagnetic interaction terms can be written

$$-e\left(\rho_S V - j_{Sk} A_k\right) - \frac{1}{2\mu} e^2 A^2 \nu_S, \tag{2.12}$$

with

$$\rho_S = \phi_1^* \phi_1 - \phi_2^* \phi_2,$$

$$j_{Sk} = \frac{1}{2\mu i}\left(\phi_1^* \partial_k \phi_1 - \partial_k \phi_1^* \phi_1 - \phi_2^* \partial_k \phi_2 + \partial_k \phi_2^* \phi_2\right). \tag{2.13}$$

3. The limit for spinors

The Dirac lagrangian density in the presence of an electromagnetic field is

$$\mathcal{L}_F = \psi^*\left(i\mathcal{D}_t + c\alpha^k i\mathcal{D}_k - \beta mc^2\right)\psi. \tag{3.1}$$

Introducing the upper and lower components of ψ, $\psi = \begin{bmatrix} \psi_+ \\ \psi_- \end{bmatrix}$, $\mathcal{L}_F$ becomes

$$\mathcal{L}_F = \psi_+^* i\mathcal{D}_t \psi_+ + \psi_-^* i\mathcal{D}_t \psi_- + c\psi_+^* i\mathcal{D}_k \sigma^k \psi_-$$

$$+ c\psi_-^* i\mathcal{D}_k \sigma^k \psi_+ - mc^2(\psi_+^* \psi_+ - \psi_-^* \psi_-),$$

σ^k being the Pauli matrices. This lagrangian contains only first-order time derivatives, so that we can directly proceed to the diagonalization of the kinetic terms. Note, however, that there are two terms proportional to c which explode for $c \to \infty$. These terms do not constitute a problem in the standard non-relativistic reduction, where one is only concerned with positive energy states. In that case one takes $\psi_- \sim 1/c$. But we want to perform the limit preserving the charge symmetry of the theory. This means that the above terms of order c must be cancelled by others, which can only come from the last term in the lagrangian.

The above is achieved by the transformation

$$\psi_+ = \psi_1 + \frac{1}{2mc} i \mathcal{D}_k \sigma^k \sigma^2 \psi_2^*,$$

$$\psi_+^* = \left(\psi_1 + \frac{1}{2mc} i \mathcal{D}_k \sigma^k \sigma^2 \psi_2^* \right)^*,$$

$$\psi_- = \sigma^2 \psi_2^* - \frac{1}{2mc} i \mathcal{D}_k \sigma^k \psi_1,$$

$$\psi_-^* = \left(\sigma^2 \psi_2^* - \frac{1}{2mc} i \mathcal{D}_k \sigma^k \psi_1 \right)^*, \tag{3.2}$$

which gives

$$\mathcal{L}_F \underset{c \to \infty}{\to} \mathcal{L}_F^{(0)} = \psi_1^* i \mathcal{D}_t \psi_1 + \psi_2^* i \mathcal{D}_t^* \psi_2 + \psi_1^* \frac{\mathcal{D}^2}{2m} \psi_1$$

$$+ \psi_2^* \frac{\mathcal{D}^{*2}}{2m} \psi_2 - mc^2 (\psi_1^* \psi_1 + \psi_2^* \psi_2), \tag{3.3}$$

with a constant jacobian.

In the derivation of eq. (3.3) we have used the fact that

$$\left(\frac{i}{2mc} \mathcal{D}_k \sigma^k \sigma^2 \psi_2^* \right)^* i \partial_t \left(\frac{i}{2mc} \mathcal{D}_h \sigma^h \sigma^2 \psi_2^* \right)$$

is not of order $1/c^2$, but

$$\left(\frac{i}{2mc} \mathcal{D}_k \sigma^k \sigma^2 \psi_2^* \right)^* (i \partial_t - mc^2) \left(\frac{i}{2mc} \mathcal{D}_h \sigma^h \sigma^2 \psi_2^* \right)$$

is.

$\mathcal{L}_F^{(0)}$ is the same function of the anticommuting ψ-fields as $\mathcal{L}_S^{(0)}$ of the ϕ-fields, apart from the self-interaction and the mass term which is necessarily negative for fermions. In particular, the interaction with the electromagnetic field is the same, there being no remainder of the spin. We will therefore use the same symbols of eqs. (2.12) and (2.13) for the electromagnetic interaction terms, by replacing the subscript S by F, namely

$$\nu_F = \psi_1^* \psi_1 + \psi_2^* \psi_2, \tag{3.4}$$

and so on.

4. The limit for gauge fields

The lagrangian density for the gauge fields is

$$\mathcal{L}_R = \tfrac{1}{2}\mathcal{F}_{0i}^2 - \tfrac{1}{4}\mathcal{F}_{ij}^2,\tag{4.1}$$

$$\mathcal{F}_{0i} = \frac{1}{c}\partial_t\mathcal{Q}_i - \partial_i\mathcal{Q}_0,$$

$$\mathcal{F}_{ij} = \partial_i\mathcal{Q}_j - \partial_j\mathcal{Q}_i.\tag{4.2}$$

If we let $c \to \infty$, other things being finite, we lose the gauge invariance of $\mathcal{F}_{0i}$. In order to have a gauge-invariant limit, it is necessary to make the change of variables

$$\mathcal{Q}_0 = -V, \qquad \mathcal{Q}_i = cA_i.\tag{4.3}$$

Under such a transformation

$$\mathcal{F}_{0i} = \partial_t A_i + \partial_i V \overset{\text{def}}{=} F_{0i},$$

$$\mathcal{F}_{ij} = c\left(\partial_i A_j - \partial_j A_i\right) \overset{\text{def}}{=} cF_{ij},\tag{4.4}$$

$$\mathcal{L}_R = \tfrac{1}{2}F_{0i}^2 - \tfrac{1}{4}c^2 F_{ij}^2.\tag{4.5}$$

Omitting the measure over the matter fields and assuming a gauge-fixing condition

$$f(V, A_i) = 0,\tag{4.6}$$

the generating functional can be written

$$W = \int [\mathrm{d}V][\mathrm{d}A_i]\prod_{xt}\delta[f(V, A_i)]$$

$$\times \exp\left(i\int \mathrm{d}t\,\mathrm{d}^3x\left[\tfrac{1}{2}F_{0i}^2 - \tfrac{1}{4}c^2F_{ij}^2 + \mathcal{L}_S + \mathcal{L}_F\right]\right).\tag{4.7}$$

We will confine ourselves to the Coulomb and Landau gauges. In both cases

$$f(V, A_i) \underset{c\to\infty}{\to} c^{-1}\delta(\partial_i A_i).\tag{4.8}$$

If we now observe that, apart for a constant factor

$$\exp(ic^2x^2) \underset{c\to\infty}{\to} \delta(x),$$

we see that the limit of the generating functional is

$$W^{(0)} = \int [dV][dA_i] \prod_{xt} \delta(\partial_i A_i) \delta(F_{ij})$$

$$\times \exp\left(i \int dt\, d^3x [\mathcal{L}_R^{(0)} + \mathcal{L}_S^{(0)} + \mathcal{L}_F^{(0)}] \right), \tag{4.9}$$

$$\mathcal{L}_R^{(0)} = \tfrac{1}{2} F_{0i}^2. \tag{4.10}$$

Under Galilei transformations of velocity v_k

$$A_i \to A_i, \qquad V \to V + v_k A_k,$$

$$F_{ij} \to F_{ij}, \qquad F_{0i} \to F_{0i} + v_k F_{ki}. \tag{4.11}$$

The constraint $F_{ij} = 0$ is therefore a Galilei-invariant constraint, and under this constraint $\mathcal{L}_R^{(0)}$ is a Galilei- and gauge-invariant lagrangian.

Probably this lagrangian has escaped previous systematic studies of the Galilei group because the possibility of constraints has not been considered.

Let us note that, as far as Galilei invariance is concerned, there is another possible lagrangian density [7]

$$\mathcal{L}_R' = -\tfrac{1}{4} F_{ij}^2.$$

With $\mathcal{L}_R'$, however, it is not possible to write down any gauge-invariant lagrangian of radiation plus matter.

The integration over A_i can be enormously simplified [14]. Let us introduce a spatial lattice by

$$A_i(x,t) \to A_i(k,t) = \frac{1}{\omega_k} \int_{\omega_k} d^3x\, A_i(x,t), \tag{4.12}$$

where ω_k is the volume of the cell centered at the site k. We can transform the "single particle" coordinates (4.12) into "intrinsic" and "centre-of-mass" coordinates:

$$A_i(k,t) = \frac{1}{\sqrt{\Omega}} q_i(t) + \xi_i(k,t),$$

$$q_i(t) = \frac{1}{\sqrt{\Omega}} \sum_k \omega_k A_i(k,t), \tag{4.13}$$

Ω being the quantization volume. We do not need an explicit choice for the ξ_i. What

is relevant here is that the δ-functions in eq. (4.9) involve only intrinsic coordinates:

$$\int [\mathrm{d}A_i] \prod_{xt} \delta(\partial_i A_i) \delta(F_{ij}) \propto \int [\mathrm{d}\xi_i] \prod_t \mathrm{d}q_i(t),$$

$$\prod_{xt} \delta(\xi_i) \propto \int \prod_t \mathrm{d}q_i(t),$$

so that we finally obtain

$$W^{(0)} = \int \prod_t \mathrm{d}q_i(t) [\mathrm{d}V] \exp\left(i \int \mathrm{d}t \left\{ \tfrac{1}{2}\dot{q}_i^2 + \int \mathrm{d}^3x \left[(\partial_i V)^2 + \mathcal{L}_S^{(0)} + \mathcal{L}_F^{(0)} \right] \right\} \right).$$

(4.14)

In the above equation it is understood that $A_i = q_i / \sqrt{\Omega}$. The measure over q_i is written explicitly to remind us that q_i depends only on t.

The integration over V gives the Coulomb interaction. All the remaining terms involving the gauge fields correspond to the hamiltonian [8]

$$H_R = \tfrac{1}{2}p_i^2 + \tfrac{1}{2}\left(\omega_S^2 + \omega_F^2 \right)q_i^2 - q_i\left(\omega_S I_{Si} + \omega_F I_{Fi} \right),$$

(4.15)

with

$$\omega_S^2 = \frac{e^2}{\mu\Omega} \int \mathrm{d}^3x\, \nu_S, \qquad \omega_F^2 = \frac{e^2}{m\Omega} \int \mathrm{d}^3x\, \nu_F,$$

$$I_{Si} = \left(\int \mathrm{d}^3x\, \nu_S/\mu \right)^{-1} \int \mathrm{d}^3x\, j_{Si}, \qquad I_{Fi} = \left(\int \mathrm{d}^3x\, \nu_F/m \right)^{-1} \int \mathrm{d}^3x\, j_{Fi}.$$

(4.16)

This hamiltonian describes photons of zero momentum and energy $\omega \sim 1/\Omega$. It gives rise to divergences in perturbation theory due to vanishing energy denominators for $\Omega \to \infty$. In the relativistic case the divergences associated with virtual and real photons cancel out leaving us with a finite correction which exponentiates. The emission of real photons is the same as that of the classical current corresponding to incoming and outgoing particles. The same is true in the present case [8] showing that radiative corrections can be evaluated accurately provided the velocities of the charged particles are small.

5. Spontaneous symmetry breaking and mass generation

In the previous section we have seen that galilean QED describes only soft photons. Processes like electron-photon scattering belong to the relativistic domain.

According to eqs. (4.16), however, the energy of the photons becomes finite in the presence of a number of particles proportional to the volume Ω. Since the momentum of these photons is still zero, such a finite energy is the photon mass. Creation of these photons is described as plasma excitations in many-body language.

The mass of the photon is spontaneously generated if the plasma is spontaneously generated, i.e., if the plasma is the limit for $c \to \infty$ of the vacuum of the relativistic theory with broken symmetry.

We will now consider the Goldstone and the Schwinger mechanisms for the breaking.

For $\sigma = 1$, $\mathcal{L}_S$ has a normal vacuum, while for $\sigma = -1$

$$\langle \Phi \rangle = \frac{\mu c}{\sqrt{2\lambda}}, \tag{5.1}$$

and the low-energy spectrum is determined by the presence of a massless scalar, the Goldstone boson [2].

For $\sigma = 1$ also $\mathcal{L}_S^{(0)}$ has a normal vacuum, while for $\sigma = -1$, $\mathcal{L}_S^{(0)}$ is the Bogoliubov model [17] of superfluidity for a charged gas. As far as we know this model has not been studied, but we do not think that the peculiar feature of the Bogoliubov model depends on the charge. This feature is well known, but we will describe it here very briefly in order to emphasize how faithful the non-relativistic model can be to its relativistic parent.

Due to the negative chemical potential, the bosons undergo a Bose-Einstein condensation

$$\langle \phi \rangle = \sqrt{\frac{2\mu}{\lambda}}\, c, \tag{5.2}$$

which breaks the gauge symmetry (2.11). This value differs from the relativistic one, eq. (5.1), only due to the scaling of the fields in the transformation (2.6). The condensate is formed of bosons of zero momentum. The excitations are due to the possibility for a pair of bosons to escape from the condensate with opposite momenta. The corresponding spectrum is linear in the momentum at small momenta, as appropriate to a massless relativistic particle, the Goldstone boson. This Goldstone boson provides the theoretical explanation for superfluidity.

Let us now turn to the Schwinger mechanism. If the electric charge is large enough, the vacuum becomes unstable with respect to creation of particle-antiparticle bound states. The process of creation is stopped by the Pauli repulsion for spinors and by the contact self-repulsion for scalars. In the absence of this repulsion, i.e., for non-renormalizable scalar electrodynamics, the hamiltonian is not bounded from below [18].

This is the many-body content of the mechanism that Schwinger conjectured for spinor QED. This dynamical mechanism can be combined with the Goldstone

mechanism. Choosing $\sigma = -1$, we have

$$\langle \phi \rangle \neq 0, \qquad \langle \nu_S \rangle \neq |\langle \phi \rangle|^2. \qquad (5.3)$$

This is a concrete example of a possibility already considered in the relativistic context [19].

We must remember, however, that in the dynamical case it is not sufficient that ω_S or ω_F be finite for the photon to acquire a mass. It is also necessary that the density fluctuations should not destroy the effect. This point will be discussed separately.

It is obvious that the Schwinger mechanism can also be used in non-abelian gauge theories, once the bound-state problem has been solved in these theories.

Finally, the Schwinger mechanism can be relevant to supersymmetry, and it would be a good test for the mentioned conjecture that bosons and fermions should condense at the same time [13,12]. In principle one can have boson-boson, fermion-fermion and boson-fermion bound states, and one should see whether all these bound states are formed at the same time, or which one is formed first.

6. The limit for non-abelian gauge theories

The extension of the content of sects. 2 and 3 to the non-abelian case is straightforward. One has simply to insert for $\mathcal{D}_t$ and $\mathcal{D}_k$ in eqs. (2.8), (2.9), (3.2) and (3.3) the covariant derivatives for scalars and spinors in the appropriate representation.

A few words are instead required for the gauge fields. Eqs. (4.4) must be replaced by

$$F_{0i}^a = \partial_t A_i^a + \partial_i V^a + g f^a{}_{bc} A_i^b V^c,$$

$$F_{ij}^a = \partial_i A_j^a - \partial_j A_i^a + g f^a{}_{bc} A_i^b A_j^c, \qquad (6.1)$$

where f_{abc} are the structures constants of the gauge group normalized according to

$$[t_a, t_b] = i f^c{}_{ab} t_c, \qquad (6.2)$$

and t^a are the hermitian matrices representing the group generators.

Also in this case the Coulomb and Landau gauge conditions coincide in the limit. The generating functional remains that given by eq. (4.9), apart from the insertion of the Faddeev-Popov determinant [20]. This is given by

$$\langle x | M_{ab} | y \rangle = \begin{cases} \left[\delta_{ab} \Delta - \dfrac{g}{c} f_{abc} \mathcal{C}_i^c \partial_i \right] \delta^4(x-y), & \text{in the Coulomb gauge,} \\[2ex] \left[\delta_{ab} \square - \dfrac{g}{c} f_{abc} \mathcal{C}_\mu^c \partial_\mu \right] \delta^4(x-y), & \text{in the Landau gauge.} \end{cases} \qquad (6.3)$$

Both expressions give, in the limit

$$\langle x|M_{ab}^{(0)}|y\rangle = [\delta_{ab}\Delta - gf_{abc}A_i^c\partial_i]\delta(t_x - t_y)\delta^3(x-y), \tag{6.4}$$

This shows the equivalence, actually the identity, of the Coulomb and Landau gauges in the limit. The extra terms which, according to Christ and Lee, must be introduced in the action in the Coulomb gauge [21] must therefore have a vanishing limit and should describe relativistic effects.

The constraints

$$F_{ij}^a = 0 \tag{6.5}$$

are the integrability conditions which allow the transformation

$$A_k^a t_a = \partial_k \exp\left[i\theta^b(xt)t_b\right]\exp\left[-i\theta^c(xt)t_c\right]. \tag{6.6}$$

There is obviously a great simplification in passing from A_k^a to θ^a, but $\mathcal{L}_R$ remains rather complicated in the non-abelian case.

7. Conclusion

We have shown that the $c \to \infty$ limit of quantum gauge theories can be performed conserving all the symmetries of the relativistic lagrangians. The low-energy properties of the resulting galilean theories agree with the properties of their relativistic parents and exhibit their many-body content.

The emphasis has been put on showing such an agreement rather than in deriving new results. We hope, however, that new results can be obtained with the present method especially in connection with the dynamical breaking of supersymmetry and colour confinement. Note that the reliability of the results can be checked by a $1/c$ expansion. The way to do this is in principle obvious because there are terms of order $1/c$ after the functional transformations in all the lagrangians, with the exception of $\mathcal{L}_R$. How to generate the expansion for the latter has been shown in a previous paper [14].

The approximation introduced by the limit can thus be kept under control, while in the limit it is in some cases possible to obtain non-perturbative results or results to all orders in the coupling constants [18].

There are many points which require further investigation. We have ignored renormalization counter terms, we have not discussed the unicity of the $c \to \infty$ limit, we have not treated Yukawa couplings and massless scalars and spinors. Concerning the last point, we only mention that, in analogy with the case of abelian gauge bosons, it is possible to obtain lagrangians with constraints

$$\mathcal{L}_S^{(0)} = \tfrac{1}{2}(\partial_t\phi)^2, \qquad \text{with constraint } \partial_i\phi = 0;$$

$$\mathcal{L}_F^{(0)} = \psi^*i\partial_t\psi, \qquad \text{with constraint } \partial_i\psi = 0.$$

Quantum field theory reduces in these cases to ordinary quantum mechanics.

I would like to thank C. Natoli for many illuminating discussions on the many-body problems.

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