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Radiative Corrections to Z_0 Jet Asymmetries in e^+e^- Annihilation.

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Summary. — Electromagnetic and finite-width effects are discussed for $q\bar{q}$ jets produced in e^+e^- annihilation near the Z_0 pole. Angular asymmetries are also considered for inclusively produced π^+ and K^+ with large fractional momenta.

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The observation of hadronic jets produced in e^+e^- annihilation at very high energy provides a test of the standard theory of electroweak interactions (1). In fact, the production cross-section of $q\bar{q}$ jets and the corresponding angular asymmetries are very sensitive to the weak vector and axial quark coupling, particularly for energies near the intermediate-boson mass (2). Compared to the

(1) S. GLASHOW: *Nucl. Phys.*, **22**, 579 (1961); S. WEINBERG: *Phys. Rev. Lett.*, **19**, 1264 (1967); A. SALAM: *Proceedings of the VIII Nobel Symposium* (Stockholm, 1968); S. GLASHOW, J. ILIOPOULOS and L. MAIANI: *Phys. Rev. D*, **2**, 1285 (1970).
(2) J. ELLIS and M. K. GAILLARD: *Physics with very high energy* e^+e^- *colliding beams*; L. CAMILLERI, D. CUNDY, P. DARRIULAT, J. ELLIS, J. FIELD, H. FISCHER, E. GABATHULER, M. K. GAILLARD, H. HOFFMANN, K. JOHNSEN, E. KEIL, F. PALMONARI, G. PREPARATA, B. RICHTER, C. RUBBIA, J. STEINBERGER, B. WIIK, W. WILLIS and K. WINTER: CERN 76-18 (1976).

pure leptonic processes, *e.g.* $e\bar{e} \to \mu\bar{\mu}$, the quark angular asymmetries have the obvious advantages of being larger at PETRA-PEP energies, and enhanced in statistics by a factor $R = \sigma(e\bar{e} \to \text{hadrons})/\sigma(e\bar{e} \to \mu\bar{\mu})$. On the other hand, the observable effects depend on the hadronization properties of the quark jets and are limited by experimental problems of particle identification. Various method of jet analysis have been suggested (3) which are suitable for the purpose of measuring the electroweak hadronic asymmetries in the near-future experiments.

Experimental investigation of these effects, however, requires an accurate evaluation of radiative corrections which can play an important role, as indeed found in the leptonic channels $e\bar{e} \to \mu\bar{\mu}$ (4) and $e\bar{e} \to e\bar{e}$ (5).

The aim of the present paper is to extend to $e\bar{e} \to q\bar{q}$ jets the analysis of electromagnetic and finite-width effects carried out previously in the leptonic channels (4,5). As discussed there, in the energy range below and not too far beyond the Z_0 pole the most relevant corrections come from soft photons and finite-width effects, after having absorbed the largest weak corrections into a redefinition of the mass and the width of the Z_0. Our results are then obtained by generalizing the study performed for μ pair production (4) to the case in which the produced particles have charges and axial vector couplings different from those of the annihilating pair. We can thus omit a detailed derivation of our formulae which can be easily obtained from ref. (4).

For the reaction

$$e^-(p_1) + e^+(p_2) \to q_f(p_3) + \bar{q}_f(p_4)$$

we define

$$s = W^2 = 4E^2 = (p_1 + p_2)^2 , \tag{1a}$$

$$z = \cos\theta = \hat{p}_1 \cdot \hat{p}_3 = \hat{p}_2 \cdot \hat{p}_4 , \tag{1b}$$

$$a = \sin\frac{\theta}{2}, \qquad b = \cos\frac{\theta}{2}, \tag{1c}$$

$$\beta_e = \frac{4\alpha}{\pi}\ln\left(\frac{W}{m_e} - \frac{1}{2}\right), \tag{2a}$$

$$\beta_f = \frac{4\alpha}{\pi}Q_f^2\left(\ln\frac{W}{\mu} - \frac{1}{2}\right), \tag{2b}$$

(3) D. H. SCHILLER: *Z. Phys. C*, **3**, 21 (1979); G. SCHIERHOLZ and D. H. SCHILLER: DESY 79/29 (1979), unpublished; M. J. PUHALA, Z. J. REK and B. L. YOUNG: *Phys. Rev. D*, **23**, 89 (1981); M. J. PUHALA, Z. J. REK, B. L. YOUNG and X. T. ZHU: *Phys. Rev. D*, **25**, 95 (1982).

(4) M. GRECO, G. PANCHERI-SRIVASTAVA and Y. SRIVASTAVA: *Nucl. Phys. B*, **171**, 118 (1980); **197**, 543 (1982) (E).

(5) M. CONSOLI, M. GRECO and S. LO PRESTI: *Phys. Lett. B*, **113**, 415 (1982). See also F. ANTONELLI, M. CONSOLI, G. CORBÒ and O. PELLEGRINO: *Nucl. Phys. B*, **183**, 195 (1981).

$$\beta_{\text{int}} = (-Q_f)\frac{4\alpha}{\pi}\ln \operatorname{tg}\frac{\theta}{2}\,, \tag{2c}$$

$$\Delta \equiv \frac{\Delta\omega}{E}\,, \tag{3}$$

where eQ_f is the charge of the quark q_f, μ is an effective quark mass, of the order of the average transverse momentum of the jet, and $\Delta\omega$ is the energy resolution. We will discuss more in detail these quantities later.

The weak boson is taken to be a resonance of renormalized mass M and width Γ, such that the phase shift δ_R is

$$\operatorname{tg}\delta_R = \frac{M\Gamma}{M^2 - s}\,. \tag{4}$$

The lepton and quark axial vector coupling are

$$\begin{cases} a = -1\,, & v = -1 + 4\sin^2\theta_W\,, \quad Q_l = -1 \quad (l = e^-, \mu)\,,\\ a_f = +1\,, & v_f = 1 - \frac{8}{3}\sin^2\theta_W\,, \quad Q_f = \frac{2}{3} \quad (f = \text{up quarks})\,,\\ a_f = -1\,, & v_f = -1 + \frac{4}{3}\sin^2\theta_W\,, \quad Q_f = -\frac{1}{3} \quad (f = \text{down quarks})\,. \end{cases} \tag{5}$$

We also define

$$r_V = \frac{vv_f}{v^2 + a^2}\,, \qquad r_A = \frac{aa_f}{v^2 + a^2}\,, \tag{6a}$$

$$\chi(s) = \frac{v^2 + a^2}{4\sin^2(2\theta_W)}\,\frac{s}{s - M^2 + iM\Gamma}\,. \tag{6b}$$

Then the differential cross-section for producing a quark q_f, of charge Q_f at the angle θ, can be written as follows:

$$\frac{d\sigma_f(\theta)}{d\Omega} = C^{\text{QED}}_{\text{infra}}\left(\frac{d\sigma_{\text{QED}}}{d\Omega}\right)(1 + C^{\text{QED}}_F) + C^{\text{int}}_{\text{infra}}\left(\frac{d\sigma_{\text{int}}}{d\Omega}, V\right)(1 + C^{\text{int},V}_F) + \\ + C^{\text{int}}_{\text{infra}}\left(\frac{d\sigma_{\text{int},A}}{d\Omega}\right)(1 + C^{\text{int},A}_F) + C^{\text{res}}_{\text{infra}}\left(\frac{d\sigma_{\text{res}}}{d\Omega}\right)(1 + C^{\text{res}}_F)\,, \tag{7}$$

where the Born cross-sections are given by [2,5]

$$\frac{d\sigma_{\text{QED}}}{d\Omega} = \frac{3\alpha^2}{4s}(1 + z^2)(Q_f^2)\,, \tag{8a}$$

$$\frac{d\sigma_{\text{int},V}}{d\Omega} = \frac{3\alpha^2}{4s}(1 + z^2)(2\operatorname{Re}\chi)(-Q_f)r_V\,, \tag{8b}$$

(8c) $$\frac{d\sigma_{\text{int,A}}}{d\Omega} = \frac{3\alpha^2}{4s}\, 2z(2\,\mathrm{Re}\,\chi)(-Q_f)r_A\,,$$

(8d) $$\frac{d\sigma_{\text{res}}}{d\Omega} = \frac{3\alpha^2}{4s}\left\{(1+z^2)\frac{v_f^2+a_f^2}{v^2+a^2} + 8zr_V r_A\right\}|\chi|^2\,.$$

$C^{(i)}_{\text{infra}}$ are the infra-red factor associated with each of the respective cross-sections and have been derived in ref. ([4]). They include the effect of soft-photon radiation, both real and virtual, to all orders in α. They take the form

(9a) $$C^{\text{QED}}_{\text{infra}} = (\Delta)^{\beta_e+\beta_f+2\beta_{\text{int}}}\,,$$

(9b) $$C^{\text{int}}_{\text{infra}} = (\Delta)^{\beta_f+\beta_{\text{int}}}\frac{1}{\cos\delta_R}\,\mathrm{Re}\left\{\exp[i\delta_R]\left[\frac{\Delta}{1+\Delta(s/M\Gamma)\exp[i\delta_R]\sin\delta_R}\right]^{\beta_e}\cdot\left[\frac{\Delta}{\Delta+(M\Gamma/s)\exp[-i\delta_R]/\sin\delta_R}\right]^{\beta_{\text{int}}}\right\},$$

(9c) $$C^{\text{res}}_{\text{infra}} = (\Delta)^{\beta_f}\left|\frac{\Delta}{1+\Delta(s/M\Gamma)\exp[i\delta_R]\sin\delta_R}\right|^{\beta_e}\cdot\left|\frac{\Delta}{\Delta+(M\Gamma/s)\exp[-i\delta_R]/\sin\delta_R}\right|^{2\beta_{\text{int}}}[1-\beta_e\delta(s,\Delta\omega)\,\mathrm{ctg}\,\delta_R]\,,$$

where

(10) $$\delta(s,\Delta\omega) = \mathrm{arctg}\frac{2\sqrt{s}\,\Delta\omega-(s-M^2)}{M\Gamma} + \mathrm{arctg}\frac{s-M^2}{M\Gamma}\,.$$

The last factor in the square bracket of eq. (9c) is responsible for the radiative tail of the resonance. More details can be found in ref. ([4]).

Finally the finite factor $C^{(i)}_F$ incorporates the rest of the corrections due to vertex, vacuum polarization and $\gamma\gamma$ and γZ box diagram terms. They are

(11a) $$C^{\text{QED}}_F = \frac{3}{4}(\beta_e+\beta_f) + \frac{\alpha}{\pi}\left(\frac{\pi^2}{3}-\frac{1}{2}\right)(1+Q_f^2) + \delta_{\text{V.P.}} + X_V(\theta)\,,$$

(11b) $$C^{\text{int,V}}_F = \frac{3}{4}(\beta_e+\beta_f) + \frac{\alpha}{\pi}\left(\frac{\pi^2}{3}-\frac{1}{2}\right)(1+Q_f^2) + \frac{1}{2}\delta_{\text{V.P.}} + \frac{1}{2}X_V(\theta)\,,$$

(11c) $$C^{\text{int,A}}_F = \frac{3}{4}(\beta_e+\beta_f) + \frac{\alpha}{\pi}\left(\frac{\pi^2}{3}-\frac{1}{2}\right)(1+Q_f^2) + \frac{1}{2}\delta_{\text{V.P.}} + \frac{1}{2}X_A(\theta)\,,$$

(11d) $$C^{\text{res}}_F = \frac{3}{4}(\beta_e+\beta_f) + \frac{\alpha}{\pi}\left(\frac{\pi^2}{3}-\frac{1}{2}\right)(1+Q_f^2) - 8\alpha\sin^2(2\theta_W)\frac{M\Gamma}{s}\cdot\frac{r_V Y_V(\theta)+r_A Y_A(\theta)}{(1+z^2)(v_f^2+a_f^2)+8zr_A r_V(v^2+a^2)}\,,$$

where $\delta_{\text{v.p.}}$ includes the corrections coming from the real part of the vacuum polarization due both to the leptons μ and τ and to the hadrons. For the latter we have used the expression given in ref. [4].

Finally the functions $X_V(\theta)$, $X_A(\theta)$, $Y_V(\theta)$ and $Y_A(\theta)$ are given by

$$X_V(\theta) = \frac{4\alpha Q_f}{\pi}\left\{\frac{1}{1+z^2}[z(\ln a^2 + \ln b^2) + a^2 \ln b - b^2 \ln a] + \right. \\ \left. + (\ln b)^2 - (\ln a)^2 + \frac{1}{2}[\mathrm{Li}_2(a^2) - \mathrm{Li}_2(b^2)]\right\}, \tag{12a}$$

$$X_A(\theta) = \frac{2\alpha Q_f}{\pi}\left\{\mathrm{Li}_2(a^2) - \mathrm{Li}_2(b^2) - \ln^2 a + \ln^2 b - \frac{1}{z}(b^2 \ln a + a^2 \ln b)\right\}, \tag{12b}$$

$$Y_V(\theta) = Q_f^2\left[z - 2z\ln(ab) + 2(1+z^2)\ln\frac{a}{b}\right] + Q_f(1+z^2)\frac{\bar{R}}{3}, \tag{12c}$$

$$Y_A(\theta) = Q_f^2\left[2z\ln\frac{a}{b} + 1\right] + 2zQ_f\frac{\bar{R}}{3} \tag{12d}$$

with $\bar{R} = \sum_{i=\text{leptons}+\text{quarks}} Q_i^2$.

Clearly our results reduce to those of μ pair production in the limit $v_f = v$, $a_f = a$ and $Q_f = -1$.

A few comments are in order, concerning the effective quark mass μ and the energy resolution $\Delta\omega$. The origin of an effective quark mass, which enters in the radiative quark factor β_f (eq. (2b)), can be traced back to the cancellation of the quark mass singularities in the physical $q\bar{q}$ cross-section, after including the QCD radiation emitted from the quark legs in the definition of the jets. Then, as is well known, the original quark mass gets replaced by a typical hadronic average transverse momentum $\langle p_\perp \rangle \sim \mu$. For this reason our formulae cannot be applied to large-$p_\perp$ events, which should be analysed by explicit use of a $q\bar{q}g$ final state. Therefore, our results refer to pure $q\bar{q}$ jets, which have been obtained from the whole of hadronic events after appropriate cuts on $p_\perp$, or T, or similar commonly used jet variables. The dependence on μ is, however, not appreciable, as indicated explicitly below.

The best way to experimentally determine the energy resolution $\Delta\omega$, as for the pure leptonic channels, is to look at the relative disalignment of the two jets. Then for typical acollinearity angles $\Delta\theta = (10\div 20)°$ one obtains values $\Delta\omega/E = 0.16\div 0.30$, which are of the same order of those used in our numerical results.

The 2-jet cross-section $d\sigma_{f\bar{f}}$ is defined as

$$d\sigma_{f\bar{f}}(\theta) = d\sigma_f(\theta) + d\sigma_{\bar{f}}(\theta) = d\sigma_f(\theta) + d\sigma_f(\pi - \theta). \tag{13}$$

Then, for a doublet of $u\bar{u}$ and $d\bar{d}$ quark pairs, we show in fig. 1 the radiative-correction factor

$$\delta_{r.c.} = (d\sigma_{u\bar{u}} + d\sigma_{d\bar{d}})/(d\sigma^0_{u\bar{u}} + d\sigma^0_{d\bar{d}}) - 1 \tag{14}$$

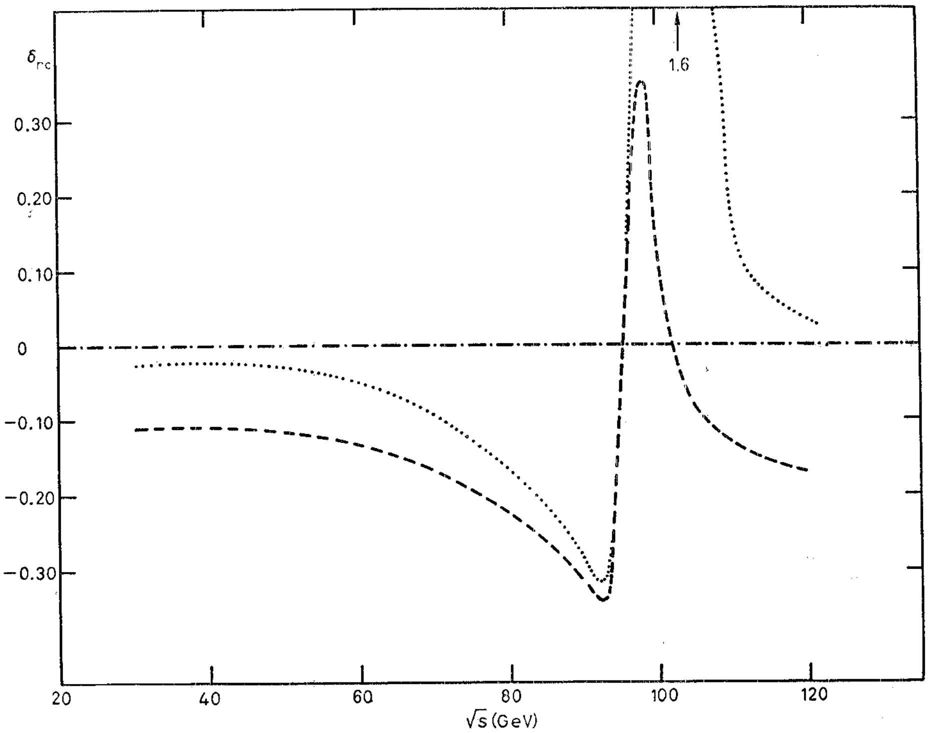

Fig. 1. – Radiative correction factor (eq. (14)) as a function of the centre-of-mass energy, for two values of the energy resolution Δ: ··· $\Delta = 0.2$, ——— $\Delta = 0.1$.

for $\Delta = 0.1 \div 0.2$ and $\mu = 0.300$ GeV, as a function of the centre-of-mass energy. An increase of μ by a factor two changes these results by $\leqslant 1\%$. With the presently accepted value $\sin^2\theta \simeq 0.22$ we have used $M = 93.2$ GeV, $\Gamma = 2.9$ GeV, which are the renormalized values of ref. (5).

As an effect of the radiation corrections, and similarly to what has been found (6) for $e\bar{e} \to \mu\bar{\mu}$, the two-jet cross-section gets modified from the usual $1 + \cos^2\theta$ distribution near the forward-backward direction. More explicitly one finds, for intermediate energies,

$$d\sigma_{2\text{-jet}}(z) \sim (1 + z^2)\left\{1 + \alpha f(z)\frac{s}{s - M^2}aa_f\right\}, \tag{15}$$

(6) M. GRECO, G. PANCHERI-SRIVASTAVA and Y. SRIVASTAVA: *Phys. Lett. B*, **80**, 390 (1979).

where $f(\cos\theta) \propto \ln^2(\sin\theta/2)$, $\ln^2(\cos\theta/2)$, $\ln(\text{tg}\,\theta/2)\ln(\Delta\omega/E)$. This effect, however, is limited to a very few percent and can be observed only by an accuracy of $\sim 1\%$ at PETRA-PEP energies, with a good detection efficiency near the forward-backward direction.

One of most common methods which have been envisaged to measure quark asymmetries is to look for leading-particle effects in the angular distributions of inclusive produced hadrons (3). We apply our formalism to such a type of measurements. If we neglect masses and transverse momenta, the cross-section for observing an hadron h at angle θ with fractional momentum $x = p^h_\parallel/p_q \simeq 2E^h/\sqrt{s}$ is given by

$$\text{d}\sigma^h(\theta, x) = \sum_q [\text{d}\sigma_q(\theta) D^h_q(x) + \text{d}\sigma_{\bar q}(\theta) D^h_{\bar q}(x)]\,, \tag{16}$$

where the sum goes over all quarks which are active at energy $\sqrt{s}$. We shall restrict ourselves to the case of pions and kaons at large x, which in turn limits the sum in eq. (16) to u, d and s parent quarks.

We shall use the following parameterization (7) for the $D^h_q(x)$'s:

$$\begin{cases} D^{\pi^+}_u(x) = D^{\pi^+}_{\bar d}(x) = (1+x)(1-x)^2/2x\,, \\ D^{\pi^+}_{\bar u}(x) = D^{\pi^+}_d(x) = D^{\pi^+}_s(x) = D^{\pi^+}_{\bar s}(x) = \dfrac{1-x}{1+x} D^{\pi^+}_u(x)\,, \\ D^{K^+}_u(x) = \dfrac{1+x}{2x}\{0.025 + 0.3(1-x)^2 - 1.25(1-x)^3 + 1.85(1-x)^4\}\,, \\ D^{K^+}_{\bar u}(x) = D^{K^+}_d(x) = D^{K^+}_{\bar d}(x) = D^{K^+}_s(x) = \dfrac{1-x}{1+x} D^{K^+}_u(x)\,, \\ D^{K^+}_{\bar s}(x) = \dfrac{1}{x}\{0.082 + 0.03(1-x)^2 + 0.35(1-x)^3\}\,. \end{cases} \tag{17}$$

Then defining the hadronic asymmetries as

$$A^h(\theta, x) = \frac{\text{d}\sigma^h(\theta, x) - \text{d}\sigma^h(\pi-\theta, x)}{\text{d}\sigma^h(\theta, x) + \text{d}\sigma^h(\pi-\theta, x)}\,, \tag{18}$$

we show in fig. 2 the integrated π^+ and K^+ asymmetries as functions of x for various energies and $\Delta\omega/E = 0.2$. The naïve asymmetries are also shown.

In conclusion we have presented a detailed study of radiative corrections for $q\bar q$ jets at energies below and near the Z_0 boson mass. Our formulae contain soft-photon effects to all orders in α and all finite terms in α. Hard-photon ef-

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(7) L. M. SEHGAL: *Proceedings of the 1977 International Symposium on Leptonic and Photon Interactions of High Energies*, edited by F. GUTBROT (Hamburg, 1977).

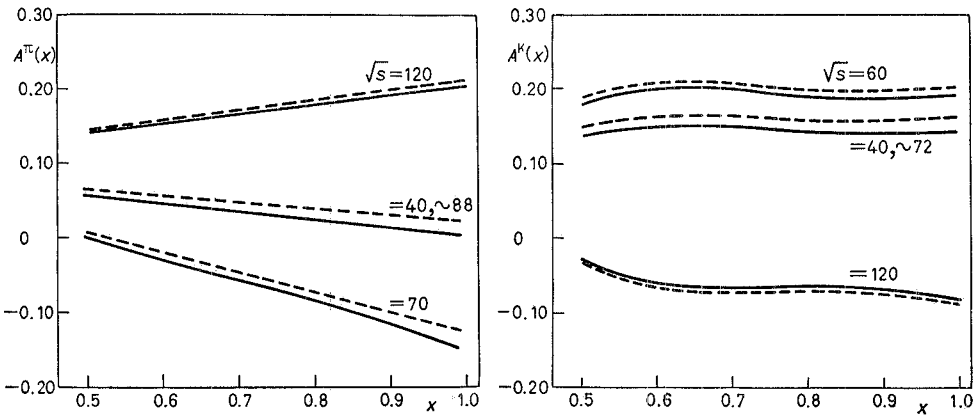

Fig. 2. – Corrected (solid lines) and uncorrected (dashed lines) integrated asymmetries for π^+ and K^+ at different energies as functions of the longitudinal hadron fractional momentum.

fects, different from the tail effect, are not included in our analysis and must be taken into account separately. As an application we have also discussed the inclusive hadron asymmetries at large fractional momenta.

After completed this work we became aware of a calculation by F. BERENDS, R. KLEISS and S. JADACH (8), where a similar analysis is presented to first order in α. Their results agree with the expansion of our eq. (7) to this order.

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(8) F. BERENDS, R. KLEISS and S. JODACH: Leiden preprint (1982).

● RIASSUNTO

Si discutono effetti elettromagnetici e di larghezza finita per produzione di jet $q\bar{q}$ nell'annichilazione e^+e^- vicino al polo dello Z_0. Sono considerate anche le asimmetrie angolari per produzione inclusiva di π^+ e K^+ di grande impulso frazionario.

Резюме не получено.

M. CONSOLI, *et al.*
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