

ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

To be submitted to
Phys. Letters

LNF-82/20(P)
16 Marzo 1982

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 e^+e^- SINGLE ARM EXPERIMENTS

INFN - Laboratori Nazionali di Frascati
Servizio Documentazione

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QCD PREDICTIONS FOR ENERGY MOMENTS IN e^+e^- SINGLE ARM EXPERIMENTS.

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ABSTRACT

We show that a new class of large infrared corrections in QCD can be easily observable by measuring the moments of the jets energy deposited in a calorimeter in single cone experiments.

There has been recently much interest in studying a class of large perturbative corrections to various hard processes in QCD⁽¹⁾. They arise near the boundaries of the phase space for multigluon emission, where the infrared singularities become competitive to the mass singularities and all of them have to be resummed. A critical signal of this kind of corrections is the appearance of squared logarithms of the appropriate scales, as for example found in the Sudakov form factors and in Drell-Yan processes.

In this paper we will study another example of large double logs, namely those observable in single arm calorimetric experiments for e^+e^- jets. In the standard double cone experiments à la Sterman-Weinberg⁽²⁾ one meets terms as $\alpha \ln \delta \ln \epsilon$ as the maximal singularity. We will show that in a single cone experiment one finds additional terms like $\alpha \ln^2 \epsilon$ which should be resummed. The solution to this problem can be simply accounted for by the use of the appropriate kinematics, as found previously elsewhere⁽¹⁾.

Let us consider a single cone of semi aperture δ around an arbitrary, but fixed, direction θ with respect to the e^+e^- beams of total energy $Q=2E$, and let $\Delta\Omega$ be the solid angle covered by this cone. We shall consider angles δ small enough to identify the angle of the calorimeter θ with the angle of a parton in the cone. We are interested in the measurement of the moments of the hadronic energy $M^{(n)}(\delta, Q)$ deposited in such a cone⁽³⁾. In the naive parton model the distribution of the energy E_C which flows in the detector is given by

$$\frac{dP}{dx} = \frac{1}{\sigma_B} \frac{d\sigma}{dx} = \delta(1-x), \quad (1)$$

where $x = E_c/E$, and

$$\sigma_B = \left(\frac{d\sigma}{d\Omega} \right)_0 \Delta\Omega = \frac{\alpha_{e.m.}^2}{4Q^2} (1 + \cos^2\theta) \Delta\Omega \sum_f 3 Q_f^2, \quad (2)$$

is the cross section for $e^+e^- \rightarrow q\bar{q}$ in the Born approximation. In the following we shall neglect hadronization effects, of order $\langle p_T^2 \rangle / E^2$.

To first order in α we can classify the events in terms of the number of the partons which enter in the cone, similarly to ref. (4). In the first case ($N=1$) we have one single parton in the cone, then

$$\begin{cases} E_c = x_i E \\ 0 < x < 1 \end{cases} \quad i = q, \bar{q}, g \quad (3)$$

with $\sum x_i = 2$. In addition we can have two partons in the case ($N=2$):

$$\begin{cases} E_c = (x_i + x_j) E \\ 1 < x < \frac{1}{1+\cos\theta} \end{cases} \quad (4)$$

To higher orders one has $N > 2$. To obtain the distribution in x one needs the partial cross sections for $e^+e^- \rightarrow q\bar{q}g$:

$$\frac{d}{dx_q dx_{\bar{q}} d(\cos\theta_q)} = \frac{3}{8} \sigma_0 \left[D_T(x_q, x_{\bar{q}}) (1 + \cos^2\theta_q) + D_L(x_q, x_{\bar{q}}) \sin^2\theta_q \right] \quad (5)$$

where $\sigma_0 = \frac{4\pi\alpha_{e.m.}^2}{Q^2} \sum_f Q_f^2$ and

$$\begin{aligned} D_T(x_q, x_{\bar{q}}) &= \frac{C_F \alpha}{2\pi} \left[\frac{x_q^2 + x_{\bar{q}}^2}{(1-x_q)(1-x_{\bar{q}})} - 2(x_q + x_{\bar{q}} - 1) \right], \\ D_L(x_q, x_{\bar{q}}) &= \frac{4C_F \alpha}{2\pi} \left[x_q + x_{\bar{q}} - 1 \right]. \end{aligned} \quad (6)$$

In the following we shall take into account only the term proportional to $(x_q^2 + x_{\bar{q}}^2) / (1-x_q)(1-x_{\bar{q}})$ because the rest is free from infrared and mass singularities and doesn't contribute to the total cross section.

From our classification of events we have

$$\frac{dP}{dx} \stackrel{(N=1)}{=} \frac{1}{2} \sum_i \iint \alpha(i) [1-\alpha(j)] [1-\alpha(k)] \delta(x-x_i), \quad 0 < x < 1 \quad (7a)$$

$$\frac{dP}{dx} \stackrel{(N=2)}{=} \frac{1}{2} \sum_{ij} \iint \theta(i) \theta(j) \delta(x-x_i-x_j), \quad 1 < x < \frac{2}{1+\cos\delta} \quad (7b)$$

where $\theta(i)$ indicates that the i -th parton is constrained in the cone, and in the above equations the various differential cross sections $d\sigma/dx_i dx_j$, suitably normalized, are understood. A factor $1/2$ has been put in front of Eqs. (7) in order to satisfy the sum rule for the first moment of the distribution⁽⁵⁾ (see later).

To regularize the distributions (7) at $x=1$ one should add the virtual contributions which, in turn, are related to the real ones, integrated over the all phase space. Then Eq. (7a) becomes

$$\begin{aligned} \frac{dP}{dx} \stackrel{(N=1)}{=} & \frac{1}{2} \sum_{i=q,\bar{q}} \iint \theta(i) \left[\delta(x-x_i) - \delta(x-1) \right] + \frac{1}{2} \iint \theta(g) \delta(x-x_g) - \\ & - \frac{1}{2} \sum_{ij} \iint \theta(i) \theta(j) \delta(x-x_i). \end{aligned} \quad (8)$$

Adding the (N=1) and (N=2) contributions and distributing the various terms in order to cancel explicitly the infrared and mass singularities one finally obtains for the n -th moment:

$$\begin{aligned} M^{(n)}(\delta, Q) = \int dx x^n \left(\frac{dP}{dx} \right) &= 1 + \frac{\alpha}{2\pi} C_F \iint \frac{x_q^2 + x_{\bar{q}}^2}{(1-x_q)(1-x_{\bar{q}})} x \\ & \times \left\{ \left(\frac{x_q^n - 1}{2} \right) \left[\theta(q) - \theta(q, g) \right] + \left(\frac{x_{\bar{q}}^n - 1}{2} \right) \left[\theta(\bar{q}) - \theta(\bar{q}, g) \right] + \frac{x_g^n}{2} \left[\theta(g) - \theta(g, q) - \theta(g, \bar{q}) \right] \right. \\ & + \frac{(x_q + x_{\bar{q}})^n - x_q^n - x_{\bar{q}}^n}{2} \theta(q, \bar{q}) + \frac{(x_q + x_g)^n - 1}{2} \theta(\bar{q}, g) + \\ & \left. + \frac{(x_{\bar{q}} + x_g)^n - 1}{2} \theta(q, g) \right\}, \end{aligned} \quad (9)$$

where $\theta(i, j) = \theta(i) \theta(j)$. As it can be easily checked, the first moment $M^{(1)}$ is correctly normalized to one.

In the approximation of δ small, and in the limit n large, the leading contributions are coming from the first two terms of Eq. (9). Then

$$M^{(n)}(\delta, Q) - 1 \approx \frac{\alpha C_F}{2\pi} \iint \frac{x_q^2 + x_{\bar{q}}^2}{(1-x_q)(1-x_{\bar{q}})} (x_q^n - 1) \left[\theta(q) - \theta(q, g) \right] \equiv I_1^{(n)} - I_2^{(n)} \quad (10)$$

The left over terms are of order $\frac{1}{n} \ln \delta$ for the contributions of the gluon in the cone and of order $\delta \ln n$ when the (q, \bar{q}) , (q, g) and (\bar{q}, g) respectively, are both in the cone.

In Eq. (10) the first term behaves as

$$I_1^{(n)} = \frac{\alpha C_F}{2\pi} \left(\frac{Q^2}{4\pi\mu^2} \right)^{-\varepsilon} \frac{1}{\Gamma(1-\varepsilon)} \int_0^1 dx (x^n - 1) \left[\frac{1+x^2}{1-x} \left(-\frac{1}{2} + \ln x^2 + \right. \right. \\ \left. \left. + \ln(1-x) + \dots \right) \right] \quad (11)$$

where we have used dimensional regularization in $d=4-2\epsilon$ dimensions.

On the other hand $I_2^{(n)}$, in the limit of small δ , factorizes in a divergent angular integral times a finite moment in x :

$$I_2^{(n)} = \frac{\alpha C_F}{2\pi} \left(\frac{Q^2}{4\pi\mu^2} \right)^{-\epsilon} \frac{1}{\Gamma(1-\epsilon)} \left\{ -\frac{1}{\epsilon} + \ln \sin^2 \frac{\delta}{2} + \dots \right\} x \int_0^1 dx (x^{n-1}) \frac{1+x^2}{1-x} \left[1-2\epsilon \ln x - 2\epsilon \ln(1-x) + \dots \right] , \quad (12)$$

Adding Eqs. (11) and (12) one finally obtains

$$M^{(n)}(\delta, Q)^{-1} \simeq \frac{\alpha C_F}{2\pi} \int_0^1 dx (x^{n-1}) \frac{1+x^2}{1-x} \ln \frac{1}{\sin^2 \frac{\delta}{2} (1-x)} . \quad (13)$$

Our result shows a new $\ln^2 n$ behaviour which is absent in the usual double cone analysis⁽²⁾. Later we shall give a simple interpretation of this behaviour. Due to the appearance of their large $\alpha \ln^2 n$ correction the problem arises of resumming together the double logs of comparable strength, namely $\ln \delta \ln n$ and $\ln^2 n$ to all orders.

This can be achieved due to the very structure of Eq. (10). As suggested by this first order calculation, the total result is essentially given by the probability that a quark (antiquark) has an energy x without any angular constraints on the gluons emitted minus the probability that both the quark (antiquark) and the gluon are in the same cone. Now the first contribution exponentiates in the manner given by the renormalization group, i.e.

$$M_1^{(n)} = \exp \left\{ \frac{C_F}{2\pi} \int_0^1 dx (x^{n-1}) \frac{1+x^2}{1-x} \int_{\lambda^2}^{Q^2} \frac{dt}{t} \alpha(t) \right\} , \quad (14)$$

where λ^2 is the infrared cutoff. Then the second term must behave in the same way, having to cancel the λ^2 singularity⁽⁷⁾, namely

$$M_2^{(n)} = \exp \left\{ \frac{C_F}{2\pi} \int_0^1 dx (x^{n-1}) \frac{1+x^2}{1-x} \int_{\lambda^2}^{Q^2 \sin^2 \frac{\delta}{2} (1-x)} \frac{dt}{t} \alpha(t) \right\} . \quad (15)$$

The upper limit in the t integral, which is fixed by the first order calculation, has a simple interpretation. In fact, in the limit $\delta \rightarrow 0$ and $x \rightarrow 1$ it gives the invariant mass t of the quark and the gluon

$$t = \frac{Q^2 x(1-x) \sin^2 \delta/2}{1-x \sin^2 \delta/2} \rightarrow Q^2 (1-x) \sin^2 \delta/2 . \quad (16)$$

This invariant mass has to be calculated for a quark and a gluon which have a relative maximal angle δ and not 2δ . With this definition in fact one consistently recovers the first order result of Eq. (13), by explicit integration over the q, \bar{q} and g Dalitz plot.

We therefore can write the n moment as

$$M^{(n)}(\delta, Q) = \exp \left\{ \frac{C_F}{2\pi} \int_0^1 dx (x^{n-1}) \frac{1+x^2}{1-x} \int_{Q^2 (1-x) \sin^2 \delta/2}^{Q^2} \frac{dt}{t} \alpha(t) \right\} . \quad (17)$$

Finally one should take into account a further kinematical effect coming from the production of quark and gluons

pairs in the gluon bremsstrahlung. This amounts to replace $\alpha(t) \rightarrow \alpha(k_{\perp}^2)$, where $k_{\perp}^2 = x(1-x)t^{(1)}$. Then Eq. (17) becomes

$$M^{(n)}(\delta, Q) = \exp \left\{ \frac{C_F}{2\pi} \int_0^1 dx (x^n - 1) \frac{1+x^2}{1-x} \int_{Q^2(1-x)^2 \sin^2 \delta/2}^{Q^2(1-x)} \frac{dk_{\perp}^2}{k_{\perp}^2} \alpha(k_{\perp}^2) \right\}, \quad (18)$$

which is our final result.

A few comments are in order. First, if one neglects the $(1-x)$ dependence in the last integral of Eq. (18), one recovers the results of ref. (8) in the large n limit, namely

$$\tilde{M}^{(n)}(\delta, Q) = \exp \left\{ \frac{C_F}{2\pi} \int_0^1 dx (x^n - 1) \frac{1+x^2}{1-x} \int_{Q^2 \delta^2}^{Q^2} \frac{dk^2}{k^2} \alpha(k^2) \right\}. \quad (19)$$

From a phenomenological point of view Eq. (18) differs drastically from the naive results (19). This is shown in Fig. 1 where we plot, for fixed $Q=30$ GeV, two moments, $M^{(6)}$ and $M^{(10)}$, as a function of δ .

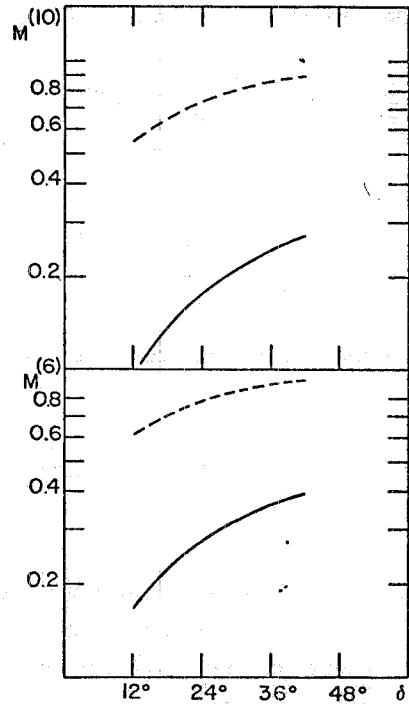


FIG. 1 - Moments $M^{(6)}$ and $M^{(10)}$ as a function of δ . The full and dotted lines refer to Eqs. (18) and (19) respectively.

Consistently with the current phenomenology, we have used $\alpha(k_{\perp}^2) = 12\pi/25 \ln \left[(k_{\perp}^2 + M^2)/\Lambda^2 \right]$ with $\Lambda \approx 0.3$ GeV and $M \approx 1$ GeV. Roughly speaking the same numerical agreement between Eqs. (18-19) is achieved with the replacements

$$\Lambda^2 \sim \Lambda^2 n, \quad \delta^2 \sim \delta^2/n \quad (20)$$

In Fig. (2) we show the combined plot of $M^{(6)}$ and $M^{(10)}$ for the same values of δ . As it can be seen, this

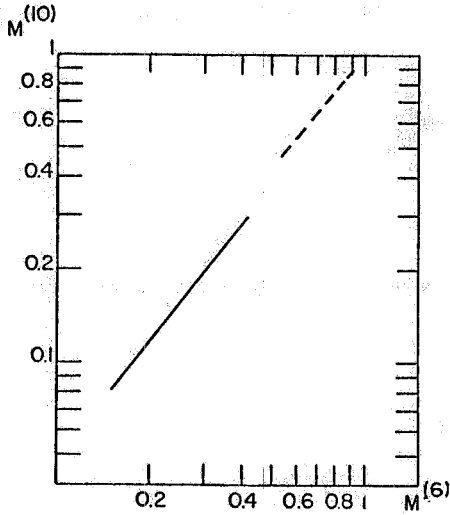


FIG. 2 - Plot of $\log M^{(10)}$ versus $\log M^{(6)}$ for various δ . Full and dotted lines as in Fig. 1.

representation doesn't show large differences between Eqs. (18-19), suggesting therefore that this kind of analysis is not a significant test of QCD.

We finally give a Sterman-Weinberg type formula for the fraction $f(\epsilon, \delta)$ of events with all the energy but at most a fraction ϵ inside a single cone of half angle δ . To first order, we obtain for $x \sim 1$

$$\frac{dP}{dx} = \delta(1-x) + \frac{\alpha C_F}{2\pi} \left[\frac{1+x^2}{1-x} \ln \frac{4}{\delta^2(1-x)} \right]_+ \quad (21)$$

and

$$f(\epsilon, \delta) = \int_{1-\epsilon}^1 dx \frac{dP}{dx} = 1 - \frac{\alpha C_F}{\pi} \left[\ln \frac{4}{\delta^2} \left(\ln \frac{1}{\epsilon} - \frac{3}{4} \right) + \frac{1}{2} \ln^2 \epsilon \right] \quad (22)$$

The exponentiated result reads then as

$$\frac{dP}{dx} = \frac{1}{2\pi} \int_{-\infty}^{\infty} db e^{ib(1-x)} \exp \left\{ \frac{C_F}{2\pi} \int_0^1 dx \frac{1+z^2}{1-z} \int_{Q^2(1-z)^2 \sin^2 \delta/2}^{Q^2(1-z)} \frac{dk_{\perp}^2}{k_{\perp}^2} \alpha(k_{\perp}^2) \left[e^{-ib(1-z)} - 1 \right] \right\} \quad (23)$$

which cannot be solved analytically unless $\alpha(k_{\perp}^2)$ is taken fixed and equal to some effective value $\bar{\alpha}$. This approximation has been already discussed in ref. (9) and gives a rough estimate of the distribution. A better approximation can only be obtained via numerical integration. A very rough estimate gives $f(\epsilon, \delta) \sim M^{(n=1/\epsilon)}(\delta)$.

In conclusion we have shown that a new class of large perturbative corrections is present and can be revealed in single cone calorimeter type experiments in e^+e^- annihilation. The observation of such a behaviour can give a new support to general ideas of resumming large infrared corrections.

We acknowledge useful discussions with F. Barreiro and R. Petronzio.

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