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MONTE CARLO CALCULATION OF ENERGY SPECTRUM AND SPATIAL DISTRIBUTION OF PHOTONS FROM POSITRON ANNIHILATION

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ABSTRACT

A Monte Carlo computing program has been used to calculate the photon spectrum and emittance from positron annihilation and bremsstrahlung.The positron energy spread, the energy loss and the multiple scattering in the annihilation target have been taken into account. Moreover the positron emittance, the positron incidence angle and the finite angular acceptance of the photon collimation channel have been explicitly considered.

Experimental results, obtained with a pair spectrometer and a beam profile monitor, are in good agreement with calculations. In particular it is shown that the positron emittance has a crucial influence on the shape of the photon spectrum and on the absolute value of the photon flux.

1. - INTRODUCTION

The measurement of photonuclear cross-sections using the quasi monochromatic photon beam from the annihilation in flight of positrons in a target of low atomic number is a well established technique[1,2] and has many advantages as regards as using a continuous bremsstrahlung beam. In order to determine shape and absolute value of different photonuclear cross sections, an accurate knowledge of the photon flux N(k) is needed.

In this paper we drescribe a computer program, which uses a Monte Carlo technique, to calculate the photon energy spectra both from positron annihilation and from bremsstrahlung. The program also calculates the angular and transversal distributions of the photon beam at different selectable distances from the annihilation target. The calculation takes into account the lateral extent, the divergence and the energy spread of the incident positron beam. Moreover, the energy loss due to the ionization and the multiple scattering of positrons in the annihilation target, the finite angular acceptance of the photon collimator channel and the central value of the photon beam collection angles are also explicitly considered.

Several calculations of the photon spectrum have been previously performed[3,4], but no one has included in a proper way, that is without making any approximation, all the above effects.

We will first recall the cross section formulae of different processes used in the calculation; then, the general scheme of the Monte Carlo program, the checks and the comparison with an analytic calculation will be described. Finally, the importance of the inclusion of the positron beam emittance will be stressed and the comparison with the experimental results obtained with the Frascati beam, will be given.

2. - FORMULAE USED IN THE CALCULATION OF THE PHOTON SPECTRUM

The predominant reactions when a high energy positron ($\gtrsim 5$ MeV) collides on a target are the following:

$$e^+ + e^- \rightarrow 2\gamma \qquad (1)$$

$$e^+ + e^- \rightarrow e^+ + e^- + \gamma \qquad (2)$$

$$e^+ + Z \rightarrow e^+ + Z + \gamma \qquad (3)$$

Reactions (2) and (3) constitute an unaivoidable background source to the monochromatic photon from the two-body final state (1). The relative background from (2) and (3) scale as Z(Z+1) with the target material. For this reason it is desiderable to use hydrogen as the target material. Although the total bremsstrahlung cross section is orders of magnitude greater than the total annihilation cross section, the bremsstrahlung angular intensity decreases with increasing angle much more rapidly than the annihilation one. Therefore, it is possible to increase the ratio annihilation/bremsstrahlung at the expenses of the intensity, by collecting photons at angles as large as possible.

When a positron of energy E annihilates, the energy k of the photon emitted at a laboratory angle ϑ is given by

$$k = mc^2/(1 - \delta \cos\vartheta),$$

where $\delta = \sqrt{(\gamma-1)/(\gamma+1)}$, with $\gamma = E/mc^2$ ($mc^2 = 0.51$ MeV is the electron mass). When $\vartheta \ll 1$ and $\gamma \gg 1$, the two-photon differential cross section (reaction (1) is given by[5]

$$\frac{d\sigma}{d\Omega} = \frac{r_e^2}{2} \cdot \frac{1}{(1+z)^2}\left(\frac{2\gamma}{1+l} + \frac{1+l}{2\gamma} + \frac{4l}{(1+l)^2}\right), \qquad (4)$$

where: $r_e = e^2/mc^2 \simeq 2.82 \cdot 10^{-13}$ cm is the classical radius of the electron, $z = \gamma\vartheta^2/2$ and $l = \gamma^2\vartheta^2$.

For the process (3) the Schiff cross section, which is very accurate for $\vartheta \ll 1$, has been used[6]

$$\frac{d\sigma}{d\Omega dk} = \frac{2\alpha r_e^2}{\pi}\,\frac{\gamma^2}{k}\,Z^2\left\{\frac{16\,l(1-\eta)}{(1+l)^4} - \frac{(2-\eta)^2}{(1+l)^2} + \left[\frac{2-2\eta+\eta^2}{(1+l)^2} - \frac{4l(1-\eta)}{(1+l)^4}\right] \ln\left[\frac{\left[2\gamma(\eta^{-1}-1)\right]^2}{(1+S^2)}\right]\right\}, \qquad (5)$$

where $\eta = k/E$, α is the fine structure constant and S is the screening factor:

$$S = \frac{2\gamma(\eta^{-1}-1)\,Z^{1/3}}{111(1+l)} .$$

For the process (2) the expression (5) with Z=1 has been used when $\vartheta \ll \sqrt{2/\gamma}$. Otherwise the following fit of the Swanson calculation has been used(7)

$$\frac{d\sigma}{d\Omega dk} = \frac{2\alpha r_e^2}{\pi}\,\frac{\gamma^2}{k}\,F\left[\frac{1}{\gamma^4\vartheta^4}+\frac{1}{4\gamma^2}\right]\left\{2(2-2y+y^2)\ln\left[2\gamma(y^{-1}-1)\right]-3+3y-y^2\right\} , \tag{6}$$

where $y=k/k_m$; with $k_m=mc^2\gamma/(1+z)$, and F is a scaling function, defined in ref. (7), which reduces the cross section to 0 for $k=k_m$:

$$F = \left\{(1-y\exp\left[-3(1-y)^{1/3}\right]\right\}\left\{1-0.1\sin\left[\cos^{-1}\left(\frac{1-z}{1+z}\right)\right]\right\}$$

We have used the Nigam and Scott correction(8) to the Moliere's calculations in order to compute the probability $P(\vartheta_s)d\vartheta_s$ for a positron to be multiply scattered through a cone ϑ_s, $\vartheta_s + d\vartheta_s$:

$$P(\vartheta_s)\,d\vartheta_s = \frac{2\vartheta_s}{\langle\vartheta_s^2\rangle}\exp\left(-\frac{\vartheta_s^2}{\langle\vartheta_s^2\rangle}\right)d\vartheta_s , \tag{7}$$

where:

$$\langle\vartheta_s^2\rangle = \chi^2(-0.17 + 1.13\ln\Omega)$$

with

$$\chi^2 = 0.157\,\frac{Z(Z+1)}{A}\,\frac{t}{E^2} \qquad \Omega = \chi^2 \Big/ \left[0.595\,\frac{Z^{2/3}}{E^2}\right]$$

where E is the positron energy in MeV, t and A are the thickness (in g/cm^2) and the atomic weight of the target.

For energy collision losses it has been used the theory of Landau, with If we call F (E, t, Δ) dΔ the probability that a positron of incident energy E has suffered, after traversing a thickness t of matter, an energy loss between Δ and $\Delta + d\Delta$, we can write, apart from a normalization factor:

$$F(E,t,\Delta)\,d\Delta = \exp\left[-\alpha^+(\lambda + \ln\alpha^+)\right]\varphi(\lambda)d\lambda \tag{8}$$

where

$$\alpha^+ = \zeta\left[2-(\gamma+1)^{-2}\right] , \quad \lambda = \frac{\Delta}{\zeta(E-mc^2)} - \left[\ln\frac{\zeta(E-mc^2)mc^2}{(h\nu_p)^2}+1.114\right]$$

$$\zeta = \frac{2\pi r_e^2 mc^2 N_o Z}{(E-mc^2)\,A}\,t$$

with N_o= Avogadro number,

$\nu_p = \left(\frac{N_o\,\varrho\,Z\,e^2}{\pi mA}\right)^{1/2}$: plasma frequency of medium of density ϱ .

$\varphi(\lambda)$ is the Landau universal function, for which we used the analytical expression given by Blunck and Leisegang[10].

For radiation energy losses, the probability distribution diverges for low energy loss, which means that the most probable energy loss by radiation is zero. On the other hand it is also known that for the mean energy losses, it is valid the relation:

$$\frac{(d\xi/dx)_{rad}}{(d\xi/dx)_{coll.}} \cong \frac{E \cdot Z}{1600\ mc^2}$$

which shows the predominance of collision on radiation energy loss, for low Z and intermediate positron energy. So we have neglected radiation losses. Care must however be taken on extending this approximation to higher energy.

3. - THE MONTE CARLO PROGRAM

In this section we describe the main lines of the calculation and the Monte Carlo sampling techniques adopted. In the following, we use a coordinate system in which the z axis is coincident with the end straight section of the positron beam handling system.

The incident positron angle on the annihilation target is extracted according to a gaussian distribution, for which the centre ϑ_γ and the standard deviation are deduced from given positron beam deflection and divergence. The x,y coordinates of the incident point on the annihilation target are chosen according to two gaussian distributions, with centres at x=y=0 and standard deviations deduced from experimental values of the beam profiles. The annihilation or radiation point inside the target is then extracted with uniform law in the target thickness. The energy losses due to ionization and the multiple scattering of each positron in the target, before the photon production, are extracted according to the given distribution laws, eqs. (7) and (8). Finally the photon emission angle is selected according to an uniform distribution and is weighted for the appropriate differential cross section value.

If the photon escaping from the target crosses all the lead collimators, its energy, its flight direction cosines and the coordinates of the impinging point on the x-y planes, for given distances from the annihilation target, are recorded. For a sake of simplicity the lead collimator transparency to photons has been assumed to be equal to 0. We have evaluated the order of magnitude of this approximation and we have found that the effect is negligible[11]. As an example, for E=180 MeV, and ϑ_γ =0.5°, the collimator real transparency increases the annihilation and bremsstrahlung total number of photons of +1.04% and +3.03%, respectively.

Special attention has been paid to the computation speed. In particular we have resorted to the following expedients;

a) in order to minimize photon losses, the photon emission angles were extracted within a variable solid angle (reduced by respect to the geometric one) whose value and orientation were calculated by taking into account the positron flight direction and the photon production point in the target;
b) for each positron a number of photons proportional to the selected solid angle was extracted;
c) each collected photon was regarded both as an annihilation photon and as a bremsstrahlung one. In the annihilation spectrum, the energy channel arising from the one-to-one correspondence with the angle of the photon was increased of a quantity equal to the cross section of the event. In the bremsstrahlung spectrum, being any energy available at a given angle, all the energy channels were increased of the proper cross section values.

In the order to be able to calculate the absolute value of the spectrum, special attention was paid to the overall normalization.

The program has as its input parameters the energy, the rms energy spread, the emittance and the central value of the incident angle of the positron; the thickness, the density and the material of the annihilation target; the photon channel characteristics (number, shape and relative distances of the collimators).

The program calculates the annihilation and the bremsstrahlung energy spectra and the photon beam emittance at different selectable distances from the annihilation target.

On a medium scale computer, as a DEC-VAX 11/780, about 60' CPU are required to run the program in order to reach a statistical error better than 1% and less than 20 K bytes of memory location are busy.

4. - CHECKS AND COMPARISON WITH OTHER CALCULATIONS

As we have stressed, any complete analytic calculation of the three-dimensional development of the process would involve considerable mathematical difficulties. Therefore, in order to compare the Monte Carlo results with an analytic calculation, we have run the Monte Carlo program for the idealised case of an incident monochromatic positron beam having negligible divergence and lateral extent. Among the previous calculations the Mancini and Sanzone's[4] is realistic enough to warrant a quantitative comparison. The calulation has been carried on a hydrogen target, 0.0118 radiation length thick, for a geometric photon collection solid angle $\Delta\Omega_0=5.0 \times 10^{-5}$ sr centered at $\vartheta_\gamma = 1°$, and a positron energy E=100 MeV.

A comparison between the results obtained from the two programs is shown in Fig. 1. The agreement between the spectra gives confidence on the accuracy of both methods of calculation.

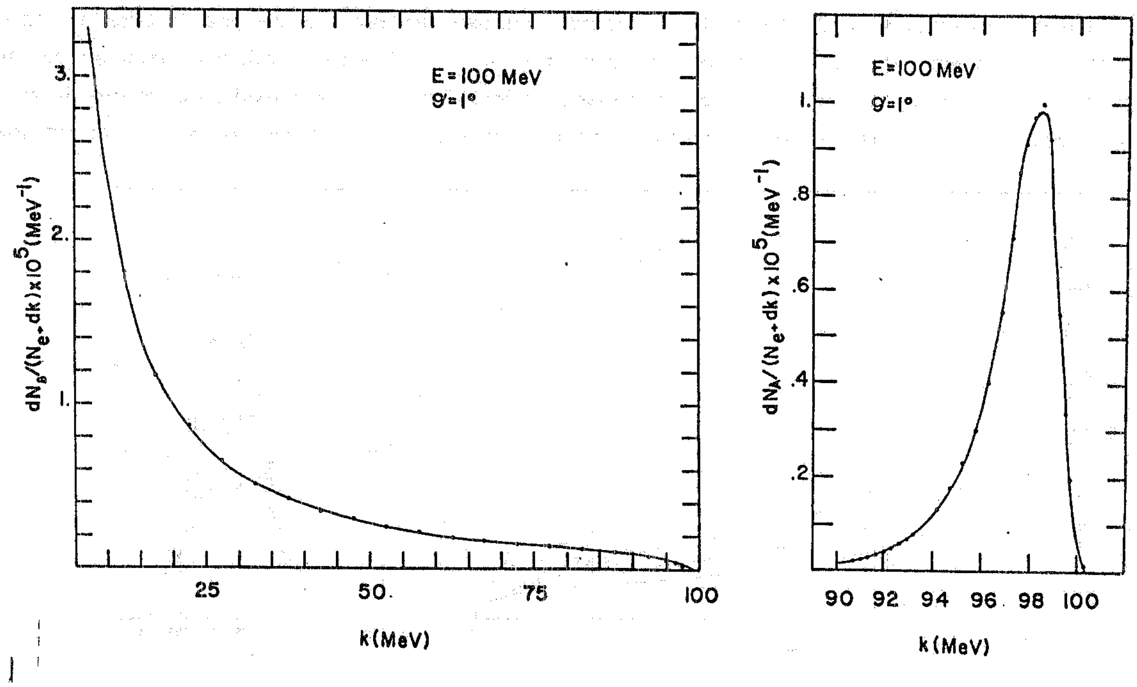

FIG. 1 - Calculated photon spectra per incident positron from monochromatic, divergenceless, point like positron bremsstrahlung a) and annihilation b) on a liquid hydrogen target. Positron total enrergy E=100 MeV, target thickness 0.018 radiation in lenght, geometric photon collecting solid angle $5\ 10^{-5}$ sr centred at $\vartheta_\gamma = 1°$. Solid curves are analytic calculations, points are Monte Carlo results.

To check the absolute normalization of the Monte Carlo results it is convenient to use a different distribution law for the photon emission angle ϑ, the only change being the physical meaning of the calculated yield. In particular, if we run the program for a uniform ϑ distribution, the photon yield must be equal to the geometric solid angle $\Delta\Omega_o$, defined by the collimators. A comparison of the values, $\Delta\Omega$, obtained with the program, with the geometric values, $\Delta\Omega_o$, is given in Table I.

TABLE I - Comparison of the evaluated photon collection solid angle $\Delta\Omega$ with the geometrical one, $\Delta\Omega_o$. ϑ_γ is the mean photon collection angle.

ϑ_γ (mr)	$\Delta\Omega_o$ (msr)	$\Delta\Omega$ (msr)
0.0 ± 4.0	0.515×10^{-4}	$(0.518 \pm 0.007) \times 10^{-4}$
17.5 ± 3.9	0.473×10^{-4}	$(0.472 \pm 0.006) \times 10^{-4}$

5. - INFLUENCE OF THE POSITRON EMITTANCE ON THE PHOTON SPECTRUM

As we have previously said, the Monte Carlo program has been written with the goal to take into account, between the other effects, the incident positron beam emittance. The influence of this parameter on the photon beam spectrum was not explicitly considered in previous calculations. This approximation results to be very crude, as shown in Fig. 2 where the bremsstrahlung and the annihilation photon spectra, calculated for several

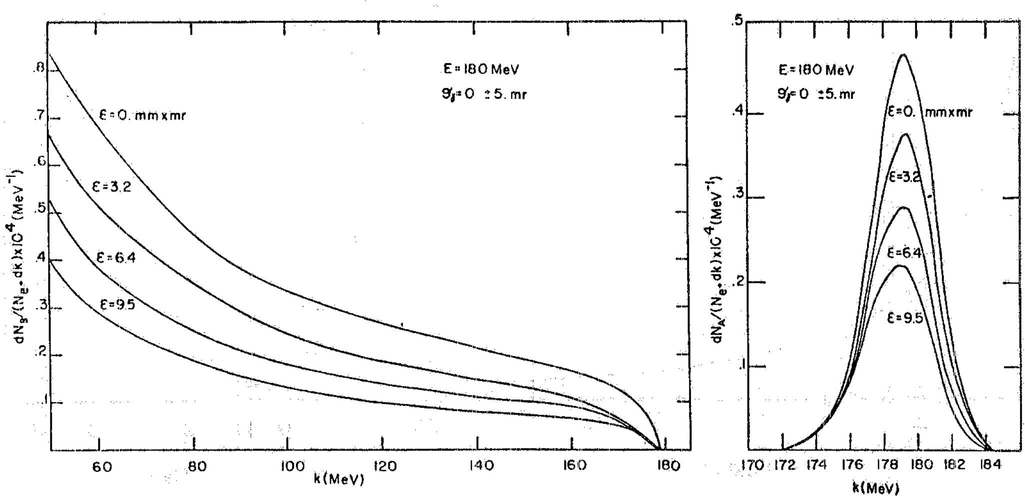

FIG. 2 - Influence of positron emittance ε on the bremsstrahlung a) and annihilation b) photon spectra. The calculations were performed in the following conditions: positron energy 180 MeV, positron energy spread 1.5% (FWHM); mean photon collecting solid angle 7.85×10^{-5} sr centred at 0°; hydrogen target 0.0118 radiation lenght thick.

positron emittance values, are given. From the figure it is evident that the positron emittance has a decisive influence on the shape of the photon spectrum, on the ratio N_A/N_B (annihilation to bremsstrahlung photons) and on the absolute value of the photon flux.

6. - COMPARISON WITH EXPERIMENT

Comparison between calulated and experimental photons spectra are given in Fig. 3. Data refer to the Frascati measurements, obtained with a pair spectrometer(12), in the following conditions:

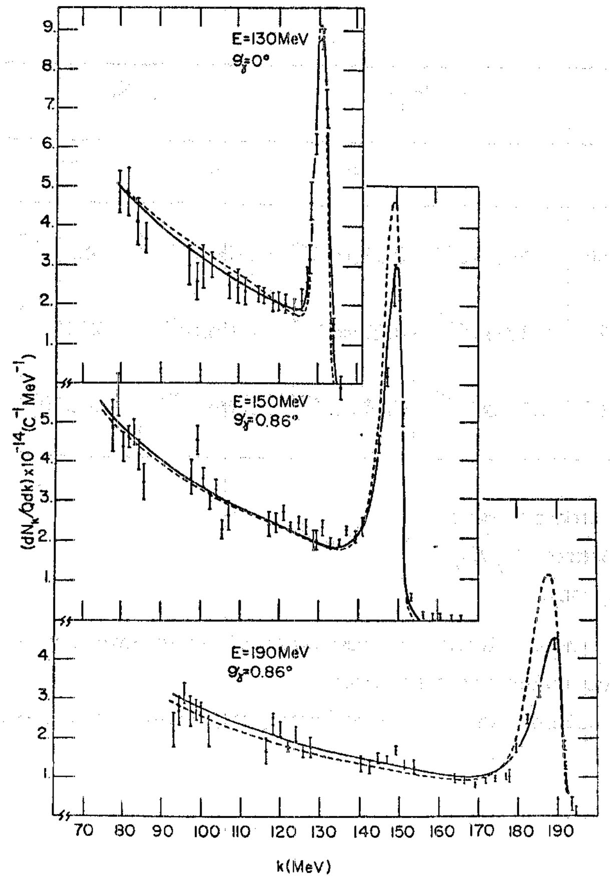

FIG. 3 - Comparison between calculated and measured photon spectra. Q is the quantameter collected charge. Experimental points are values obtained at Frascati with a pair spectrometer. Dotted and solid line curves refer to Monte Carlo calculations for a positron beam having negligible and measured emittance, respectively.

- positron beam: energy 190 MeV, 150 MeV and 130 MeV; radial emittance 18 mm x mrd, vertical emittance 15 mm x mrd; energy spread 1.5% (FWHM);
- annihilation target: liquid Hydrogen 0.0118 radiation length thick;
- geometric configuration of the photon beam collimation channel: five contiguous cylindrical lead collimators, diameter 0.9-1.0-1.05-1.15-1.22 cm respectively, 10.0 cm long each; distance from the annihilation target and the first collimator 101 cm.

The full line curves refer to the Monte Carlo results. As it can be seen, the shape of the spectrum is always well reproduced. Moreover, the integral

$$Q = \int_{2\,\mathrm{MeV}}^{k_M} S(k)\, k\, N(k)\, dk$$

(where k_M is the maximum photon energy value, S(k) is the quantameter sensitivity(13), and N(k) is the photon flux) evaluated by starting from the Monte Carlo results, is in agreement, within the 2.5%, with the measured quantameter value. The dotted line curves are the Monte Carlo results for a positron beam with a neglible emittance. Moreover in Table II are given the experimental and the calulated values of:

TABLE II - Influence of positron emittance ε on the photon flux. E is the positron energy; ϑ_γ is the central photon collection angle; Q and F are the quantameter and the Farady cup collected charges; N_A/N_{e^+} is the total annihilation photon number per incident positron; and N_A/N_B is the ratio annihilation/bremsstrahlung (the bremsstrahlung spectrum is integrated from 2 MeV up to E). The calculations are performed for the two radial and vertical positron emittance conditions: a) $\varepsilon_r=\varepsilon_v=0$ mm x mr and b) $\varepsilon_r=18$ mm x mr, $\varepsilon_v=15$ mm x mr.

E	ϑ_γ	Q/F			N_A/N_{e^+}		N_A/N_B	
(MeV)	(°)	exp.	a)	b)	a)	b)	a)	b)
130.0	0.	$0.0790^{+0.0020}_{-0.0050}$	0.1520	0.0740	1.291×10^{-4}	0.527×10^{-4}	0.69×10^{-2}	0.58×10^{-2}
150.0	0.86	$0.0174^{+0.0012}_{-0.0025}$	0.0210	0.0176	0.321×10^{-4}	0.236×10^{-4}	1.47×10^{-2}	1.25×10^{-2}
190.0	0.86	0.0175 ± 0.0020	0.0150	0.0180	0.242×10^{-4}	0.194×10^{-2}	1.98×10^{-2}	1.27×10^{-2}

- the ratio between the integral Q and the total positron current F;
- the number of annihilation photons per incident positron, N_A/N_{e^+};
- the ratio N_A/N_B = annihilation/bremsstrahlung photons.

From the Fig. 3 and from the Table II it is once more evident the importance of taking into account the positron emittance for a good agreement between measurement and calculations.

Finally in Fig. 4 a comparison between measured (full line) and calculated (histogram) photon radial profile is

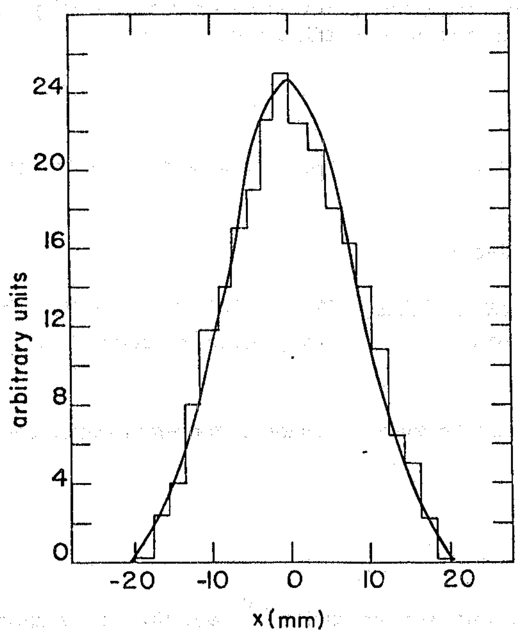

FIG. 4 - Comparison between measured (full line) and calculated (histogram) photon radial profile.

given. The measurements have been performed at Frascati with a multiwire chamber[14], set at a distance of 300 cm from the annihilation target. Experimental conditions: positron energy 200 MeV, positron radial emittance 12 mm x mr; mean photon collecting angle 0°. As can be seen, the agreement is fairly good.

In conclusion the described program is complete and versatile. It calculates correctly both the emittance and the energy spectrum of the quasi monochromatic photon beam. Moreover it gives the absolute value of the photon flux per unit of charge collected by the quantameter.

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