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BHABHA SCATTERING AROUND THE Z_0 POLE

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ABSTRACT.

An accurate investigation of electromagnetic and finite width effects for Bhabha scattering near the Z_0 pole is presented. Analytical expressions are given which contain all finite first order corrections as well as soft photons effects resummed to all orders. Weak interactions are only considered to renormalize the mass and the width of the vector boson. Some numerical results are also presented.

Experimental investigation of electroweak effects in high energy e^+e^- annihilation will present a crucial test for the standard model of weak interactions. Comparison with experiments, however, requires an accurate evaluation of radiative corrections which play an important role, as previously remarked by many authors[(1-3)]. The reaction $e^+e^- \to e^+e^-$ is particularly interesting for its large cross section providing a direct monitor of the beam luminosity. So far a complete calculation of weak and electromagnetic first order radiative corrections to this process, in the framework of the Weinberg-Salam model, has been performed in ref. (2). The validity of this calculation, however, does not extend to the energy range around the neutral vector boson pole since finite width effects were not taken into account. Furthermore weak corrections were shown in ref. (2) to play a minor role as compared to pure electromagnetic effects below and not too far beyond the Z_0 resonance.

The aim of this paper is to discuss Bhabha scattering in this energy range by improving the treatment of pure electromagnetic effects given in ref. (2) and properly taking into account the finite width of the vector boson along the same lines of ref. (3). As discussed there for the reaction $e^+e^- \to \mu^+\mu^-$, this kind of corrections are of primary importance and can sizably change

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the naive expectations. The most relevant weak corrections can be absorbed into a redefinition of the mass and width of the Z_0, as suggested in refs. (4,5). The left over terms, of order $\frac{\alpha}{\pi}\ln(\frac{s}{M^2})$, which are negligible for $s \sim M^2$ and very cumbersome to be treated analytically, will not be considered here.

Soft photons effects are resummed to all orders, with no restriction on the relative magnitude of the energy resolution $\Delta\omega$ and the Z_0 width Γ. Hard photon effects, other than the tail effect, are not included.

We give a brief account of the derivation of our formulae. The relevant virtual graphs are shown in Fig. 1.

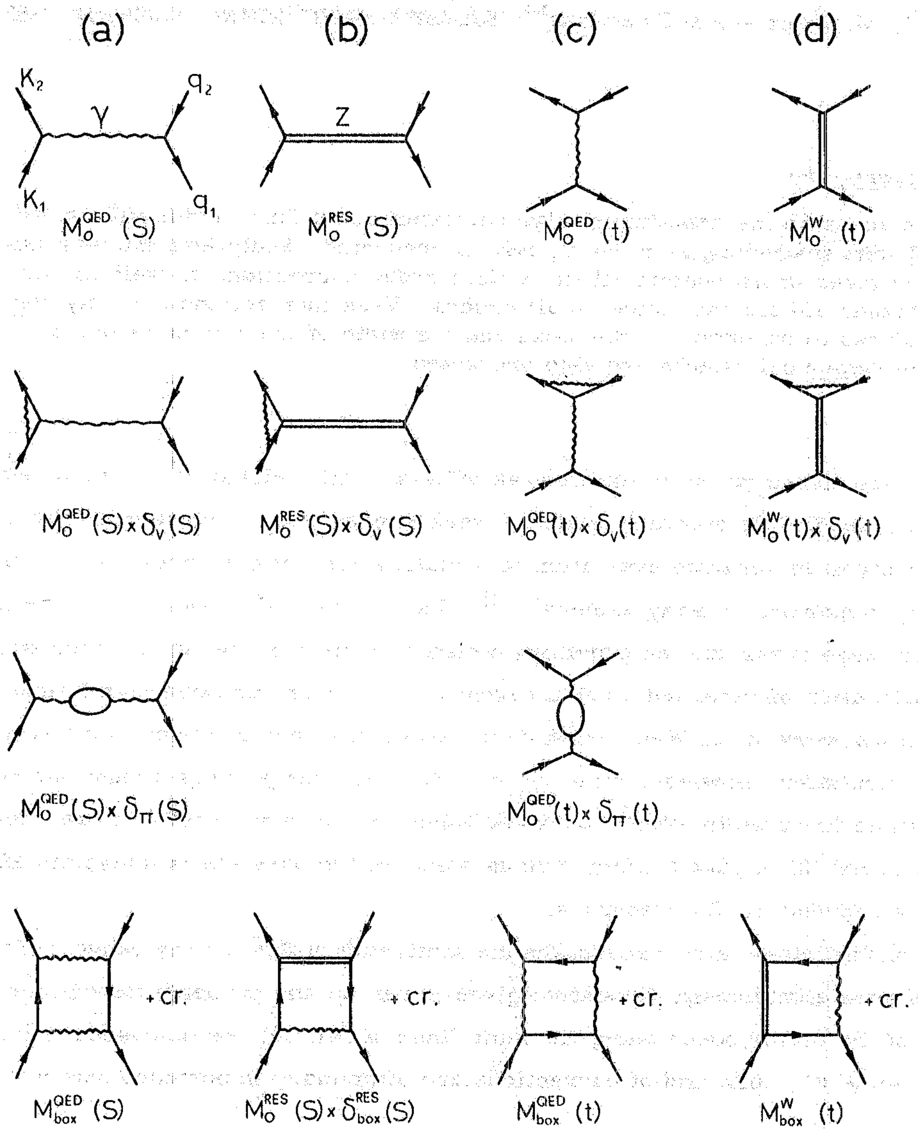

FIG. 1 - Virtual graphs in s and t channel.

With the same notation we have

$$M(s,t) = M_o^{QED}(s)\left[1+\delta^{QED}(s)\right] - M_o^{QED}(t)\left[1+\delta^{QED}(t)\right]$$
$$+ M_o^{RES}(s)\left[1+\delta^{RES}(s)\right] - M_o^{W}(t)\left[1+\delta^{W}(t)\right] + M_{box}^{QED}(s) - M_{box}^{QED}(t)\,, \tag{1}$$

where

$$M_o^{QED}(s) = \frac{e^2}{s} J_\mu(s)J'_\mu(s)\,, \quad M_o^{QED}(t) = \frac{e^2}{t} J_\mu(t)J'_\mu(t)$$

$$M_o^{RES}(s) = \frac{e^2}{s-M_R^2}\left[f_V J_\mu(s)+f_A A_\mu(s)\right]\left[f_V J'_\mu(s)+f_A A'_\mu(s)\right]$$

$$M_o^{W}(t) = \frac{e^2}{t-M^2}\left[f_V J_\mu(t)+f_A A_\mu(t)\right]\left[f_V J'_\mu(t)+f_A A'_\mu(t)\right] \tag{2}$$

$$M_{box}^{QED}(s) = \frac{2\alpha^2}{s}\left[J_\mu(s)J'_\mu(s)\,(V_1(s)+iV_2(s))+A_\mu(s)A'_\mu(s)(A_1(s)+iA_2(s))\right]$$

$$M_{box}^{QED}(t) = \frac{2\alpha^2}{t}\left[J_\mu(t)J'_\mu(t)(V_1(t)+iV_2(t))+A_\mu(t)A'_\mu(t)(A_1(t)+iA_2(t))\right],$$

with

$$J_\mu(s) = \bar{v}(k_2)\gamma_\mu u(k_1) \qquad J'_\mu(s) = \bar{u}(q_1)\gamma_\mu v(q_2)$$
$$A_\mu(s) = \bar{v}(k_2)\gamma_\mu\gamma_5 u(k_1) \qquad A'_\mu(s) = \bar{u}(q_1)\gamma_\mu\gamma_5 v(q_2)$$
$$J_\mu(t) = \bar{u}(q_1)\gamma_\mu u(k_1) \qquad J'_\mu(t) = \bar{v}(k_2)\gamma_\mu v(q_2) \tag{3}$$
$$A_\mu(t) = \bar{u}(q_1)\gamma_\mu\gamma_5 u(k_1) \qquad A'_\mu(t) = \bar{v}(k_2)\gamma_\mu\gamma_5 v(q_2),$$

and

$$f_V = \frac{4\sin^2\theta_W - 1}{4\sin\theta_W\cos\theta_W}\,, \quad f_A = \frac{-1}{4\sin\theta_W\cos\theta_W}\,,$$

θ_W being the weak mixing angle.

Moreover the following notations will be used

$$s = (k_1+k_2)^2 = 4E^2\,, \qquad t = (k_1-q_1)^2 = -s\,\frac{(1-\cos\theta)}{2}$$

$$z = \cos\theta, \; a = \sin\frac{\theta}{2}, \quad b = \cos\frac{\theta}{2}$$

$$\beta_e = \frac{2\alpha}{\pi}\left(\ln\frac{s}{m^2} - 1\right), \quad \beta_{int} = \frac{4\alpha}{\pi}\ln\frac{a}{b}$$

$$\Delta \equiv \frac{\Delta\omega}{E} = \text{fractional energy resolution}$$

The weak boson is taken a resonance of mass M and width Γ, with $M_R^2 = M^2 - iM\Gamma$ and phase shift $\delta_R(s)$ where $\mathrm{tg}\,\delta_R(s) = \dfrac{M\Gamma}{M^2 - s}$.

The radiative factors δ's in eq. (1) are defined as follows (see Fig. 1):

$$\begin{aligned}
\delta^{QED}(x) &= 2\delta_v(x) + \delta_\pi(x) \qquad\qquad (x = s,t)\\
\delta^{RES}(s) &= 2\delta_v(s) + \delta^{RES}_{box}(s)\\
\delta^{W}(t) &= 2\delta_v(t) + \frac{4\alpha}{\pi}\ln b\,\ln\frac{2E}{\lambda} + i2\alpha\,\ln\frac{2E}{\lambda} + \delta^{W}_{box}(t)
\end{aligned} \tag{4}$$

where the γ-Z box diagrams contributions $\delta^{RES}_{box}(s)$ and $\delta^{W}_{box}(t)$ are given in eqs. (9). The vertex and vacuum polarization parts in eq. (4) are

$$\begin{aligned}
\delta_v(s) \equiv \delta^R_v(s) + i\delta^I_v(s) &= \left[-\frac{1}{2}\beta_e\ln\frac{2E}{\lambda} + \frac{\alpha}{2\pi}\left(\frac{1}{2}\ln^2\frac{s}{m^2} - \ln\frac{s}{m^2}\right)\right.\\
&\quad \left. + \frac{3}{8}\beta_e + \frac{\alpha}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{4}\right)\right] + i\frac{\alpha}{\pi}\left[\pi\ln\frac{2E}{\lambda} - \frac{3}{4}\pi\right]\\
\delta_v(t) &= -\frac{1}{2}\beta_e\ln\frac{2E}{\lambda} - \frac{2\alpha}{\pi}\ln a\,\ln\frac{2E}{\lambda} + \frac{\alpha}{2\pi}\left(\frac{1}{2}\ln^2\frac{s}{m^2} - \ln\frac{s}{m^2}\right)\\
&\quad + \frac{3}{8}\beta_e + \frac{\alpha}{\pi}\left(\frac{3}{2}\ln a - \ln^2 a\right) + \frac{\alpha}{\pi}\left(\frac{\pi^2}{12} - \frac{1}{4}\right)
\end{aligned} \tag{5}$$

and[(x)]

(x) The hadronic contribution to the photon self-energy has been computed with the "effective" light quark masses introduced in ref. (4) by using dispersion relations.

$$\delta_\pi(s) \equiv \delta_\pi^R(s)+i\delta_\pi^I(s) = \frac{\alpha}{3\pi}\sum_{i=\ell,q} Q_i^2(\ell n\,\frac{s}{m_i^2} - \frac{5}{3}) + i(-\frac{\alpha}{3}\sum_{i=\ell,q} Q_i^2)$$

$$\delta_\pi(t) = \frac{\alpha}{3\pi}\sum_{i=\ell,q} Q_i{}^2(\ell n\,(\frac{-t}{m_i^2}) - \frac{5}{3}) \qquad (6)$$

with

$$Q_\ell^2 = 1, Q_i^2 = \frac{4}{3}\ (\text{up}),\ Q_i^2 = \frac{1}{3}\ (\text{down})$$

The QED box diagrams contributions are

$$V_1(s) = -8\ell n\,\frac{a}{b}\,\ell n\,\frac{2E}{\lambda} - z\,(\frac{\ell n^2 a}{b^4} + \frac{\ell n^2 b}{a^4}) + \frac{\ell na}{b^2} - \frac{\ell nb}{a^2} \equiv$$

$$\equiv -8\ell n\,\frac{a}{b}\,\ell n\,\frac{2E}{\lambda} + V_1^f(s)$$

$$V_2(s) = 2\pi\left[2\ell n\,\frac{a}{b} - \frac{1}{2}\,z\,(\frac{\ell na}{b^4} + \frac{\ell nb}{a^4}) - \frac{z}{1-z^2}\right] \qquad (7)$$

$$A_1(s) = -z\,(\frac{\ell n^2 a}{b^4} - \frac{\ell n^2 b}{a^4}) + \frac{\ell na}{b^2} + \frac{\ell nb}{a^2}$$

$$A_2(s) = 2\pi\left[-\frac{1}{2}\,z\,(\frac{\ell na}{b^4} - \frac{\ell nb}{a^4}) + \frac{1}{1-z^2}\right]$$

and

$$V_1(t) = 8\,\ell n\,b\,\ell n\,\frac{2E}{\lambda} + 8\,\ell n\,a\,\ell n\,b + \frac{\pi^2}{4}\,(1-b^4) + \frac{1-b^4}{b^4}\,\ell n^2 a$$

$$+ (1-b^4)\,\ell n^2\frac{a}{b} + \frac{a^2}{b^2}\,\ell n\,a + a^2\ell n\,\frac{a}{b} \equiv 8\ell n\,b\,\ell n\,\frac{2E}{\lambda} + V_1^f(t)$$

$$A_1(t) = -\frac{\pi^2}{4}\,(1-b^4) + (\frac{1-b^4}{b^4})\ell n^2 a - (1-b^4)\,\ell n^2\frac{a}{b} + \frac{a^2}{b^2}\ell na - a^2\ell n\,\frac{a}{b} \qquad (8)$$

$$V_2(t) = 4\pi\,\ell n\,\frac{2E}{\lambda} + \pi\left[4\ell na + \frac{1-b^4}{b^4}\,\ell na + \frac{a^2}{2b^2}\right] \equiv 4\pi\,\ell n\,\frac{2E}{\lambda} + V_2^f(t)$$

$$A_2(t) = \pi\left[\frac{1-b^4}{b^4}\,\ell na + \frac{a^2}{2b^2}\right]$$

Finally the γ-Z box diagrams terms $\delta^{RES}_{box}(s)$ and $\delta^{W}_{box}(t)$ appearing in eqs.(4) are given by

$$\delta^{RES}_{box}(s) = \frac{2\alpha}{\pi}\left[\ell n\frac{b}{a}\;\ell n\frac{(M_R^2-s)^2}{M_R^2\;\lambda^2} + \frac{1}{2}sp(a^2)-\frac{1}{2}sp(b^2)-\ell n^2a+\ell n^2b\right]$$

$$\delta^{W}_{box}(t) = \frac{\alpha}{\pi}\left[-4\ell na\;\ell nb + 2\ell n^2b-sp(b^2) + \frac{5}{12}\pi^2+2\ell n(a^2+1)(i\pi+2\ell nb)\right] \tag{9}$$

and

$$sp(x) = -\int_0^x \frac{dt}{t}\;\ell n(1-t)$$

Let's briefly comment the above formulae. Most of the results in eqs.(4-9) are already known and can be found in the literature(1,3). In particular they include vertex corrections, vacuum polarization parts and box diagrams in the s-channel. The contributions of QED box diagrams in the t-channel were previously given(6), at the level of cross section, for Bhabha scattering including a pure vector resonance in the t-channel. We give in eqs.(2-8) the full amplitude which is needed in the most general case. Finally the γ-Z box diagrams in the t-channel (Fig. 1d) have been computed in the approximation of retaining only "QED-like" terms. That means we have not included those contributions which are of the same order of magnitude of the left over weak corrections and which vanish in the limit of $k \to 0$, k being the photon internal momentum. This is exactly the same approximation used in extracting the resonating part in the γ-Z s-channel box diagrams (Fig. 1b). This amount to approximate $M^W_{box}(t)$ as

$$M^W_{box}(t) = M^W_0(t)\;\frac{\alpha}{\pi}\left\{\ell n\;\frac{(-t+M^2)^2}{M^2\lambda^2}\;(i\pi+2\;\ell nb)+2\ell n^2b-sp(b^2)-\right.$$
$$\left. -\;4\ell na\;\ell nb + \frac{2}{3}\;\pi^2\;-\;sp\;\left(\frac{s+M^2}{M^2}\right)\right\} \tag{10}$$

and in the limit $s \sim M^2$

$$M^W_{box}(t) \simeq M^W_0(t)\;\left[4\;\frac{\alpha}{\pi}\;\ell nb\;\ell n\;\frac{2E}{\lambda}\;+\;i2\alpha\;\ell n\;\frac{2E}{\lambda}+\delta^W_{box}(t)\right] \tag{11}$$

Now we turn to discuss the contributions of the real photon emission diagrams (see Fig. 2).

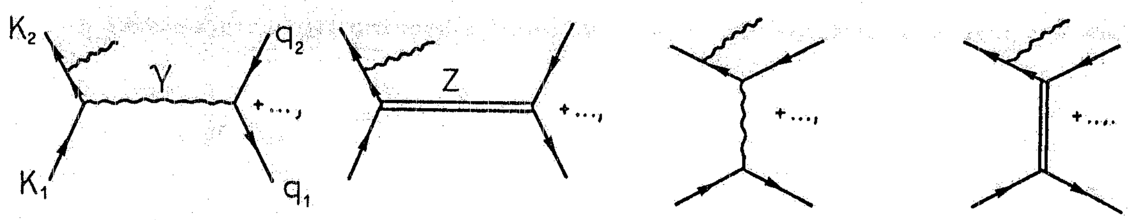

FIG. 2 - Bremsstrahlung diagrams.

The first order bremsstrahlung contributions, in the soft photon approximation, can be grouped, following ref. (3) in three different classes: "QED-like" terms, pure resonant terms and interference with the resonating amplitude. To this aim it is useful to define the various lowest order cross section as follows:

$$d\sigma_o[\gamma(s),\gamma(s)] = \frac{\alpha^2}{4s}(1+z^2) \equiv d\sigma_o(1) \tag{12.1}$$

$$d\sigma_o[\gamma(s),\gamma(t)] = -\frac{\alpha^2}{4s}\, 2\,\frac{(1+z)^2}{1-z} \equiv d\sigma_o(2) \tag{12.2}$$

$$d\sigma_o[\gamma(t),\gamma(t)] = \frac{\alpha^2}{4s}\frac{2}{(1-z)^2}\left[(1+z)^2+4\right] \equiv d\sigma_o(3) \tag{12.3}$$

$$d\sigma_o[\gamma(s),Z(t)] = -\frac{\alpha^2}{4s}\, 2R'(t)(1+z)^2(f_V^2+f_A^2) \equiv d\sigma_o(4) \tag{12.4}$$

$$d\sigma_o[\gamma(t),Z(t)] = \frac{\alpha^2}{4s}\frac{2}{1-z}\, 2R'(t)\left[(f_V^2+f_A^2)(1+z)^2+4(f_V^2-f_A^2)\right] \equiv d\sigma_o(5) \tag{12.5}$$

$$d\sigma_o[Z(t),Z(t)] = \frac{\alpha^2}{4s}\, 2R'^2(t)\Big[(1+z)^2\left[(f_V^2+f_A^2)^2+4f_V^2f_A^2\right] + 4\left[(f_V^2+f_A^2)^2-4f_V^2f_A^2\right]\Big] \equiv d\sigma_o(6) \tag{12.6}$$

$$d\sigma_o[Z(s),\gamma(s)] = \frac{\alpha^2}{4s}\, 2R'(s)\left[f_V^2(1+z^2)+f_A^2 2z\right] \equiv d\sigma_o(7) \tag{12.7}$$

$$d\sigma_o[Z(s),\gamma(t)] = -\frac{\alpha^2}{4s}\, 2R'(s)\,\frac{(1+z)^2}{1-z}\,(f_V^2+f_A^2) \equiv d\sigma_o(8) \tag{12.8}$$

$$d\sigma_o[Z(s),Z(t)] = -\frac{\alpha^2}{4s} R'(s)2R'(t)(1+z)^2\left[(f_V^2+f_A^2)^2+4f_V^2f_A^2\right] \equiv d\sigma_o(9) \tag{12.9}$$

$$d\sigma_o[Z(s),Z(s)] = \frac{\alpha^2}{4s}\left[R'^2(s)+I'^2(s)\right](f_V^2+f_A^2)^2\left[1+z^2+\frac{4f_V^2f_A^2}{(f_V^2+f_A^2)^2}\, 2z\right] \equiv d\sigma_o(10)\,, \tag{12.10}$$

and

$$R'(t) = \frac{1}{2}\frac{s}{M^2-t}\ ;\ R'(s)+iI'(s) \equiv \frac{s}{s-M_R^2}. \tag{12.11}$$

Then in terms of these elementary cross sections the bremsstrahlung terms read as

$$d\sigma(1\gamma) = \delta^{QED}(1\gamma)\sum_{i=1}^{6} d\sigma_o(i) + \delta^{int}(1\gamma)\sum_{i=7}^{9} d\sigma_o(i)+\delta^{RES}(1\gamma)d\sigma_o(10)\,, \tag{13}$$

with

$$\delta^{QED}(1\gamma) = (2\beta_e+2\beta_{int})\ln\frac{2E}{\lambda} + (2\beta_e+2\beta_{int})\ln\Delta - \frac{2\alpha}{\pi}B(m^2) + \frac{2\alpha}{\pi}F(a,b) \tag{14.1}$$

$$\delta^{int}(1\gamma) = (2\beta_e+2\beta_{int})\ln\frac{2E}{\lambda} + (\beta_{int}+\beta_e)\ln\Delta + \mathrm{Re}\left[\frac{e^{i\delta_R(s)}}{\cos\delta_R(s)}(\beta_e+\beta_{int})\ln\left(\frac{\Delta}{1+\frac{\Delta s}{M\Gamma}e^{i\delta_R(s)}\sin\delta_R(s)}\right)\right] - \frac{2\alpha}{\pi}B(m^2) + \frac{2\alpha}{\pi}F(a,b) \tag{14.2}$$

$$\delta^{RES}(1\gamma) = (2\beta_e+2\beta_{int})\ln\frac{2E}{\lambda} + \beta_e\ln\Delta - \beta_e\,\delta(s,\Delta\omega)\cot\delta_R(s) + (\beta_e+2\beta_{int})\ln\left|\frac{\Delta}{1+\frac{\Delta s}{M\Gamma}e^{i\delta_R(s)}\sin\delta_R(s)}\right| - \frac{2\alpha}{\pi}B(m^2) + \frac{2\alpha}{\pi}F(a,b)\,, \tag{14.3}$$

and

$$B(m^2) = \frac{\pi^2}{3} - \ln\frac{s}{m^2} + \frac{1}{2}\ln^2\frac{s}{m^2}$$

$$F(a,b) = 2\ln^2 a+sp(b^2) - 2\ln^2 b-sp(a^2) \tag{15}$$

$$\delta(s,\Delta\omega) = \mathrm{arctg}\,\phi_a + \mathrm{arctg}\,\phi_b$$

$$\phi_a = \frac{(2\Delta\omega\sqrt{s}+M^2-s)}{M\Gamma}\,, \qquad \phi_b = \frac{s-M^2}{M\Gamma}\,.$$

By adding the virtual and real corrections and by exponentiating the soft parts as in ref. (3) we obtain the final result,

$$d\sigma_{tot}(e^+e^-\to e^+e^-) = \sum_{i=1}^{10} C^{(i)}_{infra}\, d\sigma_o(i)\,[1+C_F^{(i)}] \tag{16}$$

where

$$C^{(i)}_{infra} = (\Delta)^{(2\beta_e+2\beta_{int})} \qquad (i=1,\ldots.6) \tag{17.1}$$

$$C^{(i)}_{infra} = (\Delta)^{(\beta_e+\beta_{int})} \frac{1}{\cos\delta_R(s)} \mathrm{Re}\left\{ e^{i\delta_R(s)} \left[\frac{\Delta}{1+\frac{\Delta s}{M\Gamma}\, e^{i\delta_R(s)}\sin\delta_R(s)}\right]^{\beta_e} \times \left[\frac{\Delta}{\Delta+\frac{M\Gamma}{s}\, e^{-i\delta_R(s)}/\sin\delta_R(s)}\right]^{\beta_{int}} \right\} \qquad (i=7,8,9) \tag{17.2}$$

$$C^{(10)}_{infra} = \Delta^{\beta_e} \left|\frac{\Delta}{1+\frac{\Delta s}{M\Gamma}e^{i\delta_R(s)}\sin\delta_R(s)}\right|^{\beta_e} \left|\frac{\Delta}{\Delta+\frac{M\Gamma}{s}e^{-i\delta_R(s)}/\sin\delta_R(s)}\right|^{2\beta_{int}} \times \left[1-\beta_e\delta(s,\Delta\omega)\cot\delta_R(s)\right] \tag{17.3}$$

and

$$C^{(1)}_F = \frac{3}{2}\beta_e + \frac{2\alpha}{\pi}\left[\frac{\pi^2}{3}-\frac{1}{2}\right] + \frac{2\alpha}{\pi}F(a,b)+2\delta^R_\pi(s) + \frac{\alpha}{\pi}\left[V^f_1(s) + \frac{2z}{1+z^2}A_1(s)\right] \tag{18.1}$$

$$C^{(2)}_F = \frac{3}{2}\beta_e + \frac{2\alpha}{\pi}\left[\frac{\pi^2}{12}-\frac{1}{2}\right] + \frac{2\alpha}{\pi}\left[\frac{3}{2}\ln a-\ln^2 a\right] + \frac{2\alpha}{\pi}F(a,b) + \delta^R_\pi(s) + \delta_\pi(t) + \frac{\alpha}{2\pi}\left[V^f_1(s) + A_1(s) + V^f_1(t) + A_1(t)\right] \tag{18.2}$$

$$C^{(3)}_F = \frac{3}{2}\beta_e - \frac{2\alpha}{\pi}\left[\frac{\pi^2}{6}+\frac{1}{2}\right] + \frac{4\alpha}{\pi}\left[\frac{3}{2}\ln a-\ln^2 a\right] + \frac{2\alpha}{\pi}F(a,b) + 2\delta_\pi(t) + \frac{\alpha}{\pi}\left[V^f_1(t) + \frac{b^4-1}{b^4+1}A_1(t)\right] \tag{18.3}$$

$$C^{(4)}_F = \frac{3}{2}\beta_e + \frac{2\alpha}{\pi}\left[\frac{\pi^2}{12}-\frac{1}{2}\right] + \frac{2\alpha}{\pi}\left[\frac{3}{2}\ln a-\ln^2 a\right] + \frac{2\alpha}{\pi}F(a,b) + \delta^R_\pi(s) + \frac{\alpha}{2\pi}\left[V^f_1(s) + A_1(s)\right] + \mathrm{Re}(\delta^W_{box}(t)) \tag{18.4}$$

$$C^{(5)}_F = \frac{3}{2}\beta_e - \frac{2\alpha}{\pi}\left[\frac{\pi^2}{6}+\frac{1}{2}\right] + \frac{4\alpha}{\pi}\left[\frac{3}{2}\ln a-\ln^2 a\right] + \frac{2\alpha}{\pi}F(a,b) + \delta_\pi(t) + \frac{\alpha}{2\pi}V^f_1(t) + \frac{\alpha}{2\pi}\frac{(f_V^2+f_A^2)\,b^4-(f_V^2-f_A^2)}{(f_V^2+f_A^2)\,b^4+(f_V^2-f_A^2)}A_1(t) + \mathrm{Re}(\delta^W_{box}(t)) \tag{18.5}$$

$$C_F^{(6)} = \frac{3}{2}\beta_e - \frac{2\alpha}{\pi}\left[\frac{\pi^2}{6}+\frac{1}{2}\right] + \frac{4\alpha}{\pi}\left[\frac{3}{2}\ell na - \ell n^2 a\right] + \frac{2\alpha}{\pi}F(a,b) + 2\mathrm{Re}(\delta_{box}^{W}(t)) \tag{18.6}$$

$$\begin{aligned} C_F^{(7)} &= \frac{3}{2}\beta_e + \frac{2\alpha}{\pi}\left[\frac{\pi^2}{3}-\frac{1}{2}\right] + \frac{\alpha}{\pi}F(a,b) + \delta_\pi^R(s) + \frac{I'(s)}{R'(s)}\delta_\pi^I(s) \\ &+ \frac{\alpha}{2\pi}\left[V_1^f(s) + \frac{I'(s)}{R'(s)}V_2(s)\right] + \frac{\alpha}{2\pi}\,\frac{f_A^2(1+z^2)+f_V^2\,2z}{f_V^2(1+z^2)+f_A^2\,2z}\left[A_1(s)+\frac{I'(s)}{R'(s)}A_2(s)\right] \end{aligned} \tag{18.7}$$

$$\begin{aligned} C_F^{(8)} &= \frac{3}{2}\beta_e + \frac{2\alpha}{\pi}\left[\frac{\pi^2}{12}-\frac{1}{2}\right] + \frac{2\alpha}{\pi}\left[\frac{3}{2}\ell na - \ell n^2 a\right] + \frac{\alpha}{\pi}F(a,b) \\ &+ \frac{\alpha}{2\pi}\left[V_1^f(t) + A_1(t)\right] + \frac{I'(s)}{R'(s)}\,\frac{\alpha}{2\pi}\left[V_2^f(t) + A_2(t) + 3\pi\right] \end{aligned} \tag{18.8}$$

$$\begin{aligned} C_F^{(9)} &= \frac{3}{2}\beta_e + \frac{2\alpha}{\pi}\left[\frac{\pi^2}{12}-\frac{1}{2}\right] + \frac{2\alpha}{\pi}\left[\frac{3}{2}\ell na - \ell n^2 a\right] + \frac{\alpha}{\pi}F(a,b) + \mathrm{Re}(\delta_{box}^{W}(t)) \\ &+ \frac{I'(s)}{R'(s)}\left[\frac{3}{2}\alpha + \mathrm{Im}(\delta_{box}^{W}(t))\right] \end{aligned} \tag{18.9}$$

$$C_F^{(10)} = \frac{3}{2}\beta_e + \frac{2\alpha}{\pi}\left[\frac{\pi^2}{3}-\frac{1}{2}\right] \tag{18.10}$$

By taking into account all the above corrections we can define the total radiative correction δ_T through the relation

$$d\sigma_{TOT} = d\sigma_0\,[1+\delta_T] \tag{19}$$

where

$$d\sigma_0 = \sum_{i=1}^{10} d\sigma_0^{(i)} \tag{20}$$

This completes the discussion of our formulae.

For numerical calculations we have used the presently accepted value of the weak mixing angle $\sin^2\theta_W \sim 0.22$, and using this we get from the results of ref. (4) a renormalized neutral vector boson $M \simeq 93.2$ GeV$^{(*)}$.

In ref. (5) a detailed calculation of one loop corrections to the leptonic width of the Z_0 has been presented. A direct extimate of the corrections to the full width can be obtained however in more direct way. Indeed one can replace in the zeroth order relation

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(*) In ref. (4) a careful analysis of the uncertainty associated to this extimate is given.

TABLE I - Percentage radiative corrections δ_T to the lowest order cross section of Bhabha scattering for some values of the center of mass energy and the scattering angle (see eq. (19)). The fractional energy resolution used is $\Delta = 0.1$ and $M = 93.2$ GeV, $\Gamma = 2.92$ GeV.

E_{CM}	20°	30°	40°	50°	60°	70°	80°	90°	100°	110°	120°	130°	140°	150°	160°
80	-21.0	-21.7	-22.4	-23.1	-23.7	-24.2	-24.7	-25.1	-25.5	-26.1	-26.7	-27.5	-28.4	-29.4	-30.7
84	-21.2	-22.1	-23.0	-23.9	-24.8	-25.6	-26.2	-26.8	-27.4	-28.0	-28.6	-29.3	-30.1	-31.0	-32.1
88	-21.7	-23.0	-24.5	-26.0	-27.4	-28.7	-29.8	-30.6	-31.3	-31.8	-32.3	-32.9	-33.4	-34.0	-34.8
90	-22.3	-24.2	-26.4	-28.5	-30.4	-32.0	-33.2	-34.1	-34.7	-35.1	-35.5	-35.8	-36.1	-36.6	-37.1
92	-22.9	-25.9	-29.2	-32.2	-34.5	-36.2	-37.3	-38.0	-38.4	-38.7	-38.8	-39.0	-39.1	-39.3	-39.5
93.2	-16.7	-17.9	-22.7	-27.8	-31.3	-33.4	-34.3	-34.6	-34.5	-34.3	-34.2	-34.2	-34.4	-34.6	-34.9
94	-15.3	-10.7	-10.1	-14.0	-18.4	-21.5	-23.6	-24.8	-25.3	-25.5	-25.5	-25.5	-25.4	-25.3	-25.2
96	-17.8	-12.2	- 1.0	12.1	18.7	18.4	16.3	15.0	15.1	16.3	18.0	19.6	21.0	22.2	23.4
98	-20.4	-19.0	-14.6	- 5.4	6.8	16.9	23.0	27.2	31.7	37.1	42.9	48.4	53.1	57.0	60.6
100	-20.9	-21.1	-20.5	-18.3	-14.4	- 9.7	- 5.7	- 2.4	0.6	4.1	8.1	12.2	16.0	19.3	22.4
110	-20.9	-21.2	-21.3	-21.2	-20.9	-20.6	-20.3	-20.4	-20.7	-21.2	-21.8	-22.4	-23.0	-23.6	-24.4

$$\Gamma_0(Z_0 \to all) = \frac{G_F M_0^3}{24\sqrt{2}\pi} \Big[[1+(1-4\sin^2\theta_W)^2] N_\ell + 2N_\nu + + 3[1+(1-\frac{8}{3}\sin^2\theta_W)^2] N_u + 3[1+(1-\frac{4}{3}\sin^2\theta_W)^2] N_d \Big] \quad (21)$$

the bare mass M_0 with the renormalized one. This gives for $\sin^2\theta_W \simeq 0.22$ a "renormalized" width $\Gamma \sim 2.92$ GeV.

In Table I the percentage radiative correction δ_T to the lowest order cross section for $e^+e^- \to e^+e^-$ is presented in the energy range around and not too far beyond the Z_0 pole. At low energy the results are almost identical to the ones obtained in ref. (2) where also weak corrections were explicitly included. In the energy range reported in Table I these results are now substantially improved due to finite width and radiative tail effects.

Finally, if one defines

$$d\sigma^{QED}(e^+e^- \to e^+e^-) = \sum_{i=1}^{3} C^{(i)}_{infra}\, d\sigma_0^{(i)} \left[1 + C_F^{(i)}\right] \quad (22)$$

we can introduce a quantity δ_W through the relation

$$d\sigma_{TOT} = d\sigma^{QED} \left[1 + \delta_W\right] \quad (23)$$

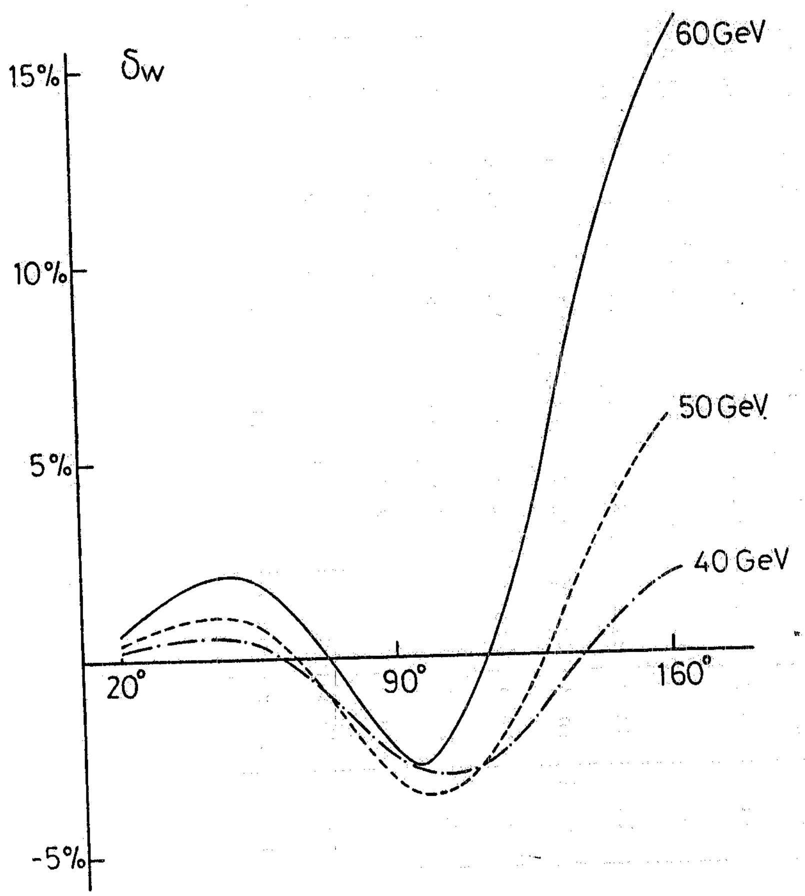

FIG. 3 - Percentage deviation δ_W from QED at intermediate energy as a function of the scattering angle.

This quantity can be of some interest at intermediate energies since at $E_{CM} = 50$ GeV the deviation is about 6% for scattering at large angles. Moreover δ_W presents a characteristic angular behaviour which could be observable. This is shown in Fig. 3, where we plot δ_W as a function of the scattering angle, at intermediate energies.

To conclude we have presented a detailed study of electromagnetic and finite width effects for Bhabha scattering near the Z_o pole. Weak interactions have been only considered to renor malize tha mass and the width of the vector boson.

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