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# THE RESONANT TIME-LIKE KAON FORM FACTOR

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The K-form factor is analyzed in terms of a family of vector mesons whose couplings are determined through our earlier model for the pion form factor. Radiative corrections are applied. The model reproduces the data quite well. For the charge radius we obtain $r_K = 0.56$F to be compared with $r_K = (0.54 \pm 0.14)$F given by the Chou–Yang model.

A resonance model for the pion form factor was presented by us [1] recently, where the contribution of the $\rho$ and its higher-mass partners was shown to be crucial to obtain the visible structure in the experimental data up to 1.6 GeV. Both radiative corrections and interference between resonances were found important. In the present paper we extend this model to discuss the kaon form factor. Recent high-quality data from Novosibirsk [2] between (1.1–1.4) GeV and higher-energy data between (1.58–2.03) GeV from Orsay [3], together with previous data from BCF [4], Frascati [5,6] and Novosibirsk [7] form a rather complete picture. They show strikingly the inadequacy of the simple VMD model in terms of the $\rho$, $\omega$ and $\varphi$ resonances. (See fig. 1.)

In our previous paper [1], we showed that the excited states of $\rho$, $\rho_1(1270)$ and $\rho_2(1540)$ were essential to account for the fine structure of the time-like pion form factor up to 1.6 GeV. The model was highly successful in the time-like region and moreover it was in excellent agreement with the absolute values of the available space-like data. Thus, we expect the kaon form factor to be also dominated by known vector

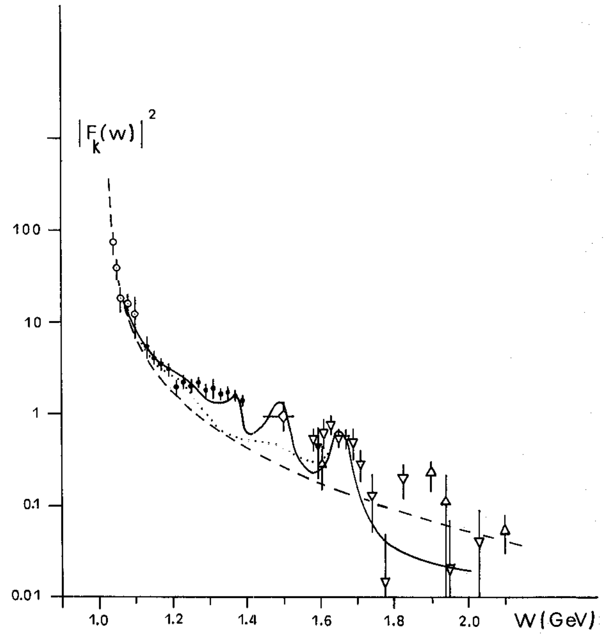


Fig. 1. The solid line is the theoretical curve for $|F_K|^2$ and the dashed line is the VDM result ($\rho$, $\omega$, $\varphi$) with $\mathcal{E}_\varphi = 0.33$. The open circles are Novosibirsk data [7]; the solid circles are from Novosibirsk [2]; the rhombi are Frascati data [6]; the nabla's are from Orsay [3]; the solid triangles are from Frascati [5] and the open triangles are Frascati BCF data [4].

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meson states. While only isovector mesons couple to $\pi^+\pi^-$ states, for $K^+K^-$ states isoscalars can also couple so that all $\rho$, $\omega$ and $\varphi$ excited states must be included.

In the energy region (1–2) GeV, evidence for $\rho_1(1270)$ and $\rho_2(1540)$ is provided by various experiments [8–11]. For $\omega$- and $\varphi$-like resonances, the situation is less clear. Evidence for an $\omega$-like state at 1.660 GeV decaying into $3\pi$ and $5\pi$ has been provided by two experiments at Orsay [11,12]. We shall call this state $\omega_2(1660)$. We shall assume the existence of an $\omega_1$ at a mass of $\approx$1.39 GeV, given by the rule $(m_{\rho_2} - m_{\rho_1}) \approx (m_{\omega_2} - m_{\omega_1})$. A suggestion for the existence of such a state has actually been provided by the Frascati–DESY collaboration [13]. $\varphi$-like states in the mass region (1–2) GeV, expected [14] at ~1.5 GeV and ~1.8 GeV as Regge recurrences have not yet been identified. Since we shall not venture beyond ~1.8 GeV, our model will only contain the resonances given in table 1.

The widths for $\rho_1$, $\rho_2$ and $\omega_2$ are as quoted in refs. [9–11]. The $\omega_1$ width has been set equal to that for the $\omega_2$ and the $\varphi_1$ width has been chosen to be 40 MeV since the reported resonances $\varphi(1820)$ and $\varphi(2100)$ have both approximate widths $\approx$(30–40) MeV [15].

We now turn to determine the couplings of these resonances to $F_K(s)$, the kaon form factor. As usual [2], the vector dominance model (VDM) for $\rho$, $\omega$, $\varphi$ gives

$$F_K(s)|_{VDM} = \frac{1}{2}\frac{m_\rho^2}{\Delta(\rho)} + (\tfrac{1}{2} - \mathcal{C}_\varphi)\frac{m_\omega^2}{\Delta(\omega)} + \mathcal{C}_\varphi \frac{m_\varphi^2}{\Delta(\varphi)}, \quad (1)$$

where

$$\Delta(v) = m_v^2 - s - \mathrm{i} m_v \Gamma_v (s/m_v^2)\,,$$

and

$$\mathcal{C}_\varphi = g_{\varphi K\bar{K}}/g_\varphi\,.$$

Table 1

| Name | Mass (MeV) | Width (MeV) |
|---|---|---|
| $\rho$ | 776 | 155 |
| $\rho_1$ | 1270 | 110 |
| $\rho_2$ | 1540 | 220 |
| $\omega$ | 783 | 12 |
| $\omega_1$ | 1390 | 50 |
| $\omega_2$ | 1660 | 50 |
| $\varphi$ | 1019 | 4 |
| $\varphi_1$ | 1500 | 40 |

More generally, the resonance dominated form factor can be written

$$F_K(s) = F_K(s)|_{VDM} + \sum_i \frac{\mathcal{C}_i m_i^2 \exp(\mathrm{i}\varphi_i)}{m_i^2 - s - \mathrm{i} m_i \Gamma_i (s/m_i^2)}\,, \quad (2)$$

where $m_i$ and $\Gamma_i$ are the mass and width, respectively, of the $i$th resonance and $\mathcal{C}_i$ is proportional to the product of the resonance coupling to the photon and to $K\bar{K}$. For simplicity, the arbitrary phases $\varphi_i$ are chosen to be zero. This avoids the introduction of a host of extra parameters and is justified a posteriori by the good quality of our fit with $\varphi_i = 0$. Thus, each new resonance enters into $F_K$ with only one parameter $\mathcal{C}_i$. A very simple argument is introduced below to determine *all* the $\mathcal{C}_i$ in terms of the corresponding parameters (fixed previously in ref. [1]) occurring in the pion form factor. Thus, *our resulting $F_K(s)$ is very nearly a prediction and not a fit to the data.*

Because we have three families ($\rho$, $\omega$, $\varphi$) of resonances it is useful to investigate possible relationships between their couplings to the photon on the one hand and to $\pi\pi$ and $K\bar{K}$ states on the other. Consider the $n$th $\rho$-meson, $\rho_n$ and its couplings $g_{\rho_n\pi\pi}$ to $\pi^+\pi^-$ and $g_{\rho_n K\bar{K}}$ to $K\bar{K}$. It seems reasonable to suppose that the $n$ dependence of both these couplings are the same. Thus, we expect the ratio

$$\frac{\mathcal{C}_{\rho_n}(K\bar{K})}{\mathcal{C}_{\rho_n}(\pi\pi)} = \left(\frac{g_{\rho_n K\bar{K}}}{g_{\rho_n}}\right)\left(\frac{g_{\rho_n}}{g_{\rho_n\pi\pi}}\right) = \frac{g_{\rho_n K\bar{K}}}{g_{\rho_n\pi\pi}}\,, \quad (3)$$

to be *independent of n*.

A specific realization of this independence occurs in the standard quark model picture where the photon gives rise to the appropriate $q\bar{q}$ to end in $\pi^+\pi^-$ or $K^+K^-$ final states. Since, the quark–photon vertex in this scheme is the same we expect the above result to hold.

Also, the simple quark model tells us that

$$\frac{\mathcal{C}_{\omega_n}(K\bar{K})}{\mathcal{C}_\omega(K\bar{K})} = \frac{\mathcal{C}_{\rho_n}(K\bar{K})}{\mathcal{C}_\rho(K\bar{K})}\,. \quad (4)$$

Experimentally [2], we know that $\mathcal{C}_\varphi = 0.33$. Analysis of the experimental data gives $\mathcal{C}_\rho(\pi\pi) = 1.08$ as discussed in ref. [1], where it was also shown that $\mathcal{C}_{\rho_1}(\pi\pi) = -0.07$ and $\mathcal{C}_{\rho_2}(\pi\pi) = -0.06$. Combining these results with $\mathcal{C}_\rho(K\bar{K}) = 1/2$, $\mathcal{C}_\omega(K\bar{K}) = \frac{1}{2} - \mathcal{C}_\varphi$ and $\mathcal{C}_\varphi = 0.33$, we obtain finally

$$\mathcal{E}_{\rho_1} = \mathcal{E}_{\rho_2} = -0.035\,, \quad \mathcal{E}_{\omega_1} = \mathcal{E}_{\omega_2} = -0.015\,,$$

$$\mathcal{E}_{\varphi_1} = -0.020\,, \tag{5}$$

Eq. (5) thus fixes the couplings of these resonances to $F_K$. We now turn to a brief discussion of radiative corrections which have been applied. The method is identical to that used in our earlier work [1] where more details can be found. Let us call the real and imaginary parts of a given resonance contribution to $F_K(s)$ $R$ and $I$. Explicitly, we have

$$\begin{aligned}|F_K|^2 = {} & (R_\rho^2 + R_\omega^2 + R_\varphi^2 + 2R_\rho R_\omega + 2R_\omega R_\varphi + 2R_\varphi R_\rho) \\ & + (R \to I) \\ & + \sum_i (R_i^2 + I_i^2)(\sin\delta_i)^{-\beta}[1 + C_{\rm res}(i)] \\ & + 2(R_\rho + R_\omega + R_\varphi) \sum_i R_i (\sin\delta_i)^{-\beta}[1 + C_{\rm int}(i)] \\ & + 2(I_\rho + I_\omega + I_\varphi) \sum_i I_i (\sin\delta_i)^{-\beta}[1 + C_{\rm res}(i)] \\ & + \sum_{i\neq j}\sum R_i R_j (\sin\delta_i)^{-\beta}(\sin\delta_j)^{-\beta} \\ & \times [1 + C_{\rm int}(i) + C_{\rm int}(j)] \\ & + \sum_{i\neq j}\sum [I_i I_j](\sin\delta_i)^{-\beta}(\sin\delta_j)^{-\beta} \\ & \times [1 + C_{\rm res}(i) + C_{\rm res}(j)]\,, \end{aligned} \tag{6}$$

where the sum over $i$ and $j$ does *not* include $\rho$, $\omega$ and $\varphi$ and

$$\tan\delta_i = (\Gamma_i/2)(m_i - \sqrt{s})^{-1}\,, \tag{7a}$$

$$C_{\rm res}(i) = -\beta\delta_i \cot\delta_i\,, \quad C_{\rm int}(i) = \beta\delta_i \tan\delta_i\,, \tag{7b,c}$$

and

$$\beta = (2\alpha/\pi)[\ln(s/m_e^2) - 1]\,. \tag{7d}$$

Here $m_e$ is the electron mass. The form of eq. (6) is similar to that obtained for the pion case. For the sake of completeness we recall the underlying logic. We use the factor $C_{\rm int}$ to correct for the interference between the real parts of the resonances since this is exactly what happens when one considers the interference between the QED term (which is real) and a resonance [16]. On the other hand, between the imaginary parts of two resonances we use the factor $C_{\rm res}$, since this case is similar to the resonance term itself. We make no

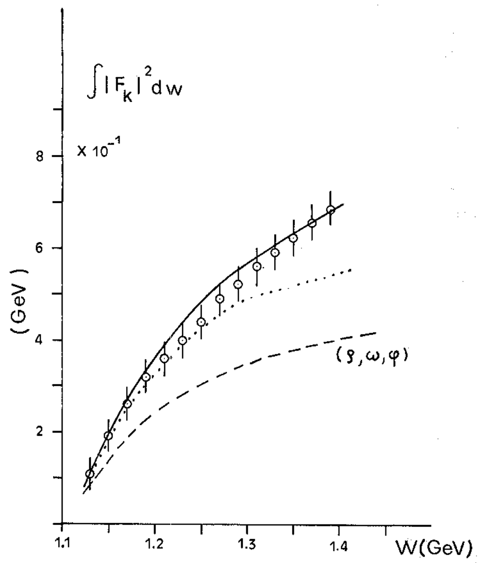


Fig. 2. The duality integral $\int |F_K|^2\, dW$ versus $W = \sqrt{s}$. The solid line is the theoretical curve, the dotted line is without the ($\omega_1$, $\varphi_1$) terms and the dashed line is the VDM prediction. The circles give the integrals over the experimental values from ref. [2].

radiative corrections for the $\rho$, $\omega$ and $\varphi$ terms because the VDM parameters (e.g., $\mathcal{E}_\varphi$) have been determined using the low-energy experimental data. Strictly speaking, eqs. (6) and (7) are valid only for energy loss $\Delta\omega \gg \Gamma$, which may not be the case here.

In fig. 1 the data are presented for $|F_K|^2$ between (1.05–2.1) GeV [2–7]. For comparison the VDM prediction (dashed line), as given by eq. (1) with $\mathcal{E}_\varphi = 0.33$, is also presented. The data are quite well reproduced by our model for the couplings.

A better way to appreciate the quality of the fit is to consider an energy integral of $|F_K|^2$ which eliminates the local fluctuations due to the radiative effects as well as the resonances. Such a comparison, up to ~1.4 GeV, is shown in fig. 2. The circles are the integrated values of the experimental points (data from Novosibirsk, ref. [2], have been used). The solid line is our complete curve, the dotted line is without $\omega_1$, $\varphi_1$ terms and the dashed line is the VDM prediction (containing $\rho$, $\omega$, $\varphi$ alone). Clearly, VDM alone is completely ruled out. The good agreement between our proposed resonance expression for this integral and the integrated experimental values (circles) is a clear indication of the need for these resonances and also, that their proposed couplings are indeed the right ones.

It is also possible to compute the prediction for $r_K = (\langle r_K^2 \rangle)^{1/2}$, the kaon charge form factor from our model. We obtain $r_K = 0.56$ fermi. Chou [17] has extracted $r_K$ from an analysis of the elastic scattering data using the Chou–Yang picture. His (grand) average value for $r_K = (0.54 \pm 0.14)$ fermi agrees quite nicely with our estimate.

In conclusion, we have presented a model for the kaon form factor similar to our pion form factor model in terms of higher vector meson states. The couplings of these resonances is determined a priori and not fitted. The agreement is very good and practically reproduces all the local structures. Radiative corrections appropriate for the resonances are quite essential for the quality of this fit. The duality integral is well saturated by our resonance model. The charge radius is also in good agreement with its estimate from an independent source. It is hoped that soon high-quality data will be available for the space-like region for kaons also, so that an absolute prediction from our model can be tested.

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