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SPIN-ISOSPIN COLLECTIVE EXCITATIONS IN LIGHT DEFORMED NUCLEI

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A B S T R A C T

Spin-isospin collective states in light oblate nuclei are related to a non-static σ-τ phase. The existence of such states in the absence of an energy splitting is shown to support the existence of the phase.

In a previous paper[1] (referred to as I), we have suggested the existence of spin-isospin excitations in light deformed nuclei related to non-static spin-isospin order. Such an order is realized in the zero-point motion by unidimensional oscillations of spin-up protons and spin-down neutrons, with respect to spin-down protons and spin-up neutrons. The direction of oscillation must coincide with the axis of spin quantization and must be parallel to the symmetry axis (longitudinal) for oblate nuclei and perpendicular to it (transverse) for prolate nuclei.

Such non-static σ-τ phase is energetically favoured by the OPE potential and would be made manifest by the existence of softened modes with enhanced M2 transition probability. More specifically, the $K = 0 \rightarrow K = 0$ transitions should be enhanced for oblate nuclei and the $K = 0 \rightarrow K = \pm 2$ ones for prolate nuclei.

This a very specific signature which, however, is of little help in experiments where the K quantum number is not determined.

The purpose of this note is therefore to discuss a characterization of non-static σ-τ phases which, even if less specific, is easier to investigate experimentally. This is the absence of splitting of the M2 resonance.

We recall, in this connection, that the collective character of the states essentially depends on the strength of the interaction in the appropriate channel with respect to the channel-independent interaction. The fact that the former be attractive or repulsive has only the effect of softening or hardening the mode and enhancing or reducing its strength, but it cannot change its collective character.

As a consequence of the repulsive correlation energy for transverse oscillations in oblate nuclei and longitudinal ones in prolate nuclei, corresponding collective states should exist at high energy and the M2 strength should be distributed between longitudinal and transverse modes.

Being interested in such a splitting of the resonance, we must be more accurate than in I, and we must take into account the fact that also the σ-τ independent part of the restoring force depends on the transverse or longitudinal character of the oscillation.

This will give rise to a major effect and will substantially change the predictions of I. We will also use a microscopic nuclear density rather than a Gaussian approximation and we will try to get an idea of the effect of short-range N-N and N-Δ interactions by the Landau parametrization. This point deserves a discussion which we postpone after the derivation of our results.

For simplicity, we confine ourselves to ^{12}C and ^{28}Si which have oblate deformation.

The total nuclear wave function is

$$\Psi_{\nu_z \nu_T K} = \Phi_{\nu_z \nu_T K}(\vec{d})\Lambda(\vec{d}) \tag{1}$$

where the intrinsic part $\Lambda(\vec{d})$ is a Slater determinant of displaced single-particle wave functions

$$\varphi_{n_z n_T m \sigma_3 \tau_3} = \varphi_{n_z}(z - \frac{1}{2} d_z \sigma_3 \tau_3)\, \varphi_{n_T m}(\vec{r}_T - \frac{1}{2} \vec{d}_T \sigma_3 \tau_3)\, \chi_{\sigma_3 \tau_3} . \tag{2}$$

The ϕ_{n_z} and $\phi_{n_T m}$ are harmonic oscillator wave functions in a cylindrical basis and $\chi_{\sigma_3 \tau_3}$ are spin-isospin wave-functions.

The shells are filled according to the following scheme

$$(\nu_z, \nu_T, m) = \begin{cases} (0,0,0);\ (0,1,\pm 1) \text{ for } {}^{12}C \\ (0,0,0);\ (0,1,\pm 1);\ (1,0,0);\ (0,2,\pm 2);\ (2,0,0) \text{ for } {}^{28}Si. \end{cases}$$

The collective motion is generated by the quantum fluctuations of the displacement parameter $\vec{d}$. In harmonic approximation, the collective Hamiltonian is

$$H = \frac{P^2}{2M} + \frac{1}{2}(C_z + K_z)d_z^2 + \frac{1}{2}(C_T + K_T)\, d_T^2 \tag{3}$$

where $M = Am/4$, C_z and C_T are the σ-τ independent restoring constants and K_z and K_T the dependent ones. These latter are determined by the equation

$$\langle \Lambda(\vec{d}) | V_{\sigma\tau} | \Lambda(\vec{d}) \rangle \sim \frac{1}{2} K_z d_z^2 + \frac{1}{2} K_T d_T^2. \tag{4}$$

$V_{\sigma\tau}$ is the σ-τ dependent part of the N-N interaction, namely the OPE plus the Landau potential

$$V_{\sigma\tau} = -\left(\frac{f}{m_\pi}\right)^2 \varrho(p^2)\, \vec{\tau}_1 \cdot \vec{\tau}_2 \left[\vec{\sigma}_1 \cdot \vec{p}\, \vec{\sigma}_2 \cdot \vec{p} / (p^2 + m_\pi^2) - g' \,\vec{\sigma}_1 \cdot \vec{\sigma}_2\right] \tag{5}$$

where

$$\frac{f^2}{4\pi} = 0.08; \quad \varrho(p^2) = \left(\frac{\Lambda^2}{\Lambda^2 + p^2}\right)^2; \quad \Lambda = 1300\ \frac{\text{MeV}}{c}, \tag{6}$$

The resulting expressions for K_z and K_T are

$$K_{z,T} = -\frac{1}{\pi^2}\left(\frac{f}{m_\pi}\right)^2 \int_{-\infty}^{+\infty} dp_z \int_0^{\infty} dp_T\, p_T\, P_{z,T} \cdot \tag{7}$$

$$\cdot \left(\frac{p_z^2}{p^2 + m_\pi^2} - g'\right) \varrho(p^2)\, G(p_z, p_T),$$

In the above equation

$$P_z = p_z^2 , \qquad P_T = \frac{1}{2} p_T^2 ,$$

$$G(p_z, p_T) = \sum_{n_{z1} n_1 m_1} \sum_{n_{z2} n_2 m_2} G_{n_{z1}}(p_z)\, G_{n_{z2}}(p_z) \cdot$$

$$\cdot\, G_{n_1 m_1}(p_T)\, G_{n_2 m_2}(p_T) , \tag{8}$$

$$G_{n_z}(p_z) = \langle \varphi_{n_z} | e^{izp_z} | \varphi_{n_z} \rangle ,$$

$$G_{nm}(p_T) = \langle \varphi_{nm} | e^{i\vec{r}_T \cdot \vec{p}_T} | \varphi_{nm} \rangle . \tag{9}$$

Finally, following the prescription of the unified theory of nuclear vibrations[2)] we put

$$C_z = \frac{A}{4} m\, \bar{\omega}_z^2 ,$$

$$C_T = \frac{A}{4} m\, \bar{\omega}_T^2 , \tag{10}$$

$\bar{\omega}_z$ and $\bar{\omega}_T$ being the oscillator frequencies. The above expressions can only be indicative. In particular the underlying quasi-boson approximation is not well justified for C_z due to the small number of single particle states available.

The first excited states are characterized by a unique quantum number ($K = 0$, for $n_T = 0$, $n_z = 1$; $K = \pm 1$, for $n_T = 1$, $n_z = 0$) and by the M2 transition strength

$$B(M2; I = K = 0 \to I, K) =$$

$$= \frac{2}{1+\delta_{K0}} \left| \langle \Phi_K \right| \mathcal{M}(\lambda = I = 2, \ \nu = K) \left| \Phi_0 \rangle \right|^2 , \qquad (11)$$

where

$$\mathcal{M}(2, K) = \frac{A}{8}\sqrt{\frac{5}{4\pi}} \sqrt{\frac{(2+K)!\ (2-K)!}{(1+K)!\ (1-K)!}} (g_p - g_n)(-1)^K d_{-K} \frac{e\hbar}{2mc} . \qquad (12)$$

Therefore

$$B(M2,\ 0 \to K) = \frac{5}{32\pi} \frac{\hbar^2}{m} A(g_p - g_n)^2 \left(\frac{1}{\hbar\omega_z} \delta_{K0} + \right.$$

$$\left. + \frac{3}{2\sqrt{2}} \frac{1}{\hbar\omega_T} \delta_{K, \pm 1} \right) \left(\frac{e\hbar}{2mc} \right)^2 f^2 m , \qquad (13)$$

Transitions $K = 0 \to K = \pm 2$ are absent (they characterize prolate deformations[1]).

The above equations will be evaluated by using the free nucleon values for g_p and g_n. These values should be renormalized according to the quenching of magnetic transitions[3] in order to compare with experiment.

We first look at the effect of the Gaussian approximation for the density. For $\bar{\omega}_z/\bar{\omega}_T = 1.5$, which corresponds to $\delta \simeq -0.4$ and $g' = \frac{1}{3}$, which cancels the δ term in the OPE potential, we obtain (Table I) $K_z = -16.5$ and -13.6 MeV fm^{-2} for ^{12}C and ^{28}Si, resp. These values should be compared to the values $K_Z = -13.3$ and -10.4 MeV fm^{-2} obtained in I, showing that the Gaussian approximation underestimates by 30% the value of K_Z.

We look next at the effect of the C_Z-C_T difference. This is found by comparison of Tables I and II and it is twofold. First there is an inversion of the energy levels which corresponds to longitudinal and transverse excitations, second an over-all reduction of collectivity (the conversion factor to be used for comparison with I is 1 W.U. = $= 15.2\ \mu^2 fm^2$).

For $g' = 0.33$, we have almost degenerate transverse and longitudinal modes, with comparable strength, while for $g' = 0.5$ only the transverse mode is collective. These situations cannot be distinguished experimentally unless the K quantum number is measured, but both characterize σ-τ correlations ($K_Z \ll K_T$).

For $g' = 0.7$, on the contrary, there are no σ-τ correlations ($K_Z \sim K_T$), and there is a well-marked splitting of the M2 resonance. Let us emphasize that the larger the ratios (K_Z/C_Z) and (K_T/C_T) are, the more reliable are the predictions of our collective model. For $g' = 0.33$, these ratios are only $\frac{1}{4}$.

We finally comment on the Landau parametrization of short-range effects. As already emphasized[4)], such a parametrization is unjustified for a static σ-τ phase. For a non-static one, it can at the best have a very rough meaning, i.e., to give an idea of the effect of an over-all reduction of the OPE potential attraction. If the short range effects were properly evaluated, in fact, they would depend on the longitudinal or transverse character of the oscillations as is the case for K_Z and

Nucleus	g'	C MeV fm^{-2}	K_z MeV fm^{-2}	ω_z MeV	B(M2;K=0→K=0) μ^2 fm^2	K_T MeV fm^{-2}	ω_T MeV	B(M2;K=0→K=±1) μ^2 fm^2
^{12}C	0.33		-16.5	8.8	250	3.4	19.0	
	0.5	22.1	- 4.5	15.6		8.4	20.5	160
	0.7		9.6	20.9	105	14.4	22.5	147
^{28}Si	0.33		-13.6	9.6	533	5.3	14.8	
	0.5	29.3	2.0	13.6		11.6	15.6	493
	0.7		20.4	17.2	299	19.1	16.9	456

TABLE I - C is evaluated according to Eq. (10) with $\bar{\omega}_z = \bar{\omega}_T$, i.e., by neglecting the C_z-C_T difference. The resulting value is slightly different from that of I, where $C = 41\ A^{-5/3}$MeV fm^{-2}. All other quantities are evaluated by putting $\bar{\omega}_z/\bar{\omega}_T = 1.5$, which corresponds to a deformation parameter $\delta = -0.4$. The values of B(M2) are omitted for K/C ~ 0.1, because in this case our collective model is not reliable.

Nucleus	g'	C_z MeV fm^{-2}	K_z MeV fm^{-2}	ω_z MeV	B(M2;K=0→K=0) μ^2 fm^2	C_T MeV fm^{-2}	K_T MeV fm^{-2}	ω_T MeV	B(M2;K=0→K=±1) μ^2 fm^2
^{12}C	0.33		-16.5	17.2	128		3.4	16.7	197
	0.5	37.9	- 4.5	21.4		16.8	8.4	18.7	176
	0.7		9.6	25.6	86		14.4	20.8	159
^{28}Si	0.33		-13.6	14.7	348		5.3	12.8	602
	0.5	50.3	2.0	17.6		22.3	11.6	14.2	542
	0.7		20.4	20.5	251		19.1	15.7	491

TABLE II - The same as Table I, with the exception of C_z and C_T which are evaluated according to Eq. (10) with $\bar{\omega}_z/\bar{\omega}_T = 1.5$.

K_T. (Even neglecting the σ-τ correlation, the Landau interaction is defined by matrix elements on the Fermi surface, which is non-spherical for a deformed nucleus) This is a major source of theoretical uncertainty, because it is not known if these effects will be co-operative with the K_z-K_T difference or not.

The previous analysis is obviously relevant to the σ-τ breathing mode [5] and more generally to precursor phenomena[6].

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