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## TRANSVERSE MOMENTUM DISTRIBUTIONS FOR DRELL-YAN PAIRS IN QCD

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Absolute  $p_{\perp}$ -distributions for Drell-Yan pairs produced in  $\pi N$  and  $p N$  collisions are studied in perturbative QCD and found in excellent agreement with data. Soft gluon effects, implemented by exact kinematics considerations, play a very important role in the analysis. A moderate value of the parton intrinsic  $\langle p_{\perp}^2 \rangle \approx 0.4 \text{ GeV}^2$  is found.

Much theoretical and experimental work has been recently devoted to the study of massive lepton-pair production in hadronic collisions [1]. It is by now well known that the absolute  $p_{\perp}$ -integrated cross section for the basic model of Drell and Yan [2] is found [1] to be renormalized by the so-called "K factor" when one uses quark densities defined in deep inelastic lepto-production. This K factor, which is approximately constant over the kinematical region explored so far, is well understood [3] in perturbative quantum chromodynamics (QCD) and this fact can be considered as a good success of the theory. The important role played by soft gluon effects in the evaluation of those corrections to the parton model results has been emphasized by various authors [4]. Furthermore it has been shown recently [5] that the inclusion of higher order soft effects slightly modifies the first-order results in the kinematical regions reasonably accessible to experiments.

On the other hand the transverse properties of the observed spectra, which should appear as the most spectacular effects of QCD, have not yet found a satisfactory theoretical explanation [1,6] <sup>+1</sup>. In fact, although the

predicted increase of  $\langle p_{\perp}^2 \rangle$  with  $S$  and  $M^2$  is qualitatively in agreement with data, one requires a quite large value of the parton intrinsic  $p_{\perp}$ , say  $\langle p_{\perp}^2 \rangle \approx 1 \text{ GeV}^2$ . Furthermore, the shape of the distributions from first-order diagrams is wrong at small and intermediate  $p_{\perp}$ , it is better for large  $p_{\perp}^2 \sim O(M^2)$ , but still the theoretical predictions lie somewhat lower than the data.

As is well known, the failure of first-order results in the small and intermediate  $p_{\perp}$  regions is not so dramatic. In fact, for  $\Lambda^2 \ll p_{\perp}^2 \ll M^2$  perturbation theory breaks down to the appearance of large  $\alpha_s^n \ln^{2n}(M^2/p_{\perp}^2)$  terms which have to be summed to all orders. This task has been essentially accomplished by Dokshitser et al. [8] in the leading double logarithmic approximation. A further improvement has been suggested by Parisi and Petronzio [9] by going into impact parameter space rather than in momentum space. More recent theoretical analyses [10] have confirmed this result, but no phenomenological use of it has been made so far.

On the other hand the same result was independently obtained by Curci et al. [11] in  $e^+e^-$  annihilation. A recent analysis [12] of total transverse momentum distributions of  $e^+e^-$  jets at PETRA energies by the PLUTO collaboration strongly supports this resummed QCD formula.

<sup>1</sup> Boursier CEA.<sup>+1</sup> For a successful non-QCD analysis, see ref. [7].

In the present letter we study the absolute  $p_{\perp}$ -distributions of lepton pairs produced in  $\pi N$  and  $pN$  collisions in the same general framework. We find excellent agreement between data [13] and the resummed QCD formula for small and intermediate  $p_{\perp}$ , with a much more reasonable value of the global quark intrinsic  $p_{\perp}$ , i.e.  $\langle p_{\perp}^2 \rangle_{\text{intr}} \sim 0.4 \text{ GeV}^2$ . At large  $p_{\perp}$  and in case of  $pN$  collisions the first-order Compton contribution nicely adds to obtain the observed tail in the  $p_{\perp}$ -distribution, much the same way as the hard gluon bremsstrahlung affects the large  $K_T e^+ e^- qq$  jet distributions. Such a term is quite small for  $\pi N$  collisions. The observed features of  $\langle p_{\perp}^2 \rangle$  dependence on the c.m. energy and the mass of the lepton pair is well reproduced also.

We stress that our results are essentially independent of free parameters. The use of exact kinematics in the transverse momentum phase space for multigluon emission is of crucial importance for our findings. This is somewhat similar to what has been found elsewhere in next-to-leading QCD corrections [4,14].

The starting formula for the full differential cross section in the soft multigluon approximation is [9,11]

$$\frac{d\sigma}{dM dy dp_{\perp}^2} \simeq \frac{d\sigma}{dM dy} \frac{dp}{dp_{\perp}^2}, \quad (1)$$

with

$$\begin{aligned} \frac{d\sigma}{dM dy} &= \frac{8\pi\alpha^2}{9MS} K \\ &\times \sum_i e_i^2 [q_i^{(1)}(\sqrt{\tau} e^y) \bar{q}_i^{(2)}(\sqrt{\tau} e^{-y}) + (1 \leftrightarrow 2)], \end{aligned} \quad (2)$$

$$\frac{dp}{dp_{\perp}^2} = \frac{1}{2} \int_0^{\infty} b db J_0(bp_{\perp}) \exp[\Delta(b, q_{\perp \max})], \quad (3)$$

where

$$\Delta(b, q_{\perp \max})$$

$$= \frac{16}{3\pi} \int_0^{q_{\perp \max}} \frac{dq_{\perp}}{q_{\perp}} \ln\left(\frac{M}{q_{\perp}}\right) \alpha(q_{\perp}) [J_0(bq_{\perp}) - 1], \quad (4)$$

$\tau = M^2/S$ ,  $\alpha(q_{\perp})$  is the running coupling constant,  $K = K(y, M, \tau)$  is the  $K$  factor, and  $q_{\perp \max}$  is the phase space limit for the emitted gluons.

So far we have neglected the effect of an intrinsic  $p_{\perp}$  in the parton distributions. We will come back to this

point later. A few remarks are in order here to clarify our starting formulae.

The approximate factorization in eq. (1) is a consequence of the soft gluon approximation <sup>‡2</sup> and allows for a great simplification in the problem. The factor  $K$  which, as said above, is quite well understood in perturbative QCD, depends also on the parametrization of the structure functions. An approximate constant value  $K = 1.8 \pm 0.2$  has been used below in comparing with all experiments with  $\pi$  and  $p$  beams at various energies, having chosen a unique parametrization of the parton densities.

The upper limit of the transverse phase space is

$$q_{\perp \max} = M(1-z)/2\sqrt{z},$$

where  $z = M^2/(x_1 x_2 S)$  is the squared ratio of the lepton pair mass to the effective invariant energy  $(\hat{S})^{1/2} = (x_1 x_2 S)^{1/2}$  of the subprocess  $q\bar{q} \rightarrow \gamma(M^2) g$ . In the usual double leading approximation one takes  $q_{\perp \max} \sim M$ , but it is very important to keep trace of the finite terms left. Finally the running coupling constant has been chosen to freeze at very small values of  $k$ , namely  $\alpha(k^2) = 12\pi/25 \ln[(k^2 + \lambda^2)/\Lambda^2]$ , in agreement with previous phenomenology at low  $k^2$  [9,11,16].

From the distribution (3), which is properly normalized to unity, one also gets [11,12] a soft contribution to  $\langle p_{\perp}^2 \rangle$  given by

$$\langle p_{\perp}^2 \rangle_{\text{soft}} = \frac{4}{3\pi} \int_0^{q_{\perp \max}^2} dq_{\perp}^2 \ln\left(\frac{M^2}{q_{\perp}^2}\right) \alpha(q_{\perp}). \quad (5)$$

In comparing with actual experiments one has to add to eq. (5) the hard contribution  $\langle p_{\perp}^2 \rangle_{\text{hard}}$ , given by the first-order Compton process, namely  $qg \rightarrow \gamma(M^2) q$ , and the parton intrinsic component  $\langle p_{\perp}^2 \rangle_{\text{intr}}$ . We therefore write for the full  $\langle p_{\perp}^2 \rangle$  of the lepton pair

$$\langle p_{\perp}^2 \rangle = \langle p_{\perp}^2 \rangle_{\text{soft}} + \langle p_{\perp}^2 \rangle_{\text{hard}} + \langle p_{\perp}^2 \rangle_{\text{intr}}. \quad (6)$$

The intrinsic  $p_{\perp}$  distribution of the partons can be included also in eq. (3), which is therefore modified as [9]

$$\frac{dp'}{dp_{\perp}^2} = \frac{1}{2N} \int_0^{\infty} b db J_0(bp_{\perp}) e^{-b^2/4A} \exp[\Delta(b, q_{\perp \max})], \quad (7)$$

corresponding to an average value  $\langle p_{\perp}^2 \rangle_{\text{intr}} = 1/A$ . The

<sup>‡2</sup> For a detailed derivation see ref. [15].

factor  $N$  properly normalized the new distribution (7), namely

$$\frac{dp'}{dp_\perp^2} \frac{dp_\perp^2}{dp_\perp^2} = 1.$$

So far we have not completely specified  $q_{\perp\max}$ , which plays an important role in the analysis. Then we estimate the average  $\langle\sqrt{z}\rangle$ , which enters in the definition of  $q_{\perp\max}$  as

$$\langle\sqrt{z}\rangle|_{y=0} = \frac{\int_0^1 dz (2\sqrt{z})^{-1} \sqrt{z} [1 - (\tau/z)^{1/2}]^{\xi} (\tau/z)^{\eta/2}}{\int_0^1 dz (2\sqrt{z})^{-1} [1 - (\tau/z)^{1/2}]^{\xi} (\tau/z)^{\eta/2}}, \quad (8)$$

having parametrized the parton densities as

$$q(x) \sim x^{\eta_1}(1-x)^{\xi_1}, \quad \bar{q}(x) \sim x^{\eta_2}(1-x)^{\xi_2}, \quad (9)$$

with  $\eta = \eta_1 + \eta_2$  and  $\xi = \xi_1 + \xi_2$ . This leads to an explicit dependence of  $q_{\perp\max}$ , and therefore of the full distribution (7), upon  $\tau$ . Although eq. (8) gets slightly modified for  $y \neq 0$ , we have used it for simplicity for all values of  $y$  needed for comparison with data.

We have then used the NA3 [13] parametrizations for the pion and the proton structure functions:

$$\begin{aligned} xV^\pi(x) &= Ax^{0.4}(1-x)^{0.9}, \\ xS^\pi(x) &= 0.12(1-x)^5, \\ xu^p(x) &= Bx^{0.5}(1-x)^{3.2}, \\ xd^p(x) &= Cx^{0.5}(1-x)^{4.2}, \\ xS^p(x) &= 0.37(1-x)^{9.4}, \end{aligned} \quad (10)$$

with  $A, B, C$  determined by the normalization conditions

$$\int V^\pi(x) dx = 1, \quad \int u^p(x) dx = 2, \quad \int d^p(x) dx = 1.$$

For the gluon distributions, which enter in the Compton term, we have taken

$$xG(x) = 3.4(1-x)^5. \quad (11)$$

Finally we have used for  $\alpha(k) \Lambda = 0.35$  GeV and  $\lambda = 1.1$  GeV and  $\langle p_\perp^2 \rangle_{\text{intr}} = 0.4$  GeV<sup>2</sup>.

Our results are shown in figs. 1–4 for  $\pi N$  and  $p N$  collisions, respectively, and compared with the experimental data. Let us discuss them in detail.

The transverse momentum distributions shown in fig. 1 are absolutely normalized through eqs. (1), (2)

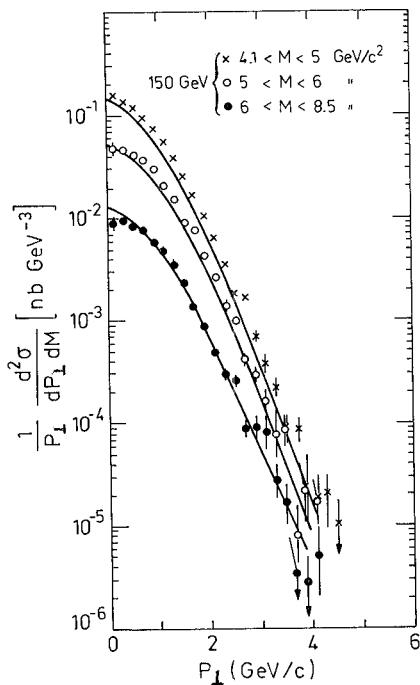


Fig. 1. Differential cross section  $p_\perp^{-1} d^2\sigma/dp_\perp dM$  in  $\pi N$  collisions at  $p_{\text{lab}} = 150$  GeV. The data are from ref. [13]. The theoretical curves are obtained for  $y = 0$ . N refers to 60% neutrons and 40% protons.

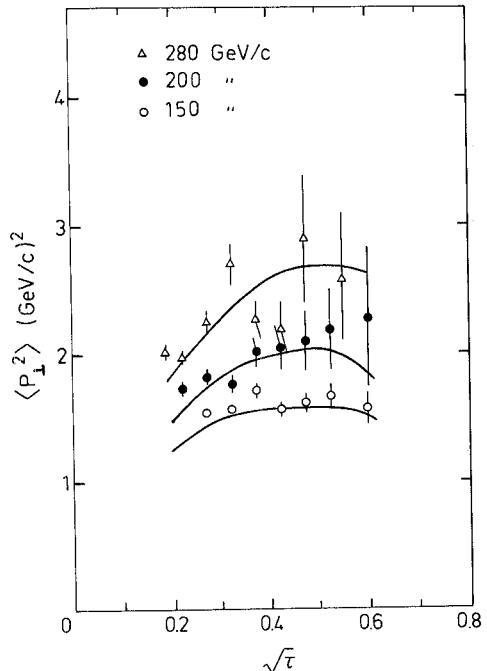


Fig. 2. Predictions for  $\langle p_\perp^2 \rangle$  from eq. (6) in  $\pi N$  collisions at  $p_{\text{lab}} = 150, 200$  and  $280$  GeV/c. The data are from ref. [13].

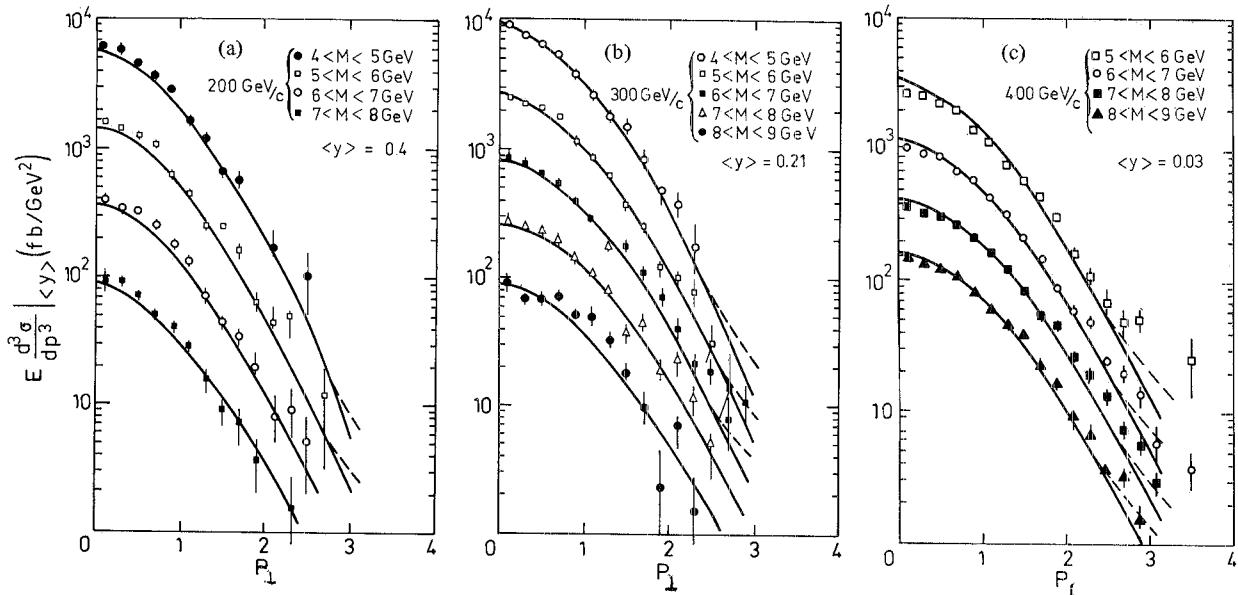


Fig. 3. Invariant cross section  $E \frac{d^3\sigma}{dp^3}$  in pN collisions at  $p_{\text{lab}} = 200$  (a), 300 (b) and 400 GeV/c (c). The data are from ref. [13]. The theoretical curves are: soft + intrinsic (full line); soft + intrinsic + hard (dashed line).

and (7). The hard Compton term, obtained from ref. [17] without any intrinsic  $p_\perp$  effects, gives a very small contribution at the tail of the distributions for low  $M$  values, which is obtained from ref. [18], if of the order

In fig. 2  $\langle p_\perp^2 \rangle$  is plotted [eq. (6)] versus  $\sqrt{\tau}$ . The hard term, which is obtained from ref. [18], if of the order of  $0.1 \text{ GeV}^2$  for  $M \sim 4-6 \text{ GeV}$  and is negligible for higher masses. The  $S$  dependence of  $\langle p_\perp^2 \rangle$ , which is evident from the figure, almost entirely comes from eq.

(5) via  $q_{\perp \max}$ .

In fig. 3 we show the  $p_\perp$ -distributions at various energies for pN collisions. The normalization is again absolute. The Compton term, which is added to the soft contribution, is comparable to it for large values of  $p_\perp^2$ , giving an important tail effect.

Finally  $\langle p_\perp^2 \rangle$  is plotted versus  $M$  in fig. 4. In this case the hard component is of order of  $0.2 \text{ GeV}^2$  for  $M \sim 4-6 \text{ GeV}$  and  $0.1 \text{ GeV}^2$  for higher masses [18]. The effective  $S$  dependence of  $\langle p_\perp^2 \rangle$  is also evident here, for the same reasons explained above.

Our results are quite stable upon a reasonable variation of  $\Lambda$ ,  $\lambda$  and  $\langle p_\perp^2 \rangle_{\text{intr}}$ . We have not tried to make a best fit to all data, including small changes in the parametrization of the structure functions. Our aim was in fact to prove the overall consistency of the idea of

summing soft gluons effects also in the Drell-Yan process.

To conclude, we have shown that the observed features of the transverse momentum distributions of Drell-Yan pairs are very well described in perturbative QCD. The multiple gluon soft contributions play a very important role in this respect. Finally a quite reasonable value of the partonic intrinsic  $\langle p_\perp^2 \rangle$  is suggested from our analysis.

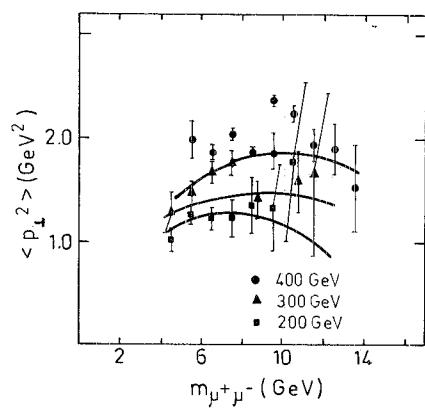


Fig. 4. Predictions for  $\langle p_\perp^2 \rangle$  in pN collisions at  $p_{\text{lab}} = 200, 300$  and 400 GeV. The data are from ref. [13].

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