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DYNAMICS OF TRAVERSING AN AVOIDED LEVEL CROSSING

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Recognizing the close formal similarity between the Landau-Zener model and the Froissart-Stora model, one can perceive in the literature a generalized Landau-Zener formula. It allows to take account of the limited safe-range of the applied electric field-jump in traversing rapidly an avoided energy level crossing.

Very recently, Rubbmark, Kash, Littman and Kleppner[1] (RKLK), using a selected pair of Stark levels of lithium Rydberg states, were able to demonstrate the Landau-Zener (LZ) effect[2,3] unambiguously. In their clean and convincing experiment, the two-level atoms were initially prepared in either the ground or the excited state via pulsed laser excitation, in the presence of an electric field. Then, by sweeping as rapidly as possible the applied electric field through the avoided crossing, the system was prevented from being completely inverted. The response of the system was observed by measuring the final populations of the two states. All the relevant physical parameters involved in the experiment were known, so that a comparison with the theoretical expectations was possible.

Since the chosen avoided crossing was close to others adjacent resonances, RKLK limited the field-jump magnitude to a relatively small value, in order to be sure that the theory of isolated resonances does apply in their experiment. On the other hand, the LZ theory assumes a linear dependence of the applied electric field over the whole time interval, $-\infty < t < +\infty$. Thus, for a correct comparison between theory and experiment, RKLK gave numerical solutions of the conventional coupled equations (i. e. the Schrödinger equation)

$$2i\dot{C}_1 = C_2 \exp(i \int_0^t \dot{\chi}\, dt')\ \omega_0 \exp(-i\Phi) \tag{1a}$$

$$2i\dot{C}_2 = C_1 \exp(-i \int_0^t \dot{\chi}\, dt')\ \omega_0 \exp(i\Phi) \tag{1b}$$

(their Eqs. (12)), assuming ω_0 = constant, Φ = constant (as in the LZ model), and, for $\dot{\chi}$, the model form

$$\dot{\chi} = \delta \tanh(\beta t)\ , \tag{2a}$$

with

$$2\delta = \alpha\tau \qquad \text{and} \qquad \beta = 2/\tau \tag{2b, c}$$

(their Eq. (18)). In eqs. above, C_1 and C_2 are the occupation numbers of the two states, ω_0 is the level separation, 2δ is the total change in energy and β may be interpreted, essentially, as the spectral width of the sweep.

Their numerical results, summarized in their Fig. 4, indicated that they operated in experimental conditions where the simple LZ model

$$\dot{\chi}_{LZ} = \alpha t = \ddot{\chi}(0)\, t \tag{3}$$

is still adequate to describe the avoided crossing traversal dynamics. This is because the applied range, 2δ, was always wide enough to accomodate generously the level separation, ω_0.

In terms of their notations, such a condition is expressed by

$$d \gg 1\ , \tag{4}$$

where

$$d = 2(2\delta/\omega_o) \tag{5}$$

(their Eq. (21)). Here we will refer to condition (4) as the "RKLK condition".

The purpose of the present letter is to point out that the case of the model form as given by Eq. (2) is solvable in terms of hyper-geometric functions, i. e. that the diabatic transition probability, P, can be given analytically in terms of the LZ parameter

$$\Gamma = \omega_o^2/(4\alpha) \tag{6}$$

(Eq. (15) of Ref. (1)) and d; or, conversely, in terms of Γ and

$$s = \omega_o\tau/2 \tag{7}$$

(Eq. (20) of Ref. (1)). The resulting very simple formula for P is presented herebelow:

$$P = \left[\sinh(p\pi)/\sinh(r\pi)\right]^2\ , \tag{8}$$

where

$$p \equiv \delta/(2\beta)\ , \qquad r \equiv (\delta^2 + \omega_o^2)^{1/2}/(2\beta)\ , \tag{8a}$$

or, in terms of the RKLK parameters,

$$p \equiv (1/8)\Gamma d^2, \qquad r \equiv p(1 + (4/d)^2)^{1/2}, \tag{8b}$$

or, alternatively,

$$p \equiv (1/8)s^2/\Gamma, \qquad r \equiv p(1 + (4\Gamma/s)^2)^{1/2}\ . \tag{8c}$$

It gives account for the consequences of a more or less complete violation of the RKLK condition, thereby allowing one to easily plan applications of pulsed-field ionization under every experimental condition.

Before we look at what is to be found in the literature on the derivation of eq. (8), we first prefer to convey the convinction of its validity. First of all, one can see very quickly that when the RKLK condition (4) is satisfied, i. e. for $p \gg 1$, it reduces precisely to the LZ diabatic transition probability,

$$P_{LZ} = \exp(-2\pi\Gamma). \qquad (9)$$

Moreover, in calculating P throughout all the ranges of the parameters indicated in Fig. 4 and Fig. 5b of Ref. (1), profiles graphically identical with those presented by RKLK have resulted. This demonstrates the suitability of our Eq. (8) and constitutes therefore the central result of our letter.

On the derivation of Eq. (8), we simply declare that Eqs. (1) with $\dot{\chi}$ given by Eq. (2) arose recently[(4)] in considering another phenomenon which is a direct analog of the RKLK experiment: the problem was the analytical calculation of the residue polarization $S_{z(t \to +\infty)} = 2P - 1$ of a polarized beam of charged relativistic spin - 1/2 particles ($S_{z(t \to -\infty)} = 1$; $S_{x(t \to -\infty)} = S_{y(t \to -\infty)} = 0$) accelerated in a synchrotron, on passing (as rapidly as possible) through a depolarization resonance. Once the close formal similarity between the LZ model[(2, 3)] and the Froissart and Stora (FS) model[(5)] (which describes the depolarization effect) is recognized, it should be easy to follow the derivation[(4)] of eq. (8). In other words, the background with which the Reader should be familiar to find that the LZ theory has its counterpart in the FS theory is supplied by the geometric representation of a two-level system, namely the Feynman-Vernon-

Hellwarth representation[6]. Evidently, in Ref. (4) we discuss the spin transition probability in terms of the Bloch-vector (S_x, S_y, S_z) in a reference frame $(\bar{i}, \bar{j}, \bar{k})$ which rotates in the physical space.

The only thing to add is that Eq. (2) is identical with eq. (6b) of Ref. (4) and that the basic equations (1) are readily converted into eqs. (8) of Ref. (4) by using

$$C_1 = g \exp(-i\Phi/2) , \qquad C_2 = f \exp(i\Phi/2). \qquad (10a,b)$$

In closing, we mention, for the sake of completeness, a couple (unique) of entirely different experiments that are an analog of the RKLK experiment: These are the successful acceleration of polarized proton beams in the Argonne ZGS[7] and of polarized electron beams in the Bonn Synchrotron[8].

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