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Thrust distributions near $T=1$

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The effect of the emission of soft quantum-chromodynamic radiation on the thrust (T) distribution for the process $e^+e^- \to q\bar{q}g$ is examined. The analysis goes beyond the usual leading-logarithmic summation analysis of the q and $\bar{q}$ jets, since we allow for the development of a hard-gluon jet and we are able to satisfy the constraints of momentum conservation and kinematics.

Leading-logarithmic analyses (LLA), i.e., a summation of all terms of the type $[\alpha_s(Q^2)\ln Q^2]^n$, have been very useful in our understanding of corrections to the Born terms in perturbative quantum-chromodynamics (QCD) treatments of hard processes.[1,2] In addition, nonleading corrections to LLA have also been shown recently[3,4] to resum in a rather simple way. The latter corrections turn out to be quite sizable, thus confirming the importance of kinematical constraints in various cases. Furthermore, the improved resummation formulas have shed much light on the so-called nonperturbative background (the soft region) in jet processes.

The above formalism has been employed to study the transverse-momentum distributions of e^+e^- hadronic jets. A recent analysis of data from the PLUTO collaboration[5] at PETRA shows a striking agreement with the corresponding theoretical expressions[6] obtained through the coherent-state formalism[7] in LLA and in which transverse-momentum conservation is explicitly taken into account.

In view of the above successes we investigate the infrared (IR) region for e^+e^- jets in another familiar variable, the thrust (T). Attempts along this line have already been made in the literature,[8] and we shall later give appropriate comparisons with this reference.

In this paper we present approximate analytical expression for the T distribution near $T \simeq 1$ for a $q\bar{q}g$ jet in e^+e^- annihilation, obtained upon summation of leading terms in $\alpha\ln[1/(1-T)]$ having explicitly accounted for (i) momentum conservation and (ii) appropriate kinematical limits. Thus, it goes beyond the leading-order calculation.[8]

Before presenting our calculations we note that our results cannot be compared directly with experimental T distributions except near $T \simeq 1$. In fact, for smaller T, sizable nonsingular α_s^2 terms need to be included.[9]

The first-order QCD correction to the basic $q\bar{q}$ process is given[10] by

$$\left(\frac{dP}{dT}\right)_0 \equiv \frac{1}{\sigma_T}\frac{d\sigma}{dT}(q\bar{q}g) = \left(\frac{4\alpha}{3\pi}\right)\left[\frac{3T^2-3T+2}{T(1-T)}\ln\left(\frac{2T-1}{1-T}\right) - \frac{3(3T-2)(2-T)}{2(1-T)}\right] \tag{1a}$$

$$\underset{(T\to 1)}{\simeq} \frac{8\alpha}{3\pi}\frac{1}{1-T}\left[\ln\left(\frac{1}{1-T}\right)-\frac{3}{4}\right] . \tag{1b}$$

The coherent-state formalism provides us with a method for obtaining corrections to the above Born term, Eq. (1), by summing the soft QCD radiation in the variable T (for $T \simeq 1$) just as has been done previously for energy-momentum distributions of quark and gluon jets.[6,7] This leads to the probability distribution in T:

$$\frac{dP}{dT} = \frac{1}{2\pi}\int_{-\infty}^{\infty} dt\, e^{it(1-T)}e^{-h(t)} , \tag{2}$$

where

$$h(t) \simeq \frac{8}{3\pi}\int_0^1 \frac{dy}{1-y}\int_{Q^2(1-y)^2}^{Q^2(1-y)} \frac{dk_\perp^2}{k_\perp^2}\alpha(k_\perp)(1-e^{-it(1-y)}) . \tag{3}$$

In Eq. (3) kinematical limits on k have been simplified and are thus only valid for large Q^2 and T. First-order expansion in α leads then to the leading term in Eq. (1b). We shall return to the second, nonleading term ($\frac{3}{4}$)

momentarily.

We now proceed to obtain an approximate analytic form for Eq. (2). Following steps similar to those developed in Ref. 3, near $T \simeq 1$, we may write

$$\frac{dP}{dT} \simeq \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} dn\, e^{n(1-T)} \exp[-(8\alpha/3\pi) H_0(n)] \ , \tag{4}$$

where

$$H_0(n) = \int_0^1 \frac{dy}{1-y} \left[\ln\left(\frac{1}{1-y} \right) \right] (1-y^n) \ . \tag{5}$$

In Eq. (4) we have approximated the running coupling constant $\alpha(k_\perp)$ by the usual $\alpha = \alpha(Q^2)$. Later on we shall improve upon this assumption. Near $T \simeq 1$ large values of n dominate in $H_0(n)$ for which we have the asymptotic expansion

$$H_0(n) \simeq \tfrac{1}{2}[\ln^2(n\gamma) + \pi^2/6] + O\left(\frac{\ln n}{n} \right) \ , \tag{6}$$

where $\ln\gamma = \gamma_E = 0.577$. The so far neglected next-to-the-leading term in Eq. (1b) can now be accounted for upon replacing $H_0(n)$ by $H(n)$:

$$H(n) = \int_0^1 \frac{dy}{1-y} \left[\ln\left(\frac{1}{1-y} \right) - \frac{3}{4} \right] (1-y^n) \underset{n \text{ large}}{\longrightarrow} \tfrac{1}{2}[\ln^2(n\gamma) - \tfrac{3}{2}\ln(n\gamma)] + \cdots \ . \tag{7}$$

In Eq. (7) the "$+ \cdots$" refers to nonsingular terms in the large-n limit. Thus all divergent terms in n have been included, save those arising from variations in $\alpha(k_\perp)$, which are omitted so far. Substituting Eq. (7) into Eq. (4), we obtain

$$\frac{dP}{dT} \simeq \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} dn\, e^{n(1-T)} \exp\left(-\frac{\beta}{2} \ln^2(n\gamma) + \frac{3}{4}\beta \ln(n\gamma) \right) \ , \tag{8}$$

with $\beta = 8\alpha/3\pi$.

Defining the scaled variable $W = (1-T)n$, Eq. (8) may be rewritten as

$$\frac{dP}{dT} \simeq \frac{1}{1-T} \exp\left[-\frac{\beta}{2} \ln^2\left(\frac{1}{1-T} \right) + \frac{3}{4}\beta \ln\left(\frac{1}{1-T} \right) - \beta(\ln\gamma) \ln\left(\frac{1}{1-T} \right) \right] I(T) \ , \tag{9}$$

where

$$I(T) = \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} dW\, e^W \exp\left[-\beta \ln\left(\frac{1}{1-T} \right) \ln W - \frac{\beta}{2} \ln^2(W\gamma) + \frac{3}{4}\beta \ln(W\gamma) \right] \ . \tag{10}$$

Keeping only the most singular terms in $I(T)$ for $T \rightarrow 1$, we obtain

$$\frac{dP}{dT} \simeq \frac{\exp\{(\frac{3}{4} - \gamma_E)\beta \ln[1/(1-T)]\}}{\Gamma(1 + \beta \ln[1/(1-T)])} \frac{\beta \ln[1/(1-T)]}{1-T} \exp\left[-\frac{\beta}{2} \ln^2\left(\frac{1}{1-T} \right) \right] \ . \tag{11}$$

The range of validity (in T) can be enlarged by inserting the full Born term [Eq. (1a)]:

$$\frac{dP}{dT} \simeq \left(\frac{dP}{dT} \right)_0 \frac{\exp\{(\frac{3}{4} - \gamma_E)\beta \ln[1/(1-T)]\}}{\Gamma(1 + \beta \ln[1/(1-T)])} \exp\left[-\frac{\beta}{2} \ln^2\left(\frac{1}{1-T} \right) \right] \ . \tag{12}$$

It is possible to give a rather intuitive meaning to the various factors in the calculated T distribution (dP/dT).

The lowest-order (Born) term distribution, $(dP/dT)_0$, is multiplied by the probability that the quark and the antiquark develop into two jets with invariant masses $M^2 \sim Q^2(1-T)$. In fact, the factor

$$\exp\{(\tfrac{3}{4})\beta\ln[1/(1-T)]-(\beta/2)\ln^2[1/(1-T)]\}$$

in Eq. (12) corresponds to an exponentiation[6,7] of the first-order probability[11]:

$$P(M^2)=\frac{8\alpha}{3\pi}\int_{M^2}^{Q^2}\frac{dm^2}{m^2}\left(\ln\frac{Q^2}{m^2}-\frac{3}{4}\right) . \qquad (13)$$

The factor

$$\frac{\exp\{-\gamma_E\beta\ln[1/(1-T)]\}}{\Gamma(1+\beta\ln[1/(1-T)])}$$

comes from the overall momentum conservation, similar to that found in the usual x distribution for the valence quark densities around $x \simeq 1$.[3,12,13] In comparison, the simply exponentiated (LLA) formula presented in Ref. 8 only contains the term $\exp\{-(\beta/2)\times\ln^2[1/(1-T)]\}$, which multiplies the singular piece of the Born term.

So far we have ignored the possibility that the hard gluon—in addition to the quark and the antiquark—may develop a jet of its own with an invariant mass $\sim Q^2(1-T)$. The corresponding probability may be incorporated in the following manner:

$$\left(\frac{dP}{dT}\right)\simeq\left(\frac{dP}{dT}\right)_0\frac{\exp\{\frac{3}{4}\beta_q+\frac{1}{2}(\frac{11}{6}-N_f/9)\beta_g\}\ln[1/(1-T)]}{\Gamma(1+(\beta_q+\beta_g)\ln[1/(1-T)])}$$
$$\times\exp\{-\gamma_E(\beta_q+\beta_g)\ln[1/(1-T)]\}\exp\{-\tfrac{1}{2}(\beta_q+\beta_g)\ln^2[1/(1-T)]\} . \qquad (14)$$

In Eq. (14) $\beta_q \equiv \beta = 8\alpha/3\pi$ and $\beta_g = 3\alpha/\pi$. Equation (14) is our final expression. Note that the factor

$$\frac{\exp\{-\gamma_E(\beta_q+\beta_g)\ln[1/(1-T)]\}}{\Gamma(1+(\beta_q+\beta_g)\ln[1/(1-T)])}$$

will appear in a perturbation expansion only at second order in $(\beta_q+\beta_g)$. This factor, however, improves considerably the falloff of the distribution for $T\rightarrow 1$.

Now we return to a discussion of the "proper" variable to accompany α. Explicit second-order calculations[9] have shown that in the two-jet limit a better convergence of the perturbation series is expected when the momentum squared, which determines the strength of α, is taken to be of the order of $Q^2(1-T)$. This is also suggested by Eq. (3). In numerical computations presented below, therefore, we have made the substitution $\alpha=\alpha[Q^2(1-T)]$.

In Fig. 1 we plot the Born distribution $(dP/dT)_0$ given by Eq. (1a) (full line) and compare it with (dP/dT) of Eq. (14) (broken line). For completeness, the first-order expansion in $(\beta_q+\beta_g)$ of our exponentiated result is also shown (dotted line). This figure shows clearly that for $T \gtrsim 0.9$ higher-order corrections given by the soft radiation are quite important. The turnover of the theoretical curve agrees quite well with the experimental results of Ref. 14, in contrast to the leading-logarithmic result of Ref. 8. However, a complete, realistic description of the experimental distributions must await suitable addition of $\alpha_s{}^2$ corrections[9] to our calculation of the singular terms in $\alpha\ln[1/(1-T)]$.

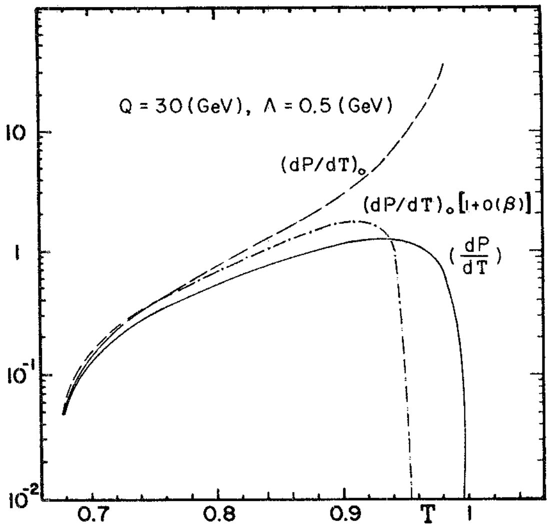

FIG. 1. Shown are (dP/dT) (continuous curve) given by Eq. (14), $(dP/dT)_0[1+O(\beta)]$ (dot-dashed curve), and $(dP/dT)_0$ (dashed curve) given by Eq. (1a). The parameters are $Q=30$ GeV and $\Lambda=0.5$ GeV.

To conclude, we have presented analytic approximation to the T distribution for a $q\bar{q}g$ jet in e^+e^- annihilation near $T \simeq 1$, obtained upon summation of leading terms in $\alpha\ln[1/(1-T)]$. We have included the evolution of a hard-gluon jet in addition to those of q and $\bar{q}$ jets. Momentum conservation is also imposed. The shape of the distribution seems to be in good qualitative agreement with the data for $T \simeq 1$.

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