ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-81/51

A.F. Grillo and Y. Srivastava: COSMOLOGICAL BLACK HOLE PRODUCTION IN GRAND UNIFIED THEORIES

Estratto da:
Nuclear Phys. B187, 271 (1981)

Servizio Documentazione
dei Laboratori Nazionali di Frascati
Cas. Postale 13 - Frascati (Roma)

Nuclear Physics B187 (1981) 271–284

COSMOLOGICAL BLACK HOLE PRODUCTION IN GRAND UNIFIED THEORIES

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Received 2 February 1981

Cosmological black hole production is reviewed with due consideration to interactions assumed relevant in the very early Universe. In the framework of the standard hot big bang and grand unified theories it is shown that primordial black holes would form too prolifically to be consistent with observations, if they are present in an extended mass range. This poses some constraints on the form of fluctuations present in the very early Universe, be they primordial or induced spontaneously by phase transitions in the cooling Universe.

1. Introduction

In the last years particle physicists have developed the ambition of being able to infer, from "known" physics at the laboratory level, information about the evolution of the very early Universe (i.e. times $\lesssim 10^{-4}$ s).

It is generally agreed that already at such early times, quantities governing the large scale evolution of the Universe (e.g., baryon/photon ratio) attained their present values. Very recently, this hope has been strengthened by the advent of the so-called grand unified theories (GUT) [1], in which all particle interactions (save gravitational) are put on the same footing in an *a priori* well defined and workable scheme.

This advance has prompted the appearance of a number of results concerning various cosmological features like baryon number generation [2], the growth and decay of fluctuations [3] and in general the very high temperature behavior of the cosmic fluid (i.e. the fluid filling the early Universe) [4].

In this paper, we shall assume a specific GU model [the simplest version of SU(5)] although the bulk of our results depend only on general properties like asymptotic

[1] Supported in part by the National Science Foundation, Washington D.C., USA.

freedom which hold in a large class of such models. We will investigate the production of black holes (bh) in such a scenario.

The possibility that extreme conditions of density could lead to appreciable production of primordial black holes (pbh), i.e. those not coming from the collapse of a star, has been envisaged some time ago by Novikov and Zel'dovich [5] and by Hawking [6]. For pbh, the mass limitation $M \gtrsim M_\odot$ does not hold and a population of mini black holes with masses as small as the Planck mass ($m_p \simeq 10^{-5}$ g) is expected to arise, due to density fluctuations in the very early Universe. On the other hand, such density fluctuations are presumably needed to explain galaxy formation.

From a purely classical point of view these pbh would still be present amongst us. However, Hawking [7] has shown that a coupling of quantum matter to the classical gravitational field of bh gives rise to unexpected results: the bh is not black, radiating all particle species with a thermal spectrum. This feature is of interest for very small (i.e. cosmological) bh only, since those arising from gravitational collapse would have a temperature far too low to be detectable.

Pbh if produced, would play a role in baryon number generation [8], galaxy formation [9], gravitational wave production [10] and perhaps could also provide enough matter to close the Universe.

We stress, however, a conceptual point which seems to be of interest. Due to their peculiar nature, pbh are never in equilibrium with the other components of the (radiation dominated) early Universe in the sense that, once formed, their number does not change any further apart from evaporation. This implies that characteristics of the pbh spectrum can, in principle, act as excellent indicators of the conditions prevailing in the Universe at the moment of their formation.

Our main result is the following: if pbh are to be produced in an extended mass range from fluctuations present at the Planck time ($t_p \sim 10^{-43}$ s) then, since the GU Universe is, thanks to asymptotic freedom, very close to an ideal gas during almost all the period we will consider (10^{-43} s $\lesssim t \lesssim 10^{-13}$ s), pbh are expected to be overproduced. This overproduction refers to the fact that there are very stringent limits due to the non-observation of their explosion. The very early Universe thus appears to be rather quiet, i.e. not far from being exactly isotropic and homogeneous.

On the other hand, fluctuations are likely to be produced also at the phase transitions during the cooling of the Universe all the way from an exactly symmetric [SU(5) say] gas of unconfined quarks, leptons and bosons to by now familiar $SU(3) \times U(1)_{EM}$ symmetric world of hadrons and leptons [11, 12]. We argue that also for such fluctuations bh are produced but not over an extended mass range. Their production at discrete mass values is allowed, however.

The plan of the paper is as follows: In sect. 2, we review the theory of pbh production, closely following the works by Carr [13, 14]. In sect. 3, we consider the thermodynamic behavior of the Universe assuming the simplest SU(5) picture. Sect. 4 is devoted to the derivation of the main results and to the comparison with observational data. In sect. 5 we discuss our results and present a speculation due to some numerical coincidences.

2. Pbh formation

We will sketch here only the main features of the pbh production mechanism since a complete treatment is available in the articles by Carr and Hawking [15] and by Carr [14].

As usual, the hypothesis will be made that the Universe is as simple as possible, which is to say that the observed large scale homogeneity and isotropy (better than 1 part in 10^4 for the background 3 K radiation) is primordial and not due to dissipative processes. There may be some reason beyond simplicity for assuming the above. In fact, were the Universe to be completely chaotic, it would be difficult to understand why the number of photons (i.e., entropy) per baryon would be a mere 10^9.

Consequently, the early Universe is described by a Friedman–Robertson–Walker metric (in terms of comoving coordinates)

$$\mathrm{d}s^2 = \mathrm{d}t^2 - R^2(t)[\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2], \tag{1}$$

where $R(t)$ obeys the Einstein equations for a perfect fluid:

$$\left(\frac{\mathrm{d}R}{\mathrm{d}t}\right)^2 = \tfrac{8}{3}\pi G\rho R^2 \tag{2a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(\rho R^3) = -3pR^2\left(\frac{\mathrm{d}R}{\mathrm{d}t}\right). \tag{2b}$$

Here ρ is the proper energy density and p is the pressure. For an ultra-relativistic gas $p = \frac{1}{3}\rho$ and we have the familiar results

$$R \sim t^{1/2}, \qquad \rho R^4 \sim \text{constant}. \tag{3}$$

More generally, if $p = f\rho$, with f a constant, eq. (2b) becomes

$$\frac{\mathrm{d}}{\mathrm{d}t}[\rho R^{3(1+f)}] = 0, \tag{4}$$

and consequently

$$R \sim t^{2/3(1+f)}. \tag{5}$$

Let us assume that at some initial time t_0 there existed overdense regions with an overdensity

$$\delta=\frac{\delta\rho}{\rho}, \tag{6}$$

on scales larger than the particle horizon.

In principle, such fluctuations could be completely arbitrary and provide initial conditions for the Universe. It has been argued, however, that there is a subset which is in some sense natural [12, 16].

They have the form

$$\delta\sim\varepsilon\left(\frac{m}{m_0}\right)^{-n}, \tag{7}$$

where m is the mass in the perturbed region, m_0 is the "horizon" mass ($m_0\sim t_0/G$) and ε could, in principle, depend on m_0. Also, a favored value for $n=\frac{2}{3}$ is obtained from theories of galaxy formation [17].

In the perturbed region the metric can be written in the form

$$\mathrm{d}s^2=\mathrm{d}\tau^2-S^2(\tau)\left[(1-kr^2)^{-1}\mathrm{d}r^2+r^2\mathrm{d}\Omega^2\right], \tag{8}$$

where k is the perturbed total energy per unit mass [16]. Choosing the proper time τ to coincide with t at the initial time t_0 and $R_0=S_0$, the equation reads

$$\begin{aligned}\left(\frac{\mathrm{d}S}{\mathrm{d}\tau}\right)^2&=\tfrac{8}{3}\pi G\rho_0(1+\delta)R_0^{3(1+f)}S^{-(1+3f)}-k\\&=\left[\tfrac{8}{3}\pi G\rho_0R_0^{3(1+f)}\right]\left[\frac{(1+\delta)}{S_0^{(1+3f)}}-\frac{\delta}{R_0^{(1+3f)}}\right].\end{aligned} \tag{9}$$

When this region begins to recollapse at

$$\begin{aligned}t_c&\sim t_0\delta^{-3(1+f)/2[1+3f]},\\S_c&\sim R_0\delta^{-1/[1+3f]},\end{aligned} \tag{10}$$

it becomes a black hole if its scale S_c is bigger than the Jeans length, $R_J=(4\pi\rho G/f)^{-1/2}$, and smaller than the particle horizon [15]. In an epoch with a "hard" equation of state (i.e. $f\neq 0$), the Jeans length is of the same order of magnitude as the Schwarzschild radius.

The above conditions for the formation of black holes can be shown [13] to be equivalent to

$$\alpha^2\left(\frac{m}{m_0}\right)^{-2/3} \geqslant \delta \geqslant \beta^2\left(\frac{m}{m_0}\right)^{-2/3}, \tag{11}$$

where the coefficients α and $\beta=\sqrt{f}$ are both of order unity.

It is important to stress that, while density perturbations are coordinate dependent, the collapse condition (11) is not [13].

We also note that since α and β are both of the same order, condition (11) severely constrains the form of fluctuations which can give rise to pbh's.

The overall probability that an initially overdense region of mass m become a black hole is given by

$$\pi(m) \sim m^{-1}\varepsilon \exp(-\beta^4/2\varepsilon^2), \tag{12}$$

if $n=\frac{2}{3}$, and

$$\pi(m) \sim \varepsilon m^{-1}\left(\frac{m}{m_0}\right)^{2/3-n} \exp\left\{-\frac{\beta^4}{2\varepsilon^2}\left(\frac{m}{m_0}\right)^{2n-4/3}\right\}, \tag{13}$$

otherwise.

The mass of the initially overdense region is not the final mass of the black hole since part of it will get redshifted away, so the final collapsed mass m_c is [13]

$$m_c \sim m^{(1+f)/(1+3f)} m_0^{2f/(1+3f)}. \tag{14}$$

A *smaller* value of f, which corresponds to a *faster* expansion then implies a *bigger* m_c for a given m. This somewhat surprising statement may be understood by noting that, in a faster expanding Universe, more mass is redshifted but the collapse takes place before [see eq. (10)].

The *present* number density of the pbh's in the interval m and $m+\mathrm{d}m$ is then

$$n(m) \sim F\varepsilon \exp\{-\beta^4/2\varepsilon^2\}\rho_0 m_0^{-2}\left(\frac{m}{m_0}\right)^{-1-(1+3f)/(1+f)}, \tag{15}$$

where ρ_0 and m_0 are the initial density of the perturbed region and the horizon mass respectively, and

$$F=\left[\frac{R(t_0)}{R(\text{now})}\right]^3,$$

is the ratio of number density now to what it was at t_0.

Eq. (15) holds for $n=\frac{2}{3}$ perturbations. If $n\neq\frac{2}{3}$ we have

$$n(m)\sim F\varepsilon\rho_0 m_0^{-2}\left(\frac{m}{m_0}\right)^{-1-(n+1/3)(1+3f)/(1+f)}\cdot$$

$$\times\exp\left\{-\frac{\beta^4}{2\varepsilon^2}\left(\frac{m}{m_0}\right)^{(2n-4/3)(1+3f)/(1+f)}\right\}. \qquad (16)$$

Thus, there is an exponential upper (lower) cut off if $n>\frac{2}{3}$ ($<\frac{2}{3}$). The case $n<\frac{2}{3}$ implies an exponentially increasing pbh which is in contradiction with observations.

These black holes would still be present if they behaved classically. However, Hawking [7] has shown that a black hole emits particles quantum mechanically with a thermal spectrum corresponding to a temperature

$$T=\frac{\hbar c^3}{8\pi GMk_B}, \qquad (17)$$

in a time [18]

$$\tau=10^{-26}M^3N^{-1}, \qquad (18)$$

where M is in grams, N is the number of particle species emitted and τ is in seconds. This time is much larger than the age of the Universe for astrophysical black holes, but bh's of mass $\lesssim 10^{15}$ g would be evaporating now.

Given the above, the present cumulative density of pbh of mass larger than $\bar{m}$ becomes

$$M(\bar{m})=\int_{\bar{m}}^{\infty}mn(m)\,\mathrm{d}m=\rho_0F\exp\left(-\frac{\beta^4}{2\varepsilon^2}\right)\left(\frac{\bar{m}}{m}\right)^{-2f/(1+f)}, \qquad (19)$$

provided $n=\frac{2}{3}$. (Note that only for $n=\frac{2}{3}$, are pbh produced over an extended range.)

Obviously, due to quantum evaporation

$$\bar{m}\gtrsim 10^{15}\ \mathrm{g}.$$

The quantity $M(m)$ is zero in exponentially cut off spectrum (for $n>\frac{2}{3}$) if pbh are produced at the Planck time, since they would peak around the Planck mass and evaporate very quickly.

3. Grand unified thermodynamics

In the expanding Universe, time and temperature are related through

$$T\sim t^{-1/2}, \qquad (20)$$

valid in a radiation dominated era, the proportionality factor depending upon the number of species of ultra-relativistic particles.

Around $t \sim 10^{-23}$ s, which corresponds to a temperature $T \sim 10^8$ GeV, the horizon length is of the order of the size of a typical hadron. Thus, if hadrons existed as individual entities at such temperatures, they would not appear as causal objects. However, already at much lower temperatures ($T \sim 200$ MeV), according to popular beliefs [19], hadrons would have split into their components. That is, a transition from a hadronic phase to a quark phase is expected at rather low temperatures (~ 200 MeV), in which quarks, gauge bosons and leptons are the "elementary" (or point-like) objects. Then, the difficulty regarding the acausal appearance of hadrons does not arise and we can safely extend our considerations to times $\lesssim 10^{-23}$ s.

An essential ingredient for a quantitative analysis of the effects of particle interactions in the very early Universe is asymptotic freedom. The attractive feature of asymptotic freedom is that the coupling constant goes to zero at large energies (or temperatures) thus enabling one to do perturbative calculations. A class of models, known as grand unified theories (GUT), is a gauge theory of quarks and leptons with a single coupling constant. For such theories, asymptotic freedom has been shown at least for the fermion/gauge sector. The situation is not clear regarding the scalar (Higgs) self-couplings. We will make the simplifying assumption, for which strong arguments can be given, that also in this sector asymptotic freedom holds.

From now on, our analysis will be carried out in the framework of some specific GUT, for which we shall obtain precise time-temperature relations. Different eras in the expanding Universe will be defined subsequently.

Up to logarithmic corrections, the energy density is given by [20]

$$\rho(t) = \frac{3}{32\pi G t^2}, \tag{21}$$

and, for N ultra-relativistic helicity states

$$\rho(T) = 0.36 N T^4, \tag{22}$$

so that

$$T \simeq 0.53 (GN)^{-1/4} t^{-1/2}. \tag{23}$$

Note the dependence on N in eq. (23).

Consider now a specific GUT, for instance the minimal realization of SU(5). Here we have 3 generations of fermions in the representations $\bar{5}$ and 10, 24 gauge bosons and a $25 \oplus 5$ of Higgs scalars. The total number of helicity states amounts to ~ 160 (for T above the masses of all particles).

At a critical temperature $T_c \sim 10^{15}$ GeV we have the first phase transition to an $SU(3) \otimes SU(2) \otimes U(1)$ symmetric world. The number of ultra-relativistic helicity states decreases to $N \sim 107$. This symmetry is broken down, at $T_c' \sim 100$ GeV, to the "real" $SU(3) \otimes U(1)_{EM}$ phase where N drops to 97. Finally, at $T \sim 200$ MeV, quarks become confined and hadrons appear.

In order to get the required thermodynamic description in these eras we need to know the grand canonical potential Ω. Given Ω, energy density and pressure can be computed through

$$\rho = \frac{1}{T} \frac{\partial}{\partial(1/T)} (T^{-1} \Omega V^{-1}), \tag{24}$$

$$p = -\Omega/V. \tag{25}$$

In refs. [4,22], Ω has been computed for QCD and SU(5) phases. The result may be written in the following general form:

$$\Omega = -VT^4\left[a - b\alpha + c\alpha^{3/2} + O(\alpha^2)\right], \tag{26}$$

α being the dimensionless fine structure constant of the relevant theory.

For instance, for the minimal SU(5) theory alluded to above, asymptotic freedom gives

$$\alpha(T) \sim \left(\frac{4\pi}{13}\right) \frac{1}{(\ln T/\Lambda)}, \tag{27}$$

where $\Lambda \sim 500$ MeV.

In eq. (26), a, b and c are numerical factors which depend upon the relevant physics in the various eras.

From eqs. (24) and (26) we can write

$$\rho \simeq 3p\left[1 + h(T)\right], \tag{28}$$

where

$$h(T) = \left(\frac{13}{12\pi}\right)\left(\frac{1}{a}\right)\alpha^2\left[b - \tfrac{3}{2}c\alpha^{1/2} + O(\alpha)\right]. \tag{29}$$

Inserting the numbers for the SU(5) phase [4]

$$a = 28.9, \qquad b = 104, \qquad c = 587.7, \tag{30}$$

we find

$$h(T \simeq 10^{15}\ \text{GeV}) \simeq -10^{-4}. \tag{31}$$

It is seen from eqs. (29) and (30) that the two leading terms in h are of the same order with opposite signs. Their approximate cancellation makes a precise determination of h rather uncertain. Luckily, however, h remains very small ($\sim 10^{-4}$) and thus its exact value is not very relevant.

With somewhat different numerics, the same result (i.e. a very small correction to the ideal gas behavior) also holds in the other phases, given asymptotic freedom and provided we stay far away from phase transition regions.

Near the critical temperatures, the picture changes drastically. The grand canonical potential becomes dominated by Higgs scalars, and in general it takes the form [23]

$$\Omega = A + B(T - T_c)^2 + C(T - T_c)^3 + \cdots . \tag{32}$$

This includes the possibility of a cosmological term (i.e. a constant in the energy density) which implies an exponentially expanding Universe near a phase transition.

4. Constraint on cosmological pbh spectrum

The results of sect. 3 hold in thermal equilibrium. But thermal equilibrium does not necessarily occur in the very early Universe. We turn to a discussion of this point.

Let us consider the SU(5) phase for which typical decay and interaction rates are [24]

$$\Gamma_D \sim \alpha M^2 N / [T^2 + M^2]^{1/2}, \tag{33}$$

$$\Gamma_I \sim \alpha^2 T^5 N / [T^2 + M^2]^2, \tag{34}$$

where $M \sim 10^{15}$ GeV. To investigate the question of equilibrium we compare the interaction rate (Γ_I) with the expansion rate of the Universe

$$H = \dot{R}/R \simeq 1.66 T^2 N^{1/2} / m_p . \tag{35}$$

We find that, down to temperatures $T \approx 10^{14}$ GeV, these two are not in equilibrium (see, however, ref. [4]).

Note that this temp ($\sim 10^{14}$ GeV) is of the same order (or below) as that of the phase transition and thus, strictly speaking, the Universe is never in a phase in which exact SU(5) interactions are in equilibrium. As has been shown in ref. [3], this leads to viscous effects in the expansion of the Universe. Viscous effects, however, should have no consequences on pbh production [25] since viscosity becomes irrelevant on scales bigger than the horizon.

We thus turn to a quantitative analysis of GUT in the collapsing region. For this purpose, let us compare the interaction rate Γ_I, eq. (34), with the expansion rate of the perturbed region [eq. (9)]:

$$\left(\frac{\dot{S}}{S}\right)^2 = S^{-2}\left[\tfrac{8}{3}\pi G\rho_0 R_0^{3(1+f)}\right]\left[\frac{(1+\delta)}{S^{1+3f}} - \frac{\delta}{R_0^{1+3f}}\right]. \tag{36}$$

Given

$$\rho S^{3(1+f)} = \rho_0 R_0^{3(1+f)}, \qquad \rho = 0.36NT^4,$$

eq. (36) can be rewritten as

$$\left(\frac{\dot{S}}{S}\right)^2 = \left(\tfrac{8}{3}\pi G\rho_0\right)\left(R_0/S\right)^{3(1+f)}\left[1 - \left(\frac{S}{S_c}\right)^{1+3f}\right] \tag{37}$$

$$= CT^4\left[1 - \left(\frac{T_c}{T}\right)^{\gamma}\right], \tag{38}$$

where the constant

$$C = \tfrac{8}{3}\pi(0.36)NG$$

and

$$\gamma = \frac{4(1+3f)}{3(1+f)}.$$

S_c and T_c are the size and temperature respectively of the region when it stops expanding.

From eq. (10) we get

$$T_c \sim t_c^{-1/2} \sim m^{-1/2}, \tag{39}$$

where m is the pbh mass.

Confronting this expansion rate with the interaction rate, we get that

$$\Gamma_I^2 \sim (\dot{S}/S)^2$$

at a temp T_D defined by

$$T_D^2 - T_c^2 \simeq \left[\frac{3\alpha^4 N}{8\pi(0.36)G}\right] + \left(\frac{\gamma-2}{2}\right)T_c^2 \times \ln\left\{\frac{8\pi(0.36)GT_c^2}{3\alpha^4 N}\right\} + O\left((\gamma-2)^2\right). \tag{40}$$

For SU(5),

$$T_{\mathrm{D}}^2 - T_{\mathrm{c}}^2 \simeq 10^{-5} T_{\mathrm{p}}^2 .$$

As a consequence, a perturbed region which is going to become a pbh is perhaps the only place of the Universe where GU interactions (in their symmetric phase) are in equilibrium. As the temperature lowers the particle interactions stay in equilibrium with the expansion of the Universe until the density decreases too much.

We can then envisage the following scenario for pbh production:

(a) $t_p \leq t \leq t_1$, where $t_1 \sim 10^6 t_{\mathrm{p}}$ is roughly the time when SU(5) breaks. In this period, GU interactions are either out of equilibrium and hence not relevant for pbh formation or, if in equilibrium are described by an equation of state

$$p = \tfrac{1}{3}\rho ,$$

up to logarithms. Masses of pbh produced in this epoch are in the range

$$m_{\mathrm{p}}(\sim 10^{-5}\ \mathrm{g}) \lesssim m \lesssim 10^6 m_{\mathrm{p}} . \tag{41}$$

(b) $t_2 \leq t \lesssim 10^{30} t_{\mathrm{p}}$, where $t_2 \sim 10^{10} t_{\mathrm{p}}$. For temperatures such that

$$10^{-16} T_{\mathrm{p}} \lesssim T \lesssim 10^{-6} T_{\mathrm{p}} , \tag{42}$$

the breakdown from SU(5) to SU(3) × SU(2) × U(1) has already taken place and these interactions are in equilibrium. Moreover, the superheavy bosons have already decayed. The further transition to the "real" world (i.e. SU(3) × $\mathrm{U(1)_{EM}}$) is expected to occur around 10^{2-3} GeV.

In the region covered by eq. (42) the number of ultra-relativistic helicity states is $N = 106.75$. Thus, eqs. (23) and (42) correspond to a time interval

$$10^{10} t_{\mathrm{p}} \lesssim t \lesssim 10^{30} t_{\mathrm{p}} . \tag{43}$$

Also in this epoch, discarding logarithmic terms, the cosmic fluid is well approximated by a perfect ultra-relativistic gas and the produced pbh have mass

$$10^{10} m_{\mathrm{p}}(\sim 10^5\ \mathrm{g}) \lesssim m \lesssim 10^{30} m_{\mathrm{p}}(\sim 10^{25}\ \mathrm{g}) . \tag{44}$$

We note in particular that pbh of $m \sim 10^{15}$ g, which would be evaporating now, fall in this range.

For the above two mass ranges, the Carr analysis gives

$$n(m) \sim \left(\frac{\rho_0}{m_{\mathrm{p}}^2}\right)\left(\frac{m}{m_{\mathrm{p}}}\right)^{-5/2} \times \begin{cases} \eta_{\mathrm{a}} \\ \eta_{\mathrm{b}} \end{cases} , \tag{45}$$

where we have put $m_0 = m_{\mathrm{p}}$.

In the absence of a theory of the phase transition between the two epochs, η_a is not directly related to η_b.

Black holes in (a) evaporate very quickly so that the only constraint on η_a is that such pbh do not produce more than the presently observed number of 3 K photons [14]. This gives the bound

$$\eta_a \lesssim 10^{-7}. \tag{46}$$

It has been shown elsewhere [26] that this essentially excludes baryon number generation through pbh evaporation.

As for epoch (b) there are two observational constraints on pbh production. First, those pbh with mass bigger than 10^{15} g would still be present in the Universe and contribute to its total mass density. In this connection, measurements of the deceleration parameter indicate that the density of the Universe cannot exceed twice the critical density [27], which implies [14]

$$\xi_b \equiv \frac{\eta_b}{F} \lesssim 10^{-17}. \tag{47}$$

The second and stronger limitation derives from the fact that pbh of mass around 10^{15} g would be evaporating now and the emitted photons would be detectable. Their non-observation implies that pbh density cannot be bigger than 10^{-8} times the critical density, which leads to

$$\xi_b \lesssim 10^{-25}. \tag{48}$$

5. Discussion and conclusions

The above results can be recapitulated in the following way.

During most of its evolution, from Planck time up to 10^{-13} s (say), the Universe at large can be described by the ideal relativistic equation of state $p = \frac{1}{3}\rho$, up to logarithmic corrections. Therefore, the mass spectrum of pbh falls with a $(-\frac{5}{2})$ power law. For presently evaporating pbh, experimental limits are very severe [eq. (48)] so that the fraction of the Universe going into pbh must have been very small at the production time. But since, as we have seen, the mass spectrum is quite smooth we reach the more general conclusion that *for all the relevant mass ranges pbh are never cosmologically important*.

A few remarks are in order.

The above analysis only regards pbh produced on an extended mass range and consequently, density fluctuations of the form $\varepsilon(m/m_p)^{-2/3}$.

Eqs. (15) and (46) give a bound on ε

$$\varepsilon \lesssim 0.065.$$

If, moreover, the hypothesis is made that the functional form of the number density, eq. (15), is not substantially changed as one crosses the phase transition region, it is possible to identify η_a with η_b. Then, the stronger condition (eq. (48)) can now be involved to give the bound

$$\varepsilon \lesssim 0.032 .$$

If the equation of state changes from $p = \frac{1}{3}\rho$ (valid both before and after the phase transition) to $p = f\rho$ (with f a constant) during the phase transition, the effect on the pbh mass spectrum is not important provided the period is not too long. However, the assumption of a constant f is unrealistic since no simple equation of state is likely for such a period.

Lastly, we entertain the possibility that phase transitions themselves produce appropriate fluctuations [11,12] which lead to production of bh. Also for this case then, we would have the same bh mass spectrum but with a different ε which refers to fluctuations at the phase transition time. The previous bounds apply here as well.

The above analysis does not hold for bh with an exponentially decreasing mass spectrum so that, for instance, a sizeable number of bh of mass around $10^{(6-7)} m_p$ could have been produced at the phase transition.

It is interesting to note that bh in this mass range can be responsible for baryon number production. See, for instance ref. [28].

Finally, a numerical coincidence is pointed out.

The cooling Universe is believed to have undergone a phase transition from unconfined quarks and gluons to hadrons. This probably took place for $T \sim (100\text{–}200)$ MeV. If this phase transition did produce fluctuations which in turn give rise to bh they would peak around $M \sim 10^{33}$ g $\sim M_{\odot}$. According to some authors [9] statistical fluctuations on the number of bh around this mass could be sufficient to bind the observed galaxies.

It is a pleasure to thank D. Nanopoulos for helpful discussions. Y.S. would like to thank the hospitality of the Theory Group at Frascati.

Note added in proof

After submission of the present paper, we have been informed [29] that a very stringent limit on the density pbh can be obtained by considering their evaporation into magnetic molecules.

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