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F. Palumbo: SPONTANEOUS SUPERSYMMETRY BREAKING FROM SUPERCONDUCTIVITY.

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ABSTRACT

It is shown that fermion condensation, unlike boson condensation, always implies supersymmetry breaking. This is the converse of the conjectured property that spontaneous supersymmetry breaking always implies superconductivity.

It has been conjectured(1) that spontaneous breaking of supersymmetry implies superconductivity. This conjecture has been illustrated(1) by a class of nonrelativistic models where supersymmetry is realized nonlinearly and only fermions are involved. It is also supported by a nonrelativistic model of interacting chiral multiplets(2). In this model both bosons and fermions are in a coherent state when supersymmetry is spontaneously broken. Moreover a fermion condensate breaks supersymmetry irrespective of the presence of a boson condensate.

Here we show that this is a general property in relativistic supersymmetry, i.e. if some fermion bilinear has a non-vanishing expectation value, supersymmetry is spontaneously broken. This can be viewed as the converse of the mentioned conjecture that spontaneous supersymmetry breaking implies superconductivity. Note the difference with respect to boson condensation, which does not necessarily imply supersymmetry breaking. In the course of our analysis we will also discuss boson condensation as a consequence of fermion condensation.

Let us consider the case of the supersymmetric interaction of scalar-spinor matter with massless multiplets of spin-particle content ($\frac{1}{2}$, 1) gauging an internal symmetry group G. Let us start by the Abelian case G=U (1) without flavors. The gauge U(1) transformation is realized on a doublet of chiral multiplets $S_a=(Z_a, \chi_{aL}, H_a)$ as follows

$$S_a = \Lambda \varepsilon_{ab} S_b \qquad \varepsilon_{12} = -\varepsilon_{21} = 1; \quad \varepsilon_{11} = \varepsilon_{22} = 0 \tag{1}$$

The vector potential is a real pseudoscalar superfield transforming as

$$\delta V = i/g\,(\Lambda - \Lambda^*) \tag{2}$$

under gauge transformation of chiral parameter Λ.

In the Wess-Zumino gauge the interaction part of the Lagrangian involving the gauge field is the D-component of the vector superfield

$$W = S_a S_a^* (1 + \frac{1}{2} g^2 V^2) + ig\, V\, \varepsilon_{ab}\, S_a^* S_b + \xi\, V, \tag{3}$$

$$\begin{aligned} W_D = & -\frac{i}{2} g\, D\, \varepsilon_{ab}\, Z_a Z_b^* - \frac{1}{2} g^2\, v_\mu\, Z_a Z_a^* \\ & + g\, v_\mu\, \varepsilon_{ab} (Z_a^* \overleftrightarrow{\partial}_\mu Z_b - \chi_{aL}\, \lambda_\mu\, \chi_{bR}) \\ & + g\, \varepsilon_{ab} (\lambda_L\, \chi_{aL}\, Z_b^* + \lambda_R\, \chi_{aR}\, Z_b) - \xi D. \end{aligned} \tag{4}$$

An arbitrary interaction of the form $\left[f(S_a S_a) \right]_F$ can also be present.

There are two possible condensates

$$\langle \lambda_L \chi_{aL} \rangle \overset{\text{def}}{=} D_a, \tag{5}$$

$$\langle \chi_{aL} \chi_{bL} \rangle \overset{\text{def}}{=} H_{ab}. \tag{6}$$

Suppose that for at least one value of a $D_a \neq 0$. In such a case, since Z_b appears at least quadratically everywhere but in the terms involving D_a, the minimum of the potential must occur at $\langle Z_b \rangle \neq 0$, so that

$$\varepsilon_{ab} \langle \lambda_L \chi_{ab} Z_b \rangle = \varepsilon_{ab} D_a \langle Z_b \rangle \neq 0. \tag{7}$$

Let us therefore consider the vacuum expectation value of the variation of the λ -component of W

$$\langle \delta W_\lambda \rangle = -\frac{i}{g} \langle D \rangle \; \varepsilon_{ab} \langle Z_a \rangle \; \langle Z_b^* \rangle + \\ + g \; \varepsilon_{ab} (D_a \langle Z_b^* \rangle + D_a^* \langle Z_b \rangle) - \xi \; D \quad . \tag{8}$$

Now there are two possibilties

$$\langle D \rangle \neq 0, \tag{9}$$

$$\langle D \rangle = 0. \tag{10}$$

In the first case supersymmetry breaking follows from

$$\langle \delta \lambda \rangle = \langle D \rangle \neq 0. \tag{11}$$

In the second case at the minimum

$$< \delta W_\lambda > = 2|g| \left(|D_1| \, |<Z_2>| + |D_2| \, |<Z_1>| \right) \neq 0 \qquad (12)$$

showing that supersymmetry is broken.

Let us now consider the other possible condensate. The bilinear $\chi_{aL} \chi_{bL}$, $a \neq b$ does not occur in the Lagrangian. We therefore consider the chiral multiplet $S_a S_b$. The vacuum expectation value of the variation of its χ-component is

$$< \delta \left[S_a S_b \right]_\chi > = < H_a Z_b + H_b Z_a - \chi_{aL} \chi_{bL} > \qquad (13)$$

Now as before if for all a $< H_a > = 0$, breaking of supersymmetry follows from the above equation, while if for some a $< H_a > \neq 0$, it follows(5) from

$$< \delta \chi_{aL} > = < H_a > \neq 0. \qquad (14)$$

Note that in the presence of the condensate $\chi_{aL} \chi_{bL}$ only, condensation of bosons cannot be proved by the above method.

Introduction of flavors or of a non-Abelian gauge group G does not alter any of the above results.

The case of interacting chiral multiplets can be treated in exactely the same way. A detailed analysis will be presented separately in connection with a discussion of our conjecture.

After this work has been completed, I have heard that the case of the condensate $\chi_{aL} \chi_{bL}$ in the absence of Yukawa couplings and mass terms has been discussed by S. Dimopoulos and S. Raby in a different context.

REFERENCES

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(5) A similar procedure could be used also for the condensate $(\lambda_L \chi_{aL})$ by considering the vector multiplet $i(\Lambda - \bar{\Lambda})$ V, but in this way we would not gain any information about boson condensation.