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SUMMING QCD SOFT CORRECTIONS

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ABSTRACT

In this review the effect of soft gluon emission in hard process is discussed to all orders in QCD. A general formalism for resumming these large corrections is presented and detailed implications are considered for deep inelastic leptoproduction, Drell-Yan processes and e^+e^- annihilation. A short discussion of $k_\perp$ effects in Drell-Yan and e^+e^- jets is also presented.

Leading logarithmic analyses (LLA) i.e. a summation of all terms of the type $(\alpha_s(Q^2)\ln Q^2)^n$, have been very useful in understanding the corrections to the Born terms in the perturbative Quantum Chromo-Dynamics (QCD) treatments of hard processes[1]. More recently, accurate computations of non leading corrections have been performed in various processes and often found[1,2] numerically large, particularly near the boundary of the phase space. These results cast doubt on the validity of the perturbative series at present energies, unless these large non leading terms can be summed up to all orders. However the effect of higher order corrections can be possibly minimized by a judicious choice of the renormalization prescription, in other words by a better definition of the expansion parameter. Much work has been done along this line[3]. Furthermore it has been also realized the important role played by soft gluon effects and simple resummation techniques have been proposed[4,5,6] which take into account a large part of higher order corrections.

In this talk I will briefly review this latter subject, discussing in detail various hard processes where the emission of soft QCD radiation is quantitatively relevant. After recalling the main results from first order calculations and a discussion of their physical origin, I will present a general formalism for resumming these large soft corrections based on the scheme of coherent states. Next, detailed implications for deep inelastic scattering, Drell-Yan process and e^+e^- annihilation will be given. To conclude, a short discussion of $k_\perp$ effects in Drell-Yan processes as well as in e^+e^- jets will be presented. Higher twist contributions and more general mass effects will be neglected in our discussion. Further details on the topics studied here can be also found in various and more extensive reviews which are listed in ref. [1].

Let us start with deep inelastic leptoproduction. The quark densities $q_k(x,t)$, defined for example in terms of the structure function $\mathscr{F}_2(x,t)$ as

$$\mathscr{F}_2(x,t) = \sum_k e_k^2 q_k(x,t)\ , \tag{1}$$

to first order in α_s satisfy the following equation[7]

$$q_k(x,t) = \int_x^1 \frac{dy}{y}\left[\delta\left(1-\frac{x}{y}\right) + \frac{\alpha_s}{2\pi}\, t\, P_{qq}\left(\frac{x}{y}\right) + \alpha_s f_q^{DI}\left(\frac{x}{y}\right)\right] q_{ok}(y) + \\ + \text{(gluon contributions)}\ , \tag{2}$$

where $t = \ln Q^2/\mu^2$, $q_{ok}(x)$ are the bare densities, $P_{qq}(z)$ has its usual meaning[8] and $f_q^{DI}(z)$ gives the correction to the leading order result. In terms of n-moments eq. (2) can be rewritten as

$$q_k^{(n)} = 1 + \frac{\alpha_s}{2\pi} t\, c_F\, \gamma_{qq}^{(n)} + \alpha_s f_q^{DI(n)} \tag{3}$$

with $c_F = 4/3$,

$$\gamma_{qq}^{(n)} = -2 \sum_{j=1}^{n} \frac{1}{j} + \frac{3}{2} + \frac{1}{n(n+1)} = -2\ln(\gamma n) + \frac{3}{2} + O\left(\frac{1}{n}\right), \tag{4}$$

and $\ln\gamma \equiv \gamma_E = 0.5772$ is the Euler's constant. Then for $z \lesssim 1$ one finds, for the most singular terms,

$$f_q^{DI}(z) = \frac{c_F}{2\pi}(1+z^2)\left[\frac{\ln(1-z)}{1-z}\right]_+ + \cdots\cdots \tag{5}$$

or

$$f_q^{DI(n)} = \frac{c_F}{\pi} \sum_{k=1}^{n} \frac{1}{k} \sum_{j=1}^{k} \frac{1}{j} + \cdots\cdots = \frac{c_F}{2\pi}(\ln\gamma n)^2 + \cdots\cdots \tag{6}$$

The physical origin of this large correction term can be simply traced back to the appropriate use of the exact kinematics in the calculation of the emission of a real gluon. In fact the leading and next-to-leading logarithmic terms in the square bracket of eq. (2) come from the bremsstrahlung contribution to $\mathcal{F}_2$ as

$$\mathcal{F}_2 \sim \delta(1-z) + \frac{c_F}{2\pi}\left(\frac{1+z^2}{1-z}\right)\int_{\mu^2}^{k_{\perp max}^2 \sim Q^2(1-z)} \frac{dk_\perp^2}{k_\perp^2}\, \alpha_s(k_\perp) \tag{7}$$

Then if $\alpha_s \ln(1-z) \sim O(1)$, and this happens near the boundary of the phase space, a large correction is obtained to the leading order result, which simply comes approximating $k_{\perp max}^2 \sim Q^2$.

A similar result is found in the Drell-Yan process[9]. With the usual notations, the corrections to order α_s to the Born term are obtained from[7]

$$\frac{d\sigma^{l\bar{l}}}{dQ^2} = \frac{4\pi\alpha^2}{9SQ^2}\int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\left[\sum_k e_k^2\, q_k(x_1,Q^2)\,\bar{q}_k(x_2,Q^2) + (1 \to 2)\right]\cdot$$
$$\cdot\left\{\delta(1-z) + \alpha_s\,\theta(1-z)\left[f_q^{DY}(z) - 2 f_q^{DI}(z)\right]\right\} + \text{(gluon contributions)}, \tag{8}$$

where $z = \tau/x_1x_2 \equiv Q^2/Sx_1x_2$ and we have explicitly introduced the Q^2 dependence in the parton densities in deep inelastic scattering (see eq. (2)). Then the correction $\tilde{f}_q(z) \equiv \left[f_q^{DY}(z) - 2 f_q^{DI}(z)\right]$ gets two important contributions

$$\tilde{f}_q(z) = \frac{c_F}{2\pi}\left\{2(1+z^2)\left[\frac{\ln(1-z)}{1-z}\right]_+ + \pi^2\,\delta(1-z) + \cdots\right\}. \tag{9}$$

The first one, which is just twice that of eq. (5), has the same dynamical origin. Namely, as in eq. (7), one has

$$\frac{d\sigma^{l\bar{l}}}{dQ^2} \sim \delta(1-z) + \frac{2c_F}{2\pi}\left(\frac{1+z^2}{1-z}\right)\int_{k^2_{min}\sim Q^2(1-z)}^{k^2_{max}\sim Q^2(1-z)^2} \frac{dk_\perp^2}{k_\perp^2}\,\alpha_s(k_\perp). \tag{10}$$

Here the factor $(2c_F)$ is simply related to the emission of a gluon from two legs, the upper limit $k^2_{\perp max}$ is the appropriate kinematical bound for $z \sim 1$ for this process and $k^2_{\perp min}$ is the effective lower limit obtained subtracting the effect of gluon emission in deep inelastic scattering, which is already included in the definition of $q_k(x_i, Q^2)$ and $\bar{q}_k(x_i, Q^2)$ in eq. (8).

The second contribution in eq. (9), i.e. the π^2 term, has also a simple explanation. In Drell-Yan the vertex correction is proportional to $\mathrm{Re}\ln^2(-q^2) = \ln^2|q^2| - \pi^2\ (q^2 > 0)$. The $\ln^2|q^2|$ term cancels with the analogous contribution from real emission, exactly as it does in deep inelastic scattering where, however, being $q^2 < 0$ no such π^2 term is present. Working to all orders, the expected exponential form of the quark form factor[10], leads to exponentiate[4,5] this term, namely $\sim \exp\{c_F \pi\alpha(Q^2)/2\}$.

As next example of occurrence of large corrections to the usual LLA let us consider the thrust (T) distribution in the process $e^+e^- \to q\bar{q}g$. The lowest order result is[11]

$$\frac{1}{\sigma_o}\left(\frac{d\sigma}{dT}\right)_o \simeq \frac{c_F\alpha_s}{\pi}\left\{\frac{3T^2-3T+2}{T(1-T)}\ln\left(\frac{2T-1}{1-T}\right) - \frac{3(3T-2)(2-T)}{2(1-T)}\right\} \tag{11a}$$

which for $T \sim 1$ becomes

$$\frac{1}{\sigma_o}\left(\frac{d\sigma}{dT}\right)_o \underset{T\sim 1}{\sim} \frac{2c_F\alpha_s}{\pi}\left(\frac{1}{1-T}\right)\left(\ln\frac{1}{1-T} - \frac{3}{4}\right). \tag{11b}$$

Now it is easy to see that the effect of the emission of soft radiation gives corrections $\propto \alpha_s^n \ln^{2n}(1-T)$ which invalidate the lowest order result (11b).

The above examples show that the soft behaviour of the theory plays an important role in the evaluation of the corrections to the leading order results. If one is able to sum them up, then one can hope to have a better control on the residual series.

The formalism of coherent states, developed in ref. [12] is indeed a rather powerful resumming technique. It provides one with matrix elements which are free from infrared singularities at all orders in the leading log approximation. This has been obtained by extending from QED to QCD the concept of classical currents associated with the external particles to incorporate the new properties of colour and the appearance of the effective coupling constant. Furthermore the question of mass singularities can also be incorporated[13] in the formalism by including the emission of collinear radiation. Various applications can be found in the literature[5, 14].

To our purposes let us first consider, in this formalism, the valence quark densities in the region $x \lesssim 1$, in the usual LLA, i.e. when $\alpha(Q^2)\ln Q^2 \sim O(1)$ and $\alpha(Q^2)\ln(1/(1-x)) \ll 1$. Then one finds[5, 15]

$$q(x, Q^2) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} db\, e^{ib(1-x)} \exp\left\{\int_0^1 dz\, P(z)\, \xi(Q^2)\left[e^{-ib(1-z)} - 1\right]\right\} \qquad (12)$$

where $P(z) = \frac{c_F}{2}\,\frac{1+z^2}{1-z}$ and

$$\xi(k^2_{max} \sim Q^2) = \int_{\mu^2}^{k^2_{\perp max}} \frac{dk_\perp^2}{k_\perp^2}\,\alpha(k_\perp)\,. \qquad (13)$$

In eq. (12) the exponentiated factor $\left\{\int dz\, P(z)\, \xi(Q^2)\, e^{-ib(1-z)}\right\}$ corresponds the multiple real gluon emission constrained by the condition that the total energy carried out by the radiation does not exceed $(1-x)$. Then the factor (-1), coming from virtual emissions, cancels the infrared singularities at $z \to 1$. The connection with the more conventional approach becomes clearer by rewriting eq. (12) as

$$q(x, Q^2) \simeq \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} dn\, x^{-n} \exp\left[\frac{c_F}{2}\int_0^1 dz\left(\frac{1+z^2}{1-z}\right)\xi(Q^2)(z^n - 1)\right], \qquad (14)$$

having approximated, for $x \to 1$, $\ln x \sim (x-1)$.

This result explicitly shows that the moments of the distribution (12) coincide with the usual moments of the parton densities in the large n limit. The eq. (14) becomes

$$q(x,Q^2) \simeq \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} dn\, x^{-n} \exp\left\{-c_F\, \xi(Q^2)\left[\ln\gamma n - \frac{3}{4} + o\left(\frac{1}{n}\right)\right]\right\}, \tag{15}$$

and, by saddle point techniques

$$q(x, Q^2) \simeq \frac{e^{(\frac{3}{4} - \gamma_E) c_F \xi}}{\Gamma(c_F\,\xi)} (1-x)^{c_F \xi - 1}. \tag{16}$$

This result coincide with that obtained in refs.[16] by conventional diagrammatic analyses and explicitly shows the simplicity of this approach.

When the condition $\alpha(Q^2)\ln(1/(1-x)) \ll 1$ is released, the corrections to the above LLA result have been shown in ref.[5] to arise simply by taking into account the correct kinematics in eq. (12), namely $\xi(Q^2) \to \xi\left[Q^2(1-z)\right]$. Then in analogy to eq. (14), one obtains for the n-th moment of the valence densities

$$q^{(n)}(Q^2) \sim \exp\left\{\frac{c_F}{2\pi}\int_0^1 dz\left(\frac{1+z^2}{1-z}\right)\int_{\mu^2}^{Q^2(1-z)} \frac{dk_\perp^2}{k_\perp^2}\,\alpha(k_\perp^2)\left[z^n-1\right]\right\}. \tag{17}$$

For $\alpha(k_\perp) = \text{const} = \alpha_s$ this corresponds to the simple exponentiation of the first order result (eq. (6))

$$q^{(n)}(Q^2) \sim \exp\left\{\frac{\alpha_s c_F}{\pi}\left[\left(\frac{1}{2}(\ln\gamma n)^2 + \frac{\pi^2}{6}\right) + \ln\frac{Q^2}{\mu^2}\left(-\ln\gamma n + \frac{3}{4}\right) - \frac{7}{8} + o\left(\frac{1}{n}\right)\right]\right\}. \tag{18}$$

Transforming back eq. (18) by the saddle point method one finally obtains, in place of eq. (16),

$$q(x,Q^2) \simeq e^{\beta\left[\frac{3}{4}\ln\frac{Q^2}{\mu^2} + \frac{\pi^2}{12} - \frac{7}{8} + \frac{\gamma_E^2}{2}\right]} \frac{e^{-\gamma_E \beta \ln\frac{Q^2}{\mu^2 n_o}}}{\Gamma(\beta')} \cdot (1-x)^{\beta'-1}\, e^{-\frac{\beta}{2}\ln^2 n_o}, \tag{19}$$

where $\beta = c_F\alpha_s/\pi$, and n_o is the saddle point value defined by the equation

$$n_o = \frac{1}{1-x} \beta \ln\left(\frac{Q^2}{\mu^2 \gamma n_o}\right) \simeq \frac{1}{1-x} \beta' . \tag{20}$$

The effect of the running coupling constant in eq. (17) can also be included. An explicit expression for $q(x, Q^2)$ in this limit is obtained in ref. [17] by using the same techniques.

An important consequence of eq. (17) is that the Altarelli-Parisi evolution equation for the non-singlet quark density is modified[5, 6], for $x \sim 1$, as

$$\frac{dq(x, Q^2/Q_o^2)}{d\ln(Q^2/Q_o^2)} = \frac{1}{\pi} \int_x^1 \frac{dy}{y} \left\{ P\left(\frac{x}{y}\right) \bar{\alpha}\left[\frac{Q^2}{Q_o^2}\left(1 - \frac{x}{y}\right)\right]\right\}_+ q\left(y, \frac{Q^2}{Q_o^2}\right) , \tag{21}$$

where $P(z)$ is defined in eq. (12). In other words, the simple rescaling of the argument of the running coupling constant from Q^2 to $Q^2(1-z)$ takes into account the most important higher order corrections. Expanding

$$\alpha\left[(Q^2(1-z)\right] = \frac{1}{\ln\left[Q^2(1-z)\right]} = \frac{1}{\ln Q^2}\left[1 - \frac{\ln(1-z)}{\ln Q^2} + O\left(\frac{1}{\ln Q^2}\right)^2\right] = \tag{22}$$

$$= \alpha(Q^2)\left[1 + \ln(1-z)\,\alpha(Q^2) + O(\alpha^2(Q^2))\right] ,$$

one obtains the leading log term, the next-to-leading term, and so on.

Of course eq. (17) is valid for $n \gg 1$ but limited by $n < Q^2/\mu^2$ or roughly $1-x > \mu^2/Q^2$. In this very tiny region of the phase space one finds a strong damping of the form of a Sudakov form factor, which makes the leading twist contribution discussed here probably negligible with respect to higher twist effects. More details about this point can be found in ref.[6].

Phenomenological implications of the threshold behaviour of the structure functions for deep inelastic scattering, namely the question of the parametrizations to be used in analysing the experimental data which are compatible with the above dicussion, are considered in ref. [18].

Similar modifications of the leading order result in the case of the photon structure functions which, in the near future, can be measured for x enough close to one, have been recently studied in ref. [19].

We now discuss the implications of the above results in the Drell-Yan process. By taking into account the soft gluon emission at all orders, and omitting for simplicity the contribution of gluons in the initial state, eq. (8) is replaced by[5, 20)]

$$\frac{d\sigma^{l\bar{l}}}{dQ^2 dx_F} = \frac{4\pi\alpha^2}{9SQ^2} \int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\,\delta(x_1 - x_2 - x_F)\left[\sum_k e_k^2 q_k(x_1, Q_o^2)\,\cdot\right.$$
$$\left.\cdot\ \bar{q}_k(x_2, Q_o^2) + (1 \rightarrow 2)\right]\tilde{f}(z, Q^2, Q_o^2)\exp\left\{\frac{\alpha(Q^2)}{2\pi}\, c_F \pi^2\right\}, \tag{23}$$

with

$$\tilde{f}(z, Q^2, Q_o^2) = \frac{1}{2\pi}\int_{-\infty}^{\infty} db\, e^{ib(1-z)} \exp\left\{\frac{c_F}{\pi}\int_0^1 dy\left(\frac{1+y^2}{1-y}\right)\,\cdot\right.$$
$$\left.\cdot\int_{Q_o^2(1-y)}^{Q^2(1-y)^2} \frac{dk_\perp^2}{k_\perp^2}\,\alpha(k_\perp)\left[e^{-ib(1-y)} - 1\right]\right\}. \tag{24}$$

In eq. (23) we have more generally introduced the parton densities at a mass scale Q_o^2, which leads to the lower bound $k^2_{\perp min} = Q_o^2(1-y)$ in the $k_\perp^2$ integral in eq. (24). Furthermore the π^2 term coming from the mismatch in the quark form factor from space like to time like regions is exponentiated, as discussed earlier.

The easiest way to solve eq. (23) is to consider its τ^n moments. Then the r.h.s. reduces to the product of the n-th moments of the q, $\bar{q}$ and $\tilde{f}$ distributions, which can be easier transformed back. Without going into details which can be found in ref. [20)], one obtains the soft correction factor K to the naive model ($\tau \lesssim 1$)

$$K = e^{\frac{\alpha(Q^2)}{2} c_F \pi}\, \frac{\Gamma(1+\xi)}{\Gamma\left[1 + \xi + \beta\ln\left(\frac{Q^2}{Q_o^2 n_o}\right)\right]}\, e^{\beta\left(\frac{3}{4} - \gamma_E\right)\ln\frac{Q^2}{Q_o^2}}\,\cdot$$
$$\cdot\ e^{\gamma_E \beta \ln n_o}\,(1-\tau)^{\beta\ln\frac{Q^2}{Q_o^2 n_o}}\, e^{-\frac{1}{2}\beta\ln^2 n_o}\left(1 - \frac{3}{4}\beta\ln n_o\right), \tag{25}$$

where $\beta = 2\alpha_s c_F/\pi$, n_o is the saddle point value defined by

$$n_o = \frac{1}{1-\tau}\left[1+\xi+\beta\ln\left(\frac{Q^2}{Q_o^2\,\gamma\, n_o}\right)\right], \tag{26}$$

and $\xi = \xi_1 + \xi_2$, having parametrized, at $q^2 = -Q_o^2$, and for $x \sim 1$, $q(x)$ and $\bar{q}(x)$ as $(1-x)^{\xi_1}$ and $(1-x)^{\xi_2}$ respectively. In deriving this result we have kept all leading logarithmic terms in $(1-\tau)$ and ξ. Furthermore eq. (25) has been explicitly checked to first order in α_s. The phenomenological implications of eq. (25) are shown[20] in Figs. 1 and 2 for p-p and π-p collisions respectively, for

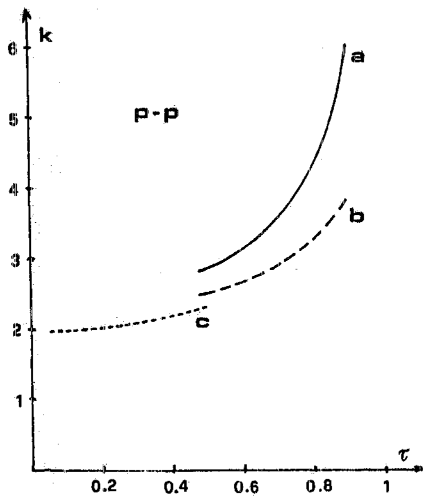

FIG. 1

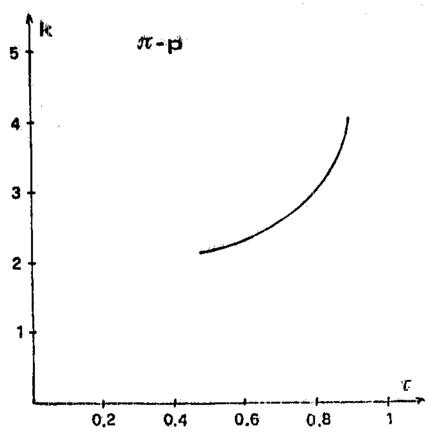

FIG. 2

for $\sqrt{s} = 30$ GeV, $\alpha_s \simeq 0.2$ and $Q^2 = Q_o^2$. In Fig. 1 the full curve (a) represents eq. (25), the dashed line (b) represents the leading log approximation of (25) (e. g. first order in α_s) and for comparison the full first order calculation of ref.[7] is also shown (dotted line (c)). As it is clear from this figure, the inclusion of soft contributions to all orders do not change significantly the first order result up to $\tau \sim 0.6-0.7$. On the other hand for larger values of τ the absolute cross section falls down so rapidly that the increasing behaviour of K will not be observable. In the case of pion-proton collisions (Fig. 2) smaller corrections are found. An improvement of the actual experimental accuracy could, for this case, reveal the τ dependence of K.

As a last example we will consider how the lowest order result (eq. (11b)) for the T-distribution in the process $e\bar{e} \to q\bar{q}g$ is modified for $T \lesssim 1$ by soft radiation. The analysis proceeds quite similarly to the previous cases. The physical idea is that the quark, antiquark and the gluon as well will develop into jets of invariant mass $\sim Q^2(1-T)$. The corresponding T distribution is found[21)]

$$\frac{dP}{dT} \simeq \left(\frac{dP}{dT}\right)_o \frac{e^{\left[\left(\frac{3}{4} - \gamma_E\right)\beta_q + \left(\frac{11}{12} - \frac{N_f}{18} - \gamma_E\right)\beta_g\right]\ln\left(\frac{1}{1-T}\right)}}{\Gamma\left[1 + (\beta_q + \beta_g)\ln\left(\frac{1}{1-T}\right)\right]} \cdot \tag{27}$$

$$\cdot\; e^{-\frac{1}{2}(\beta_q + \beta_g)\ln^2\left(\frac{1}{1-T}\right)} ,$$

where $\left(\frac{dP}{dT}\right)_o$ is given by eq. (11a), $\beta_q = \left(\frac{8\alpha_s}{3\pi}\right)$ and $\beta_g = \left(\frac{3\alpha_s}{\pi}\right)$. This result goes beyond the usual LLA, where only the term $\exp\left[-\beta_q \ln^2(1-T)/2\right]$ is found[22)] to multiply the Born term. In Fig. 3 we plot the Born distribution $\left(\frac{dP}{dT}\right)_o$ given by eq. (11a) (full line) and compare it with $\left(\frac{dP}{dT}\right)$ of eq. (27) (broken line). For completeness the first order expansion in $(\beta_q + \beta_g)$ of eq. (27) is also shown. This figure shows clearly that for large T the higher order corrections are quite important, and in qualitative agreement with the experimental results. Of course a complete comparison with data must await suitable addition of finite α_s^2 corrections, which have been shown to be sizeable[23)].

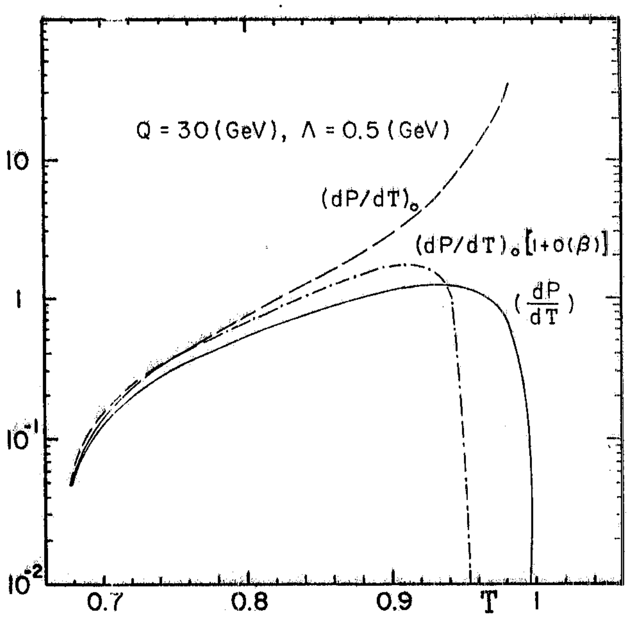

FIG. 3

We conclude this short review by discussing $k_\perp$ effects which have been most extensively studied in Drell-Yan and e^+e^- annihilation[24)]. As well known, gluon bremsstrahlung provides a non-zero transverse momentum $k_\perp$ for the lepton or the quark pair in the two processes. However, whereas for $k_\perp^2 \sim O(Q^2)$ the transverse momentum distribution is expected to be fully described by first order QCD diagrams, for $\Lambda^2 \ll k_\perp^2 \ll Q^2$ the perturbation theory breaks down due to the appearance of large $\alpha_s^n \ln^{2n}(Q^2/k_\perp^2)$ terms arising from the emission of n gluons, both soft and collinear, which have to be summed to all orders of perturbation theory.

In Drell-Yan this task has been essentially accomplished by Dokshitzer, Dyakonov and Troyan[25], who gave an expression valid in the leading double logarithmic approximation. An improvement of this result has been suggested by Parisi and Petronzio[26], by transforming to the impact parameter space, where transverse momentum conservation can be taken into account exactly. This is particularly relevant when $k_\perp \to 0$, which can be reached by emission of at least two gluons whose transverse momenta are not small and add to essentially zero momentum. They proposed

$$\left.\frac{d\sigma}{dQ\, dk_\perp^2\, dy}\right|_{y=0} = \frac{1}{2}\int_0^\infty b\, db\, J_o(b k_\perp)\, \tilde{\sigma}(b, Q, S) \,, \tag{28}$$

where

$$\begin{aligned}\tilde{\sigma}(b,Q,S) = \frac{8\pi\alpha^2}{9QS} \sum_i e_i^2 \Big[q_i^{(1)}\big(\sqrt{\tau}, \tfrac{1}{b^2}\big)\, q_i^{(2)}\big(\sqrt{\tau}, \tfrac{1}{b^2}\big) + \\ + (1 \to 2)\Big] \exp\left[\Delta(Q^2, b)\right]\end{aligned} \tag{29}$$

and

$$\Delta(Q^2, b) = \frac{16}{3\pi}\int \frac{dq_\perp}{q_\perp} \ln\left(\frac{Q}{q_\perp}\right) \alpha_s(q_\perp) \left[J_o(b q_\perp) - 1\right] . \tag{30}$$

In e^+e^- annihilation the same result has been independently obtained by Curci, Greco and Srivastava[27] for the transverse momentum distribution of a $q\bar{q}$ jet, namely

$$\frac{dP}{dk_\perp^2} = \frac{1}{2}\int_0^\infty b\, db\, J_o(b k_\perp)\, \exp\left\{\Delta(Q^2, b)\right\} \tag{31}$$

with $\Delta(Q^2, b)$ given by eq. (30).

More recent analyses[28] have confirmed in Drell-Yan the general structure of eq. (28). So far there is no detailed phenomenological analysis of recent $k_\perp$ data in Drell-Yan based on eq. (28). On the other hand a very recent analysis[29] of the total transverse momentum distribution of e^+e^- jets at various energies from the PLUTO Group strongly supports the QCD prediction (31). Deviations from this formula at the highest energies and large $k_\perp$ have been found in agreement with first order results. This is explicitly shown in Fig. 4. Thus the

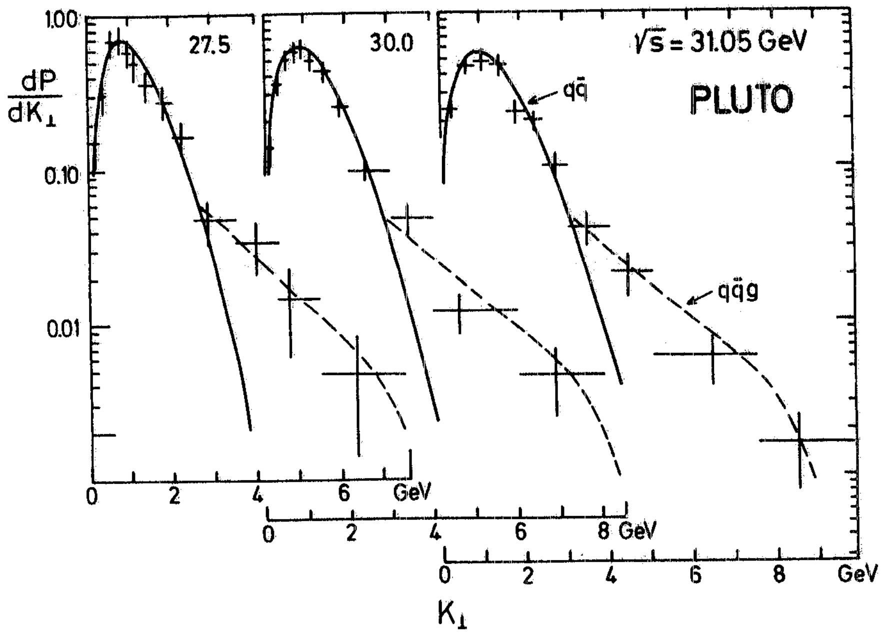

FIG. 4

resummation in $k_\perp$ of these soft effects in the double leading log approximation and using explicit momentum conservation seems in excellent agreement with experimental observations.

To conclude, we have discussed the problem of large higher order corrections which are related to the soft behaviour of the theory. The resummation of these effects have been explicitly studied in various processes. It is plausible that the residual series is then under much better control. As stated at the beginning the optimization of the convergence of this residual expansion can be further improved by an accurate choice of the renormalization prescriptions which reduce the effect of genuine higher order corrections.

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