To be submitted to
Physics Letters

ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-81/7(P)
6 Febbraio 1981

G. Curci and M. Greco:
SOFT CORRECTIONS TO THE DRELL-YAN PROCESS IN QCD.

Servizio Documentazione
dei Laboratori Nazionali di Frascati
Cas. Postale 13 - Frascati (Roma)

INFN - Laboratori Nazionali di Frascati
Servizio Documentazione

LNF-81/7(P)
6 Febbraio 1981

# SOFT CORRECTIONS TO THE DRELL-YAN PROCESS IN QCD

G. Curci
Scuola Normale Superiore, Pisa and
Istituto Nazionale di Fisica Nucleare, Sezione di Pisa, Italy

and

M. Greco
INFN, Laboratori Nazionali di Frascati, Italy.

ABSTRACT.

The infrared corrections to the Drell-Yan process are studied to all orders in QCD. Our results show that higher order terms slightly modify the first order corrections in the range of $\tau$ experimentally accessible in proton-proton collisions. Accurate measurements with pion and kaon beams should however reveal the increase with $\tau$ of the soft corrections.

The basic theoretical understanding of lepton pair production in hadronic collisions is provided by the model of Drell and Yan(1), where one is able to predict the absolute cross section in terms of the nucleon structure functions measured in deep inelastic lepton scattering. It is by now well known that Quantum Chromo-dynamics (QCD) gives corrections to this parton model result, which turn out to be particularly large at present energies(2). The important role played by soft gluon effects in the evaluation of those corrections, particular ly for large values of $\tau$ ($\tau \equiv Q^2/s$), has been recently emphasized by various authors[3]. It is therefore worthwhile to investigate quantitatively the possible relevance of higher orders in perturbation theory, also in view of phenomenological implications for pion and kaon beams, where reasonably large values of $\tau$ can be experimentally explored.

In this letter we present analytic expressions obtained upon summation of these soft corrections to all orders, which are of immediate phenomenological application. The theoretical framework is provided by a recent study[4] of this kind of effects in various hard processes in QCD, which shows in particular the relevance of using the appropriate kinematics in each reaction. Our results show that the inclusion of higher orders slightly change the first order results in the $\tau$ range experimentally accessible with today's accuracy[5]. Only for $\tau$ close to one the soft corrections, which are always positive in sign, become sizeable large but unlikely to be observed in proton-proton collisions due to the rapid falloff of the cross section. The detection of these effects is however possible by accurate measurements with pion and kaon beams.

The soundness of our estimates rests on the assumption that the exponentiation of the soft effects in its natural kinematical range of validity takes into account almost all dangerous sources of large corrections. Genuine and large $\alpha_s^2$ terms, which can possibly modify our conclusions, can only be found by performing a complete second order calculation, which is out of the aims of the present study.

The starting point is the following formula, obtained in ref. (4), which takes into account the soft gluon emission to all orders:

$$\frac{d\sigma}{dQ^2 dx_F} = \frac{4\pi\alpha^2}{9sQ^2} \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} \, \delta(x_1 - x_2 - x_F) \Big[ \sum_i e_i^2 \, q_i(x_1, Q_o^2) \cdot \\ \cdot \, \bar{q}_i(x_2, Q_o^2) + (1 \leftrightarrow 2) \Big] \tilde{f}(z, Q^2, Q_o^2) \exp\Big[ \frac{\alpha(Q^2)}{2\pi} \, c_F \pi^2 \Big], \quad (1)$$

with

$$\tilde{f}(z, Q^2, Q_o^2) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} db\, e^{ib(1-z)} \exp\left\{ \frac{c_F}{\pi} \int_0^1 \frac{dy(1+y^2)}{(1-y^2)} \cdot \int_{Q_o^2(1-y)}^{Q^2(1-y)^2} \frac{dk_\perp^2}{k_\perp^2} \alpha(k_\perp) \left[ e^{-ib(1-y)} - 1 \right] \right\}, \tag{2}$$

where $\sqrt{s}$ is the invariant mass of the hadronic system, Q the dilepton mass, $q^2 = -Q_o^2$ is the reference momentum squared where the quark and antiquark distribution functions have been defined and $z = \tau/x_1x_2$. We have disregarded the small contribution of gluons in the initial state.

The physical interpretation of eq. (1) is rather simple. After having introduced the $Q_o^2$ dependence in the parton densities in deep inelastic scattering, the corrections to the naive Drell-Yan formula come from the radiation emitted by the quark and the antiquark at the appropriate mass scale $Q^2$, after subtracting the effect of that at mass scale $Q_o^2$, which has already been included in the definitions of $q_i(x_j, Q_o^2)$ and $\bar{q}_i(x_j, Q_o^2)$. This is explicitly represented in $\tilde{f}(z, Q_o^2, Q^2)$ of ref. (2) in the formalism of coherent states. Furthermore the factor $\exp\left[\alpha(Q^2)c_F\pi/2\right]$ in eq. (1) comes from the mismatch in the quark form factor of space-like $q^2$ values for electroproduction to the time-like values for dilepton production[6].

The easiest way to solve analytically eq. (1) is to consider its $\tau^n$ moments. Then the r.h.s. reduces to the product of the n-th moment of the q's, $\bar{q}$'s and $\tilde{f}$, which can be easier transformed back. Then parametrizing the parton densities (at $q^2 = -Q_o^2$) as

$$q(x) \sim (1-x)^{\xi_1} x^{\eta_1}, \qquad \bar{q}(x) \sim (1-x)^{\xi_2} x^{\eta_2} \tag{3}$$

one obtains, in a reduced notation,

$$\int_0^1 d\tau\, \tau^{n-1} \left[\frac{d\sigma}{dQ^2 dx_F}\right]_{x_F=0} \propto \int_0^1 dz\, z^{n-1}\, \tilde{f}(z) \cdot$$

$$\cdot \int_0^1 dx\, x^{2n-2} (1-x)^{\xi} x^{\eta} = \tilde{f}^{(n)}\, B(2n+\eta-1,\, \xi+1)\,, \tag{4}$$

with $\xi = \xi_1 + \xi_2$, $\eta = \eta_1 + \eta_2$.

The $\tilde{f}^{(n)}$ moments are easily obtained by rewriting eq. (2), for $z$ close to one, as

$$\tilde{f}(z, Q^2, Q_o^2) \simeq \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} dn\, z^{-n} \exp\left\{ \frac{c_F}{\pi} \int_0^1 \frac{dy(1+y^2)}{1-y} \cdot \right.$$

$$\left. \cdot \int_{Q_o^2(1-y)}^{Q^2(1-y)^2} \frac{dk_\perp^2}{k_\perp^2}\, \alpha(k_\perp) \left[y^n - 1\right] \right\}, \tag{5}$$

and therefore

$$\tilde{f}^{(n)} = \exp\left\{ \frac{c_F}{\pi} \int_0^1 \frac{dy}{1-y} (1+y^2)(y^n-1) \int_{Q_o^2(1-y)}^{Q^2(1-y)^2} \frac{dk_\perp^2}{k_\perp^2}\, \alpha(k_\perp) \right\}. \tag{6}$$

Taking into account the effect of the running coupling constant $\alpha(k_\perp) = 1/b \ln(k_\perp^2/\Lambda^2)$, with $b = 25/12\pi$, eq. (6) can be solved exactly, leading to

$$\tilde{f}^{(n)} = \exp\left\{ - \frac{c_F}{\pi b} \sum_{j=0}^{h} \left[F(j) + F(j+2)\right] \right\}, \tag{7}$$

where

$$F(j) = \frac{1}{j+1} \ln \frac{\ln(\frac{Q^2}{\Lambda^2})}{\ln(\frac{Q_o^2}{\Lambda^2})} + \sum_{k=0}^{j} (-1)^{k+1} \frac{j!}{(k+1)!\,(j-k)!} \cdot$$

$$\cdot \left[e^{-A}\, Ei(A) - e^{-B}\, Ei(B)\right], \tag{8}$$

with $A = \frac{k+1}{2} \ln(\frac{Q^2}{\Lambda^2})$ and $B = \frac{k+1}{2} \ln(\frac{Q_o^2}{\Lambda^2})$.

Unfortunately this result is not very useful for two different reasons. First eq. (7) can be used only for $n \ll \frac{Q^2}{\Lambda^2}, \frac{Q_o^2}{\Lambda^2}$, because of the obvious limitations on $\alpha(k_\perp)$. Therefore a suitable mass regulator has to be introduced. Second, and more important, the analytical continuation of (7) to complex n, in order to transform back eq. (4), is far from being trivial.

We have then followed the simplest attitude to keep $\alpha(k_\perp)$ fixed and equal to some average value $\bar{\alpha}$, which leads us finally to rather simple analytical results. Then eq. (6) leads, for large n, to

$$\tilde{f}(n) = \exp\left\{\beta\left[\ln\frac{Q^2}{Q_o^2}\left(-\ln\gamma n + \frac{3}{4}\right) + \frac{1}{2}\ln^2\gamma n + \frac{\pi^2}{12} - \frac{7}{8} + 0\left(\frac{1}{n}\right)\right]\right\}, \quad (9)$$

where $\ln\gamma \equiv \gamma_E = 0.5772$ and $\beta = 2\bar{\alpha}c_F/\pi$. We have included all $\ln^2 n$, $\ln n$ and constant terms. For $n=5$ and, say, $Q^2/Q_o^2 = 10$, the argument of the exponential in the r.h.s. of (9) differs within 6 % from the exact result.

In the same large n limit, expanding the Beta-function in eq. (4) one finally obtains, for $\tau \lesssim 1$,

$$\left[\frac{d\sigma}{dQ^2 dx_F}\right]_{x_F=0} \simeq \frac{4\pi\alpha^2}{9sQ^2} N \exp\left\{\frac{\alpha(Q^2)}{2} c_F \pi + \beta\left[\frac{3}{4}\ln\frac{Q^2}{Q_o^2} + \frac{\pi^2}{12} - \frac{7}{8}\right]\right\} J(\tau,\xi), \quad (10)$$

with

$$J(\tau,\xi) = \frac{1}{2\pi i}\frac{\Gamma(1+\xi)}{2^{\xi+1}}\int_{-i\infty}^{i\infty} dn\, e^{n(1-\tau)-(\xi+1)\ln n - \beta\ln\frac{Q^2}{Q_o^2}\ln\gamma n + \frac{\beta}{2}\ln^2\gamma n} \quad (11)$$

and the normalization factor N accounts for the quark charges and the coefficients of the quark distributions. Finally, the last integral in eq. (11) can be performed using the saddle point method giving

$$J(\xi,\tau) \simeq \frac{\Gamma(1+\xi)}{2^{\xi+1}\,\Gamma(1+\xi+\beta\ln\frac{Q^2}{Q_o^2 n_o})} \cdot e^{-\gamma_E \beta(\ln\frac{Q^2}{Q_o^2} - \frac{1}{2}\gamma_E)} \cdot (1-\tau)^{\xi+\beta\ln\frac{Q^2}{Q_o^2 n_o}} \, e^{-\frac{\beta}{2}\ln^2 n_o} \tag{12}$$

where $n_o$ is the saddle point value defined by the equation

$$n_o = \frac{1}{1-\tau}\left(1+\xi+\beta\ln\frac{Q^2}{Q_o^2\gamma n_o}\right) . \tag{13}$$

For comparison the naive Drell-Yan formula, which is simply obtain ed by inserting $\tilde{f}(z) = \delta(1-z)$ in eq. (4), is written as

$$\left[\frac{d\sigma}{dQ^2 dx_F}\right]^{\text{naive}}_{x_F=0} \simeq \frac{4\pi\alpha^2}{9sQ^2} N \frac{1}{2^{\xi+1}} (1-\tau)^{\xi} . \tag{14}$$

Therefore, comparing eqs. (10) and (14), the correction factor C to the naive model is given by

$$C = \frac{\Gamma(1+\xi)}{\Gamma(1+\xi+\beta\ln\frac{Q^2}{Q_o^2 n_o})} \cdot e^{\frac{\alpha(Q^2)}{2} c_F \pi} \, e^{\beta(\frac{3}{4}\ln\frac{Q^2}{Q_o^2} + \frac{\pi^2}{12} + \frac{\gamma_E^2}{2} - \frac{7}{8} - \gamma_E \ln\frac{Q^2}{Q_o^2 n_o})} \cdot (1-\tau)^{\beta\ln\frac{Q^2}{Q_o^2 n_o}} \, e^{-\frac{1}{2}\beta\ln^2 n_o} , \tag{15}$$

which also includes the scaling violations arising when $Q^2 \neq Q_o^2$.

To check our result to the lowest non trivial order in $\alpha(Q^2)$ we replace eq. (2) by its leading logarithmic approximation and include also the term $\propto \pi^2$ from the form factor

$$\tilde{f}(z) = \delta(1-z)\left[1+\frac{\beta}{4}\right] + \beta\left[\frac{\ln(1-z)}{1-z}\right]_+ + \frac{3}{4}\beta\left(\frac{1}{1-z}\right)_+ , \qquad (16)$$

as obtained in ref. (2) for $Q^2 = Q_o^2$.

From eqs. (1-3-16) we then obtain

$$\left[\frac{d\sigma}{dQ^2 dx_F}\right]^{LLA}_{x_F=0} \simeq \frac{4\pi\alpha^2}{9sQ^2} N \int_\tau^1 \frac{1}{2\sqrt{\tau}} \frac{dz}{\sqrt{z}} \left(1-\sqrt{\frac{\tau}{z}}\right)^\xi \left(\sqrt{\frac{\tau}{z}}\right)^\eta \cdot$$

$$\cdot \left\{ \delta(1-z)\left(1+\frac{\beta}{4}\right) + \beta\left[\frac{\ln(1-z)}{1-z}\right]_+ + \frac{3}{4}\beta\left(\frac{1}{1-z}\right)_+ \right\} \simeq \qquad (17)$$

$$\simeq \frac{4\pi\alpha^2}{9sQ^2} N \frac{1}{2\sqrt{\tau}} (\sqrt{\tau})^\eta \left\{ (1-\sqrt{\tau})^\xi \left(1+\frac{\beta}{4}\right) + \right.$$

$$\left. + \beta \int_\tau^1 dz \left[\frac{\ln^2(1-z)}{2} + \frac{3}{4}\ln(1-z)\right] \frac{d}{dz}\left[\frac{1}{\sqrt{z}}\left(1-\sqrt{\frac{\tau}{z}}\right)^\xi\right] \right\}$$

Approximating $(1-\sqrt{\tau}/z) \simeq (1-\tau/z)/2$, for $\tau \lesssim 1$, we get finally:

$$\left[\frac{d\sigma}{dQ^2 dx_F}\right]^{LLA}_{x_F=0} \simeq \frac{4\pi\alpha^2}{9sQ^2} N \frac{(1-\tau)^\xi}{2^{\xi+1}} \left\{ \left(1+\frac{\beta}{4}\right) + \frac{\beta}{2}\left[\ln^2(1-\tau) - 2\ln(1-\tau) \cdot \right.\right.$$

$$\left. \cdot (\gamma_E + \psi(1+\xi)) + \psi^2(1+\xi) + 2\gamma_E \psi(1+\xi)\right] + \qquad (18)$$

$$\left. + \frac{3}{4}\beta\left[\ln(1-\tau) - \psi(1+\tau)\right]\right\} ,$$

having kept all logarithmic terms in $(1-\tau)$ and $\xi$.

In the same approximation we obtain from eq. (15)

$$C = (1+\frac{\beta}{4}) + \frac{\beta}{2}\left\{\ln^2(1-\tau) - 2\ln(1-\tau)\left[\gamma_E + \psi(1+\xi)\right] + \right. \qquad (19)$$
$$\left. + \ln(1+\xi)\left[2\,\psi(1+\xi) - \ln(1+\xi)\right] + 2\gamma_E \ln(1+\xi)\right\},$$

which coincides with the first factor in the r. h. s. of eq. (18) up to terms of $O(1/(1+\xi))$. The second factor, on the other hand, is a genuine hard gluon effect and thus it has to be explicity added to eq. (18).

The full correction factor K to the Drell-Yan formula becomes therefore

$$K = e^{\frac{\alpha(Q^2)}{2} c_F \pi} \frac{\Gamma(1+\xi)}{\Gamma\left[(1+\xi+\beta\ln(\frac{Q^2}{Q_o^2 n_o})\right]} e^{\beta(\frac{3}{4}-\gamma_E)\ln\frac{Q^2}{Q_o^2}} \cdot \qquad (20)$$
$$\cdot\, e^{\gamma_E\beta\ln n_o}\,(1-\tau)^{\beta\ln\frac{Q^2}{Q_o^2 n_o}}\, e^{-\frac{1}{2}\beta\ln^2 n_o}\,(1-\frac{3}{4}\beta\ln n_o)\,,$$

having kept all leading logarithmic terms in $(1-\tau)$ and $\xi$. For $Q^2 = Q_o^2$, and expanding the $\Gamma$-functions it becomes simply

$$K \simeq e^{\frac{\alpha(Q^2)}{2} c_F \pi}\, e^{\frac{\beta}{2}\ln^2(\frac{1+\xi}{1-\tau}) + \gamma_E\beta\ln(\frac{1+\xi}{1-\tau})} \cdot \left[1 - \frac{3}{4}\beta\ln(\frac{1+\xi}{1-\tau})\right] \qquad (21)$$

Eqs. (20-21) are our final result, valid for $\tau \lesssim 1$.

In Fig. 1 we plot eq. (21) (full curve, a) for the case of proton-proton collisions ($\xi \simeq 10$) at $\sqrt{s}$ = 30 GeV and with $\bar{\alpha} \simeq 0.2$ ($\beta \simeq 0.17$). The dashed line (b) represents the leading logarithmic approximation (e. g. first order in $\alpha_s$) of eq. (21). For comparison we also show (dotted line, c) the full first order calculation of ref. (7), neglecting as usual the small (~ 5 %) quark gluon contribution. As it is clear from this figure, the inclusion of soft contributions to all orders do

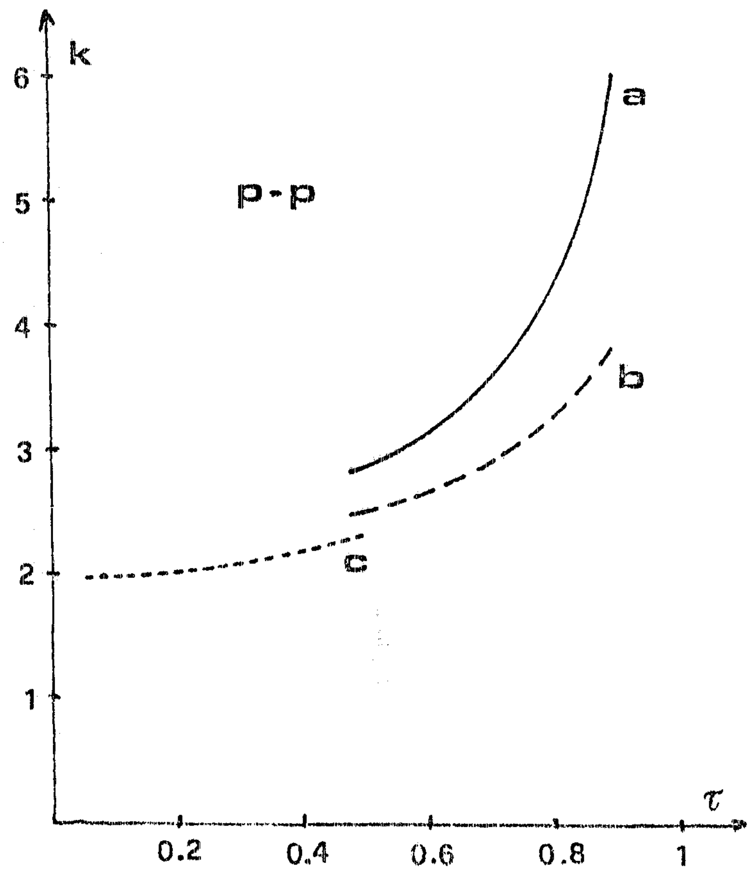


FIG. 1

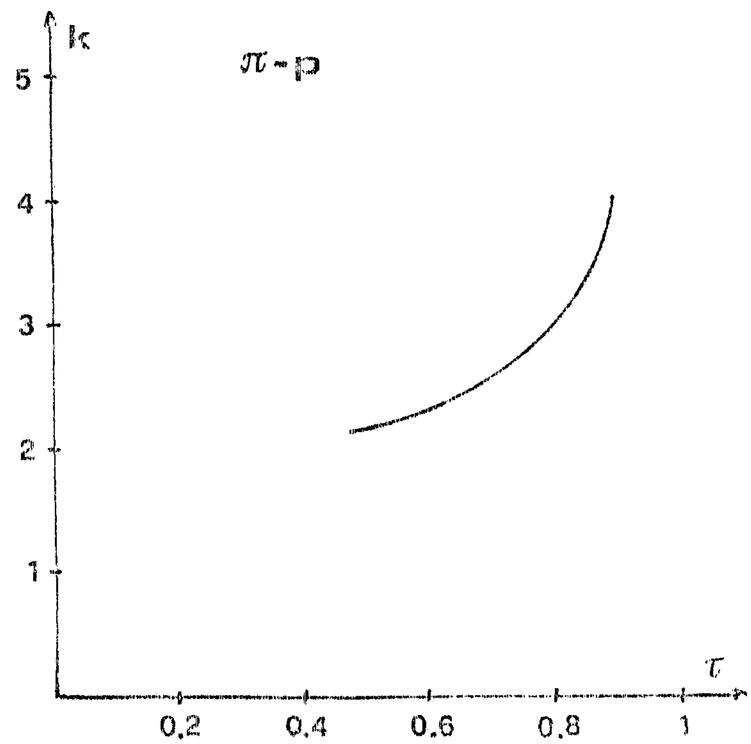


FIG. 2

not change significantly the first order result up to $\tau \sim 0.6-0.7$. On the other hand for larger values of $\tau$ the absolute cross section falls down so rapidly that the increasing behaviour of K will not be observable.

In the case of pion-proton collisions ($\xi \simeq 4$) we find smaller corrections than for p-p. With the above values of s and $\bar{\alpha}$ we show our results (eq. (21) for $Q^2 = Q_0^2$) in Fig. 2. An improvement of the actual experimental accuracy could, for this case, reveal the $\tau$ dependence of K.

To summarize, we have considered the effect of soft gluon corrections in the Drell-Yan processes. Our analysis shows that the inclusion of higher orders in the perturbative series slightly modifies the first order results in the region of large $\tau$'s reasonably accessible to p-p experiments. However an improvement of the experimental accuracy could make those effects observable in case of lepton pairs production with pion and kaon beams.

We would like to thank G. Martinelli for his contributions to the early stage of this work as well as for many discussions.

REFERENCES.

(1) - S. D. Drell and T. M. Yan, Phys. Rev. Letters 25, 316 (1970).

(2) - J. Kubar-André and F. E. Paige, Phys. Rev. D19, 221 (1979); G. Altarelli, R. K. Ellis and G. Martinelli, Nuclear Phys. B157, 461 (1979); J. Abad and B. Humbert, Phys. Letters B78, 627 (1978); B80, 286 (1979); K. Harada, T. Kaneko and N. Sakai, Nuclear Phys. B155, 169 (1979); A. P. Contogouris and K. Kripfganz, Phys. Letters B84, 473 (1979); Phys. Rev. D19, 2207 (1979).

(3) - V. N. Gribov and L. N. Lipatov, Sov. J. Nuclear Phys. 15, 458, 675 (1972); Yu. L. Dokshitser, Sov. J. Nuclear Phys. 46, 641 (1977); G. Parisi, Phys. Letters B90, 295 (1980); G. Curci and M. Greco, Phys. Letters B92, 175 (1980); D. Amati, A. Bassetto, G. Marchesini and G. Veneziano, Nuclear Phys. B173, 429 (1980).

(4) - G. Curci and M. Greco, ref. (3).

(5) - For a comprehensive experimental review see, for example, G. Matthiae, CERN Preprint EP/80-183 (1980).

(6) - G. Parisi; G. Curci and M. Greco, ref. (3).

(7) - G. Altarelli, R. K. Ellis and G. Martinelli, ref. (2).