F. Palumbo: SPIN-ISOSPIN ORDER: CRITICAL DENSITY IN NUCLEAR MATTER AND A POSSIBLE REALIZATION IN NUCLEI.

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## 1. INTRODUCTION

At the beginning of the past decade different authors[1,2] predicted an ordered phase of nuclear matter at sufficiently high density, originally called Nuclear binding by the OPEP or Pion condensate. I will refer to it here as Spin-isospin ordered phase (SIOP).

It is now widely accepted that SIOP can occur only at densities higher than normal density, which excludes SIOP in nuclei. Current research is therefore mostly devoted to predict and detect precursor phenomena of SIOP in nuclei[3], or to study the possibility of realizing SIOP in heavy-ions collisions.

The above conclusion is grounded on calculations on nuclear matter and qualitative arguments about nuclei. I want to discuss the soundness of the assumptions at the basis of these calculations and arguments explaining why in my view they should not be taken as conclusive. I will then present some recent developments related to nuclei.

## 2. NUCLEAR BINDING BY THE OPEP

The OPEP is strong and long range, but it does not contribute to the binding energy in Hartree approximation unless the nuclear system is in a SIOP. This is due to its spin-isospin dependence

$$V_{OPE} = \frac{1}{3} f^2 \mu \vec{\tau}_1 \cdot \vec{\tau}_2 \left\{ -\frac{4\pi}{3} \partial(\vec{r}) \vec{\sigma}_1 \cdot \vec{\sigma}_2 + \left[ \vec{\sigma}_1 \cdot \vec{\sigma}_2 + (1 + \frac{3}{\mu r} + \frac{3}{\mu^2 r^2}) S_T \right] \frac{e^{-\mu r}}{\mu r} \right\}. \tag{1}$$

In fact assuming the spins quantized along the z-axis

$$\langle V_{OPE} \rangle_{direct} = \frac{1}{2} \sum_{\tau_3(1)\sigma_3(1)} \sum_{\tau_3(2)\sigma_3(2)} \int d\vec{r}_1 \int d\vec{r}_2 \langle \tau_3(1)\sigma_3(1)\tau_3(2)\sigma_3(2) . $$

$$\left| V_{OPE} \right| \tau_3(1) \sigma_3(1) \tau_3(2) \sigma_3(2) \rangle \varrho_{\tau_3(1)\sigma_3(1)}(\vec{r}_1) \varrho_{\tau_3(2)\sigma_3(2)}(\vec{r}_2) , \tag{2}$$

where $\varrho_{\tau_3 \sigma_3}$ is the one-body density matrix of nucleons of isospin $\tau_3$ and spin $\sigma_3$. The spin-isospin matrix element is

$$\langle \tau_3(1)\sigma_3(1)\tau_3(2)\sigma_3(2) \mid V_{OPE} \mid \tau_3(1)\sigma_3(1)\tau_3(2)\sigma_3(2) \rangle = \frac{1}{3}f^2\mu \left\{ -\frac{4\pi}{3}\delta(\vec{r}) + \right.$$

$$\left. + \left[ 1 + (1+\frac{3}{\mu r}+\frac{3}{\mu^2 r^2})(3\frac{z^2}{r^2} - 1)\right]\frac{e^{-\mu r}}{\mu r} \right\} \tau_3(1)\sigma_3(1)\tau_3(2)\sigma_3(2) \quad . \tag{3}$$

It is convenient to introduce the spin-isospin density operator

$$S_{ik} = \bar{\psi}\tau_i \sigma_k \psi. \tag{4}$$

Eq. (2) can be rewritten in terms of the average value of $S_{33}$

$$\langle S_{33} \rangle = \sum_{\tau_3 \sigma_3} \varrho_{\tau_3 \sigma_3} \tau_3 \sigma_3 , \tag{5}$$

$$\langle V_{OPE} \rangle_{direct} = \frac{1}{2} \int d\vec{r}_1 \int d\vec{r}_2 \langle S_{33}(\vec{r}_1) \rangle \langle S_{33}(\vec{r}_2) \rangle \frac{1}{3}f^2\mu \left\{ -\frac{4\pi}{3}\delta(\vec{r}) + \right.$$

$$\left. + \left[ 1 +(1 +\frac{3}{\mu r} + \frac{3}{\mu^2 r^2})(3\frac{z^2}{r^2} - 1)\right]\frac{e^{-\mu r}}{\mu r} \right\} . \tag{6}$$

Eqs. (5) and (6) show that if $\varrho_{\tau_3 \sigma_3}$ is independent of $\tau_3 \sigma_3$, $\langle V_{OPE} \rangle_{direct}=0$. It is therefore natural to think that a SIOP should be favored at suffici-ntly high density. At high density in fact, the exchange potential energy per particle grows like the density $\varrho$, the kinetic energy per particle like $\varrho^{5/3}$ and the direct potential energy per particle like $\varrho^2$. This last term is therefore dominating, and if the OPEP were the whole N-N interaction, a SIOP would necessarily be established which would eventually lead to nuclear collapse.

Spin-isospins of the nucleons can be ordered only if the nucleons are to some extent localized. The factor ( $\frac{3z^2}{r^2}$ - 1) in eq. (6), requiring maximum asymmetry between the z-axis and the x-y plane, favors localization along one direction only, the direction of spin quantization. It turns out, as the result of a variational calculation employing the OPEP (with a monopole regularization) as the only N-N interaction, that the localization should be complete, qualifying the phase transition as a first order one. This result was already contained in the original paper[1], but it has been fully exploited by Tamagaki and coworkers[4]. Nuclear matter in SIOP is crystal-like along the z-direction as shown in Fig. 1.
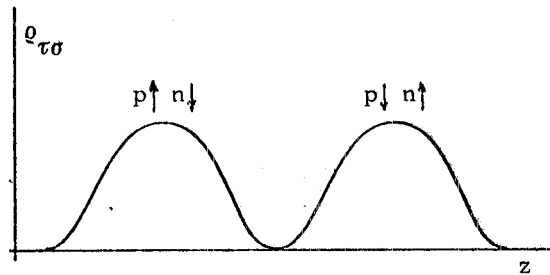


FIG. 1 - Spin-isospin density with the OPEP.

The critical density is determined by comparing the ground state energy in the normal state with the ground state energy in the ordered state, with the result $\varrho_c \sim 0.5\ \varrho_o$.

Of course use of the OPEP as the whole N-N potential is unrealistic, and one has to investigate the effect of the other components of the interaction. In view of the crystal-like character of the ordered phase particularly important is expected to be the role of the core of the N-N potential. It can be important in two ways. It can change the structure of the ordered phase and it can change the critical density.

The first possibility has been investigated[5] using the OBEP of the Ueda and Green school, and exploring different configurations. The structure which is energetically favored by these potentials is represented in Fig. 2.
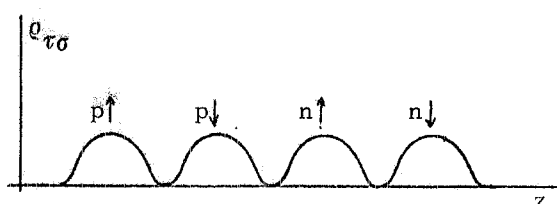
FIG. 2 - Spin-isospin density with the OBEP.

This result has been obtained by neglecting the short-range correlations in the w.f.. And this leads us to the second point: How will these correlations change the critical density?

If one assumes that the short-range correlations are the same in the normal as in the ordered phase, their effect is to increase the critical density above $\varrho_o$. This assumption, however, is far from justified. It is worse than to use the same short-range correlations for a fluid and crystal, because here short-range correlations must be spin-isospin dependent and anysotropic.

Use of this so far unjustified assumption can be avoided if one determines the critical density as the point of instability of the normal state. This is what has been done in the study of Pion condensation. It does not require the evaluation of any quantity related to the ordered state, but rests an another assumption to be discussed later.

## 3. PION CONDENSATION

The pion propagator in the nuclear medium is

$$D\ (\vec{k},\omega,\varrho) = \left[ \omega^2 - k^2 - \mu^2 - \Pi(\vec{k},\omega,\mu) \right]^{-1} \tag{7}$$

where $\vec{k}$ is the momentum, $\omega$ the energy, $\mu$ the mass and $\Pi$ the selfenergy which is a function of the nuclear density $\varrho$ . The normal state becomes unstable when the pion propagator has a pole at energy $\omega = 0$. In this case it costs no energy to produce pions.

The equivalence between pion condensation and the phase described in the previous section is established by the identity of the ground state. If there is pion condensation, the average value of the pion field is different from zero

$$(\varDelta + \mu^2)\langle\varphi_i\rangle = \frac{f}{\mu}\ \partial_k\ \langle S_{ik}\rangle \tag{8}$$

and therefore $< S_{ik} > \neq 0$. Condensation of charged pions is related to superconductivity in layers[4] and will not be discussed here.

In order to establish the equivalence completely we observe that both in pion condensation and in nuclear binding by the OPEP parity and isospin are broken. In the case of pion condensation this is obvious because the pion field is pseudoscalar and isovector. In the other case parity is broken as a consequence of breaking of translational invariance, while breaking of isospin follows from the fact that the operators $S_{ik}$ are isovectors. In fact applying the Wigner-Eckart theorem we have

$$< T\, T_z \,|\, S_{3k} \,|\, T\, T_z> \propto \frac{T_z}{T(T+1)} \; . \tag{9}$$

For symmetric nuclear matter $T_z = 0$, and $< S_{3k} >$ cannot be different from zero for a state of definite T.

This shows that isospin breaking must be a characteristic feature of (static, see below) SIOP also in nuclei, since the above argument does not depend on the system being finite or infinite. Parity breaking, on the contrary, is a consequence of $< \varphi_i > \neq 0$ only for an infinite system, where parity is defined w.r. to arbitrary points, but not for nuclei, where parity is defined only w.r. to the c.m..

Established the equivalence between Pion condensation and Nuclear binding by the OPEP, let us turn to the determination of the critical density. If the pion self-energy is evaluated in RPA neglecting short-range correlations between nucleons, the critical density is found to be lower than $\rho_0$. In the present case, however, introduction of short range correlations is much easier, as already noted, because we need to deal only with the normal state. This has been done using the Landau parameter, and this approximation has been recently checked to be very good[6]. The effect of the short range correlations is to increase the critical density up to $\rho_c \simeq 2\rho_0$.

This procedure is correct, however, only if the phase transition is of second order. This is the assumption I was talking about.

In order to appreciate the difference between first order and second order phase transitions in the present context, let us refer to a well-known case, the vapor-liquid phase transition, whose phase diagram is reported in Fig. 3.
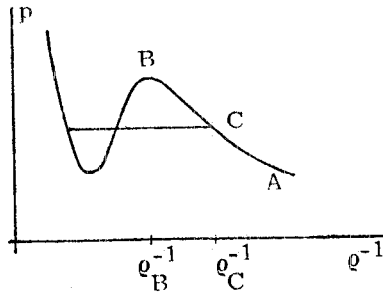


FIG. 3 - The vapor-liquid phase diagram.

This a first order phase transition, occuring at density $\varrho_c$. The critical density is determined by the crossing of two curves, giving the free energy of the vapor and liquid phase, respectively. Each curve is analytic, and the phase transition is due to the fact that the physical system is described by one analytic function before the transition and by a different one afterwards.

Actually the vapor can be made to follow the curve AB beyond the critical point C by means of an adiabatic compression. This phase of supersaturated vapor can be also described theoretically by means of the RPA. The point B will appear in this case as a second order phase transition occuring at $\varrho_B > \varrho_C$. At this point there is in fact a singularity in the second derivative of the Gibbs potential with respect to the pressure telling that the compressibility is infinite

$$k_T = - \frac{1}{V} \left. \frac{\partial V}{\partial P} \right|_T = - \frac{1}{V} \frac{\partial^2 G}{\partial p^2} \; . \tag{10}$$

A similar situation could occur in our case. A calculation of the type just described looking at the pole of the pion propagator can tell us very little concerning the possibility of a first order phase transition at lower density. This in fact is related to large quantum fluctuations around the mean field approximation of RPA. Dyugaev has studied this problem[7] with the conclusion that the phase transition is actually of first order, but at a critical density very close to the critical density it would have as a second order one. It is the estimate of the difference between these critical densities that seems to me very uncertain, due to the difficulty of properly taking into account the core effects, if they are large.

Note that the possibility I am considering is relevant also to experiment. Suppose in fact that the critical density is higher than the experimental density, that the phase transition is of first order and that one tries to reach it in heavy-ion collisions. If the experimental conditions correspond to an "adiabatic compression" the critical density for the first order phase transition is overcomed without effect until the second order phase transition is realized at higher density.

Therefore looking for the instability of the normal state is not an alternative procedure w.r. to the comparison of the energies of the normal and the ordered state, but rather a complementary one. Both should be used to determine the characther of the phase transition and the critical density.

A last remark is in order about precursor phenomena. If $\varrho_c > \varrho_o$, and the phase transition is of first order, such phenomena do not exist. We will see, however, that something very similar to them can exist in nuclei, due to their finite size.

There are many important points left which I do not have time to discuss, including the effects of isobars and the problem of the convergence of the sum of the bubble diagrams (bubbles into bubbles)[6].

## 4. SIOP IN NUCLEI

I mentioned at the beginning arguments against the existence of SIOP in nuclei[8]. Such arguments are i) that almost degenerate parity doublets should exist on account of parity breaking in nuclear matter and ii) that the levels with the quantum numbers of the pion should be lowered.

I will discuss these points after presenting a possible mechanism for SIOP in nuclei. According to my previous analysis I do not consider yet settled the value of the critical density, and therefore I will have in mind both the case that $\varrho_c > \varrho_o$ and $\varrho_c < \varrho_o$.

Let us separate the nucleus into two parts with spin-isospin order, for instance one part containing spin-up protons and spin-down neutrons, the other spin-down neutrons and spin-up protons. Let us denote by d the distance between their c.m. and by V(d) the separation energy (Fig. 4).
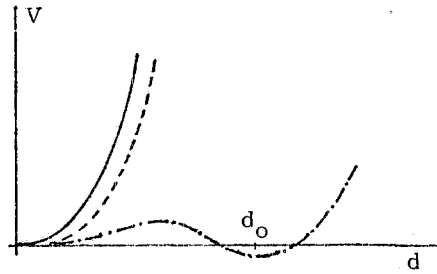


FIG. 4 - The potential separation energy: The solid line is for the disordered separation in the zero-point potion, the dashed line for the spin-isospin ordered separation in the zero-point motion, the dot-dashed line for the static separation.

This must be compared with the separation energy $V_o(d)$ of the nucleus into two parts each of which has no spin-isospin order. Such separation actually takes place in the zero-point motion, and what we investigate is whether the spin-isospin ordered separation is favored or not w.r. to the disordered separation.

If $V(d) > V_o(d)$ the disordered separation is preferred. If $V_o(d) > V(d) > 0$ the zero-point motion will take place between two spin-isospin ordered phases. We talk in this case of nonstatic order. The average value of the pion field is proportional to the average value of d, and therefore vanishes. The order parameter is $< \varphi^2 > \propto < d^2 >$.

The extreme possibility is that V(d) becomes negative with a minimum at $| d | = d_o$. In this case we have a static order with order parameter $d_o$ and an average pion field $<\varphi> \propto d_o$.

Only the case of nonstatic order has been investigated[9]. Due to the known difficulties with short-range correlations, only the OPEP has been taken into account. As a result it is not possible to predict reliably wheter the nonstatic order is actually realized. It is however possible to predict a number of characteristic features the nucleus should have if it were in the ordered phase.

It turns out that oscillations must be one-dimensional, and along the direction of spin quantization (oscillations in the perpendicular plane can also occur but not associated with spin-isospin). This direction must coincide with the symmetry axis for an oblate nucleus, and must be perpendicular to it for a prolate nucleus. In any case this kind of correlation is only possible for nuclei with $A < 60$.

The signature of this mode is the lowering of the excitation energy and the enhancement of the B(M2). Typical values are reported in the Table 1.

| A | δ | ℏω (MeV) | B(M2; K=0 → K=0) W.u. | B(M2; K=0 → \|K\|=2) W.u. |
|---|---|---|---|---|
| 12 | -0.4 | 12 | 20 | 0 |
|  | 0.4 |  | 5 | 16 |
| 28 | -0.4 | 11 | 28 | 0 |
|  | 0.4 |  | 8 | 23 |

Table 1 Excitation energy in MeV and B(M2) in W.u. for two nuclei at different values of the deformation parameter δ. For δ < 0 the nucleus is oblate and the oscillation takes place along the symmetry axis, while for δ > 0 the nucleus is prolate and the oscillation takes place along a direction perpendicular to it.

The zero-point correlation just described can coexist with other spin-isospin correlations, for instance a breathing mode of spin-up protons and spin-down neutrons against spin-down neutrons and spin-up protons. This would renormalize M2 transitions and would presumably enhance M1 transitions.

It so appears that nonstatic spin-isospin order enhances the e.m. transition amplitudes and lowers the energy of unnatural parity levels, which is considered a precursor to Pion condensation. This point needs further investigation. We see, however, that no parity doublets are to be expected. (I already observed, moreover, that parity breaking is not to be expected even in the presence of static SIOP). In addition, lowering of the levels of unnatural parity is a signature of nonstatic SIOP only, while nothing has been proved, as far as I know, concerning static SIOP. I should also emphasize that all the mentioned effects are to be expected only in deformed nuclei. I cannot see how observations concenring spherical nuclei (the famous level[8] of $^{16}$O at 12.78 MeV) can be relevant to SIOP in any of its possible realizations.

I will conclude this discussion of nonstatic order by mentioning that it is energetically more favored the higher the density. Also if it is not actually realized in nuclei, it could therefore be excited under compression.

The extreme possibility of static order according to the mechanism outlined has not yet been studied in detail. An entirely different possibility has been considered by G. Do Dang[10], who has studied a nucleus made only of spin-up protons and spin-down neutrons. The necessary density has been estimated[6] to be twice the experimental density.

I will conclude my talk by quoting an experiment which in my view can set an upper bound on the amplitude of spin-isospin density fluctuations in nuclei in an almost model independent way. If so the experiment, though very difficult, does not suffer from the ambiguities of many other tests proposed, which depend on the details of s.p. w.f., exchange currents, and so on.

The idea[11] is that nuclei in static SIOP should give rise to a coherent scattering of neutrinos at values of the momentum transfer where the coherent scattering by normal nuclei is negleageable. This is due to the axial current. This current in the Weimberg-Salam model is

$$J_\mu = \frac{1}{2} \bar{\psi} \gamma_\mu \gamma_5 \tau_3 \psi , \tag{11}$$

and its spatial components in the nonrelativistic approximation become just the order

parameter

$$J_k = -i\,S_{3k}\,.$$ <div style="text-align: right">(12)</div>

This effect could also have interesting astrophysical consequences.

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