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ON THE STRUCTURE OF THE PHASES IN LATTICE GAUGE THEORIES

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In recent years a lot of work has been concentrated on the study of non-abelian gauge theories on a lattice[1]. The introduction of a lattice is crucial do define the theory in a non-perturbative way: in lattice gauge theories it is possible to use strong coupling techniques such as the high temperature expansion[2], the numerical simulations based on the Montecarlo method[3,4] and the real space renormalization group[4,5]. Of course we have to pay a price for having all these advantages: the theory can be interpreted as the Euclidean version of a relativistic invariant local gauge field theory only in the limit in which the coherence lenght ξ goes to infinity, when it is measured in units of the lattice spacing. In the language of statistical mechanics the divergence of the correlation lenght corresponds to a second order phase transition.

If a model has only first order transitions, the absence of second order phase transitions implies that the coherence lenght never becomes infinite: the model under consideration does not have a continuum limit and the corresponding local field theory does not exist.

These remarks clearly show the importance of knowing the structures of the phases in lattice gauge theories; unfortunately at the present moment only few results are firmly estabilished. Before discussing them, let us define our notation. The gauge fields U are defined on the links of a D-dimensional hypercubic lattice and have a value in the gauge group G (not in the Lie algebra of the group). We will concentrate our attention on non-abelian Lie groups, abelian groups have their own characteristics and they behave in a rather peculiar way.

The partition function can be computed using the following hamiltonian:

$$H = -\sum_p T_r(U_p) \tag{1}$$

where the sum runs over all the plaquettes of the lattice (four links forming a square are called a plaquette) and U_p stand for the product of the four U around the plaquette p. The temperature T, at which the partition function is evaluated, is proportional to the bare coupling constant g^2.

In the high temperature phase, when $T \to \infty$ ($\beta = 1/T \to 0$), the theory is confined and the correlation lenght is proportional to β^4. At low temperature the perturbative expansion can be trusted only if the dimensions D are greater than 4; if $D < 4$ the standard perturbative expansion cannot be constructed because of strong infrared divergences; renormalization group arguments suggest that in dimensions $D = 4 + \varepsilon$, a second order transition is present at temperature $T_{II}(\varepsilon)$ proportional to ε: the value of the critical temperature and the corresponding critical exponents can be computed as a power expansion in ε. In this low temperature phase the theory is not confined: it is believed that for $D < 4$ the theory is always confined, for any value of the temperature, while if $D > 4$ the second order phase transition at $T_{II}(\varepsilon)$ separates the unconfined from the confined phase. This picture is the simplest one compatible with the firmly established results; the phase diagram in the temperature-dimensions plane would be the one shown in Fig. 1. There are only one confined and one unconfined phase, separated by a line of second order phase transitions. There are strong indications which suggest that the situation is more complex: they will be discussed later. For the moment let us stick to this naive picture.

In Fig. 1 the star denotes the upper critical dimension D_S: if $D > D_S$ the appropriate mean field theory holds at the phase transition point and the critical exponents have a trivial dependence on the dimensions. This diagram is constructed in analogy with the one known for the non linear model, shown in Fig. 2, where D_S is known to be equal to 4.

It is possible that, as it happens in the non linear σ-model, the critical exponents can be computed also in powers of $\varepsilon' = D_S - D$. In ref. 6 it has been suggested that $D_S = 8$; two ways are allowed in order to construct the ε' expansion: in the first approach one should introduce "prepotentials" in such a way that the field A_μ would become a composite field and that the gauge theory would look like the high derivative version of the non linear σ-model; in the second approach one should show that gauge theories are the equivalent of a generalized string model in which off-shell Green functions are computable and finite. It is very hard to implement these suggestions, however I believe that something can be done in this direction using the techniques introduced by Wallace and Zia[7] in the study of the

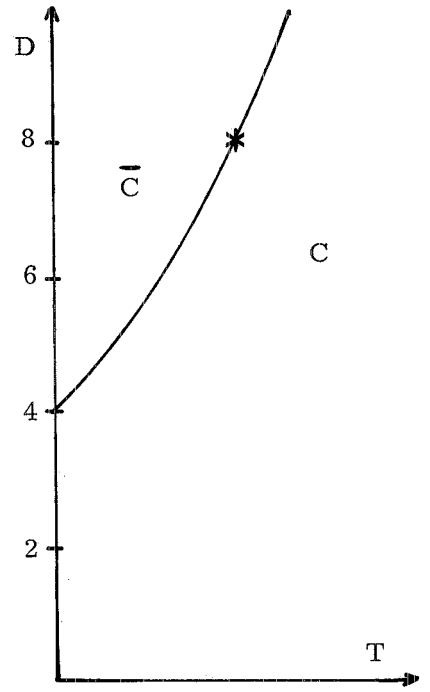

FIG. 1 The naive phase diagram for non abelian gauge theories in the temperature dimensions plane: there are two phases, the confined (C) and the unconfined one ($\bar{C}$), which are separated by a line of second order phase transitions; the star denotes the upper critical dimensions, as explained in the text.

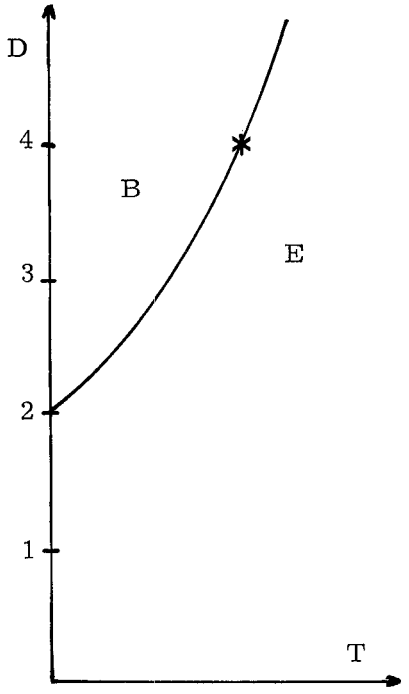

FIG. 2 The phase diagram for the non-linear O(N) invariant σ model (N > 2): there are two phases: in phase (B) the O(N) symmetry is spontaneously broken while in phase (E) the symmetry is exact.

"gas-liquid interface" in $1+\varepsilon$ dimensions.

Let us forget these speculations and let us come back to the study of the consequence of the conventional picture: contrary to the naive intuition, the correcteness of Fig. 1 does not imply that the theory is confined in the continuum limit. In order to discuss more carefully this point, it is convenient to define the surface tension σ using the expectation values of large Wilson's loops:

$$(2) \qquad \langle \Pi_{i \in C} U_i \rangle \simeq \exp(-\sigma S)$$

where S is the minimal area enclosed by the large contour C; σ is different from zero only if the lattice theory is confined; by continuity it should go to zero at the deconfinement transition ($T_{II}(\varepsilon)$. Now, if $D > 4$, two different theories can be constructed in the continuum limit: the one is obtained at T_{II}^{-}, the other at T_{II}^{+}. The first theory will be not confined for obvious reasons; for the second theory the situation is more subtle: in the continuum limit we must measure everything in units of the coherence lenght ξ(i.e. the inverse of the mass gap). Now three possibilities are given:

$$(3) \qquad \lim_{T \to T_{II}^{+}} \xi^2 \sigma \equiv R = \begin{cases} 0 \\ 0(1) \\ \infty \end{cases} .$$

If our expectations are correct and the theory is confined in the continuum limit, R should be non zero and finite; if $R = 0$ the theory in the continuum limit is not confined (or at least the potential is not linear at large distances); if $R = \infty$, the quarks are hyperconfined, i.e. their mean separation would be zero and we would never be able to discover their existence in experiments like deep inelastic scattering. In this scheeme the quantity R (which can be expressed in terms of the ratio of the glueball mass with the coefficient of the linear term of the static potential between two quarks) can be computed as an expansion in powers of β, evaluated at $\beta_c = T_{II}^{-1}$. If $D = 4$, $\beta_c = \infty$ and appropriate numerical techniques must be used to sum the high temperature expansion. In principle all the physically interesting quantities can be extracted from the high temperature expansion: the success of this program crucially depends on the correctness of the phase diagram shown in Fig. 1, i.e. from the absence of any phase transition at finite temperature at $D = 4$. (We recall to the reader that here we consider only pure gauge theory: one hopes that the fermions can be succesfully studied in perturbation theory at a later stage).

A shadow of doubt on the whole picture was trown some years ago by the paper of Balian, Drouffe and Itzykson (BDI)[8]: in a mean field approximation they found that gauge theories undergo a <u>first</u> order transition. However it was not clear which was the range of validity of their mean field approximation and their prediction was not considered with the due weight. New results obtained this years, show that the conventional picture of Fig. 1, is too naive and that the true situation is more complex.

The first published Montecarlo simulations of gauge theories[3] have shown a clear signature for a first order transition for the Z_2 gauge group in dimensions $D = 4,5$ and for SU(2) in $D = 5$ (no Montecarlo simulations have been done in non integer dimensions). For SU(2) with $D = 4$ no first order transition is observed, but there is a peak in the specific heat which may be due to a second order phase transition[4].

Apart from these "experimental" results, new theoretical results where contained in the paper by Drouffe, Sourlas and myself in which gauge theories where studied in the limit $D \to \infty$[9]: indeed it is known that in this

limit the lattice version of the non linear σ-model become soluble and the standard mean field theory holds[10].

When $D \to \infty$, the high temperature expansion can be exactly summed in a closed form, and the free energy becomes a function of the scaling variable $\tilde{\beta} = \beta/D^{1/4}$. In very high dimensions many diagrams are negligeable; the only diagrams which survive are those which at fixed power of β, have the maximal power of D; these diagrams are very simple: they corresponds to the surface of polybranched polymers of cubes. The enumeration of these diagrams is a rather simple combinatorical problem; however one must be quite careful in estabilishing the correct diagrammatical rules, in particular in computing the contribution of disconnected diagrams: we do not enter in the detailed derivation of these rules; which can be found in the original papers[8,9,11] and we present only the final results. The free energy we obtain after resumming the dominant diagrams has second order transition at $T_c \propto D^{1/4}$. This transition is characterized by the divergence of the specific heat, i.e. by the plaquette-plaquette susceptibility: at the transition point the plaquette-plaquette correlation lenght is going to infinity, but the surface tension σ remains finite ($R = \infty$ in eq.(3)). This transition is not a deconfinement transition and corresponds to the condensation into vacuum of the boxitons of ref.3, i.e. the glueballs in a more phenomenological language.

These results strongly suggest the possibility of having two different confined phases: the standard high temperature phase C_1 and a new low temperature phase C_2 in which boxiton are condensed. The possible existence of two qualitatively different phases is a rather general phenomenon which can be simply understood in geometrical terms.

Let us consider a geometrical picture in which we neglect all the group theoretical quantum numbers: gauge theories become theories of interacting surfaces, as can be readly seen from the high temperature expansion. When the surface tension decreases, the possibility of thin, long deformations of the surface increases: these hydra-like configurations are local deformations of the surface which are allowed also when the surface still keeps its local rigidity (i.e. $\sigma \neq 0$): as an explicit computation shows, these local deformations have an entropy much higher than global deformations, at least in high dimensions.

With an abuse of language one could say that the surface roughening transition happens at a temperature greater than the critical temperature at which the surface tension goes to zero.

Unfortunately we have been unable to find a good order parameter which characterizes this surphases roughening transition and we do not know if the transition between C_1 and C_2 is really a second order phase transition, or if the second order phase transition we have found is only the end of the metastable region and there is a first order transition from C_1 and C_2 at a higher temperature.

However this problem is purely academic in high dimensions: if one compare the high temperature expansion either with the low temperature expansion or with the rigorous lower bounds for the free energy of BDI, one finds a first order transition at $T = T_I \propto D^{-9}$. This BDI transition is a deconfinement transition and there is no field theory associated to it in the continuum limit; the temperature of our roughening transition is much lower (it is proportional to $D^{-1/4}$). The condensation of boxitons may happen only in the metastable phase for very high dimensions and therefore it is not relevant. The reasons for which the BDI first order transition can hardly be seen in the high temperature expansion, are explained in a small appendix on first order transitions.

Now which is the substitute of Fig. 1. We definitely do not know, we do not have enough informations. Waiting for careful analysis of longer high temperature expansions and for more accurate Montecarlo simulations I will try to put forward a new picture, which has very small chances of being correct, but it will be quite interesting to be able to disprove it. This picture is shown in Fig. 3 and 4 for the gauge groups Z_2 and SU(2) respectively.

In both Fig. 3 and 4 we indicate by full lines real phase transitions and by dashed lines virtual phase transitions in the metastable region. In Fig. 3 line (a) denotes the BDI first order transition between the confined and the unconfined phase, line (b) is the second order transition between the confined and the unconfined phase, which is supposed to be a virtual transition: the true transition (line (a)) happens at a lower temperature. Line (c) is the second (first) order roughening transition, which at high dimensions is in the metastable region, and becomes a real transition only for enough low dimensions. In Fig. 4 line (a) denotes a first order deconfinement phase transition for $D > 3$, and line(b) the second order roughening transition, which for dimensions greater than 3 is a virtual transition and comes to physical reality only for dimensions 3. The detailed form of Fig. 3 and 4 for D smaller than 4 and 3 respectively, is not very important and should not be taken too seriously.

It would be easy to find other scheemes which are as reasonable as the one presented here: for example we could suppress line (c) in Fig. 3 and declare than line (a) is a first order transition if $D > 4.5$, but a second order one if $D < 4.5$.

The diagram of the phases of gauge theories deserves further investigations and may be further surprises are waiting for us: ice, which is a relatively simple system, has a dozen of different phases.

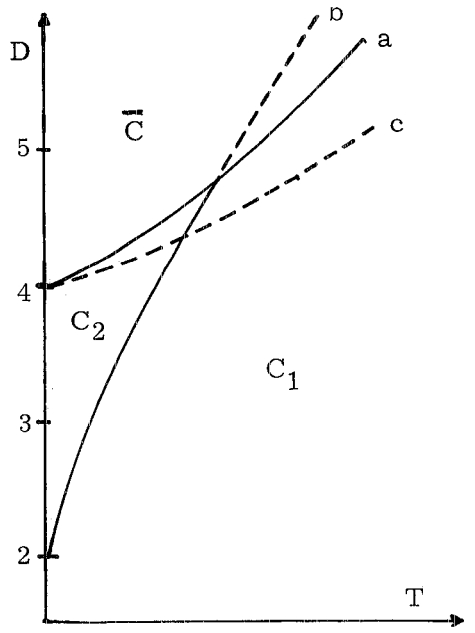

FIG. 3 The conjectured phase diagram for SU(2) gauge theories: there are two confined phases C_1 and C_2 and one unconfined phase $\bar{C}$: line (a) separates the confined from the unconfined phase and it is a first order transition, line (b) should separate the two confined phases C_1 and C_2, but it may physically realized only for low dimensions: for high dimensions it is located in the metastable region and it is masked by the first order transition line (a). Line (c) is the second order phase transition which can be seen using the renormalization group and the low temperature (low coupling) expansion; we suppose that this second order transition only indicates the end of the metastable region, the true transition (a) happens at a lower temperature.

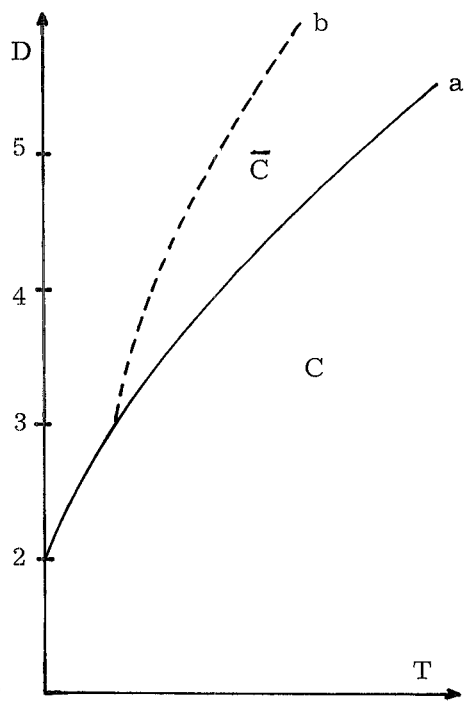

FIG. 4 The conjectured phase diagram for Z_2 gauge theories: line (a) is the only real phase transition (first order for $D > 3$) which separates the confined from the unconfined phase; line (b) is the second order phase transition which can be seen from the analysis of the high temperature expansion.

Appendix

In this appendix we recall some know (proved or conjectured) facts on first order phase transitions. A second order phase transition is characterized by the absence of latent heat, the correlation lenght goes to infinity and the free energy has a power-like singularity at the critical temperature.

Let us consider a first order phase transition from the phase A to the phase B. It is usual to define two free energies F_A and F_B, which describe the system in phase A and in phase B respectively; they satisfy the inequalities:

$$\text{(A.1)} \qquad \begin{aligned} F(T) = F_A(T) < F_B(T) \qquad & T > T_c \\ F(T) = F_B(T) < F_A(T) \qquad & T < T_c \end{aligned}$$

where $F(T)$ is the true free energy of the system.

In writing eq. (A.1) we have implicitely assumed that both F_A and F_B are analytic function at T_c: that happens in the mean field approximation and at all orders in perturbation theory. In reality both functions are only C^∞ at the transition point; it is believed that their analytic continuation below (or above) the transition has an exponentially small imaginary part due to "instanton" effects [12].

In not very sofisticated analysis there is no difference between a C^∞ function and an analytic function: the singularity at T_c of the free energy is practically invisible in the high temperature expansion unless one consider incredible high orders. The only viable method to find first order transitions consistes in constructing the two free energies F_A and F_B and to compare them. If we only consider the properties of the free energy F_A it is practically impossible to find the first order transition; as a consequence, if we forget to construct the free energy F_B, we cannot suspect the existence of the first order transition, unless we discover that F_A violates rigorously established inequalities.

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