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MULTIPLE SCATTERING IN TRANSFER REACTIONS BETWEEN HEAVY IONS AT INTERMEDIATE ENERGIES.

A. Ma/ecki
Institut of Nuclear Physics, Krakow(Poland).

B. Minetti
Istituto di Fisica Sperimentale del Politecnico di Torino, Italy.

G. Rosa
Università di Roma , and INFN, Sezione di Roma, Italy.

P. Di Giacomo and A. Reale
INFN - Laboratori Nazionali di Frascati, Italy.

1. - INTRODUCTION.

The lack of experimental knowledge of heavy ion collisions at intermediate energies, that is between ~ 10 MeV/ Nucl. and \sim GeV/Nucl data, will probably be filled in the next few years.

In this energy gap, even the qualitative features of the interaction are to date scarcely known⁽¹⁾.

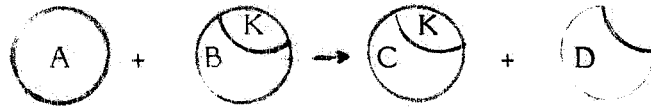
The extrapolation of both low energy collective models (e.g. Deep Inelastic Collisions) and relativistic models will hardly be exhaustive. For instance, the few-nucleon transfer reactions by peripheral collisions will probably take out a sizeable part of the interaction cross section. We are looking for a microscopic approach to

these phenomena, quite outside the framework of the popular DWBA approximation.

The Glauber model⁽²⁾ of multiple scattering provides a simple description of the elastic nucleus-nucleus collisions^(3,4). Recently⁽⁵⁾, the Glauber eikonal approach has been generalized for the collisions with rearrangement of nucleons. In this note we are studying the reactions of nucleon transfer between fast (some hundred MeV/nucleon) heavy ions.

2. THE EIKONAL APPROACH.

Let us denote A and B - the nuclei in the entrance channel, and C, D - the nuclei in the exit channel. We consider a transfer of K nucleons from B to C. (Fig. 1).



At moderately high energies one may neglect the nuclear Hamiltonians and the nuclear part of the total energy in Green's functions of the initial and final channel. Consistently, the scattering of two bound nucleons can be well described by the free nucleon-nucleon transition operators (impulse approximation).

In order to obtain a Glauber description of the reaction the free Green's function is further approximated as the eikonal propagator. The convenient choice of the eikonal axis OZ is along the vector:

$$\vec{p}_i + \left(\frac{C}{A} p_i^2 - \vec{p}_i \cdot \vec{p}_f \right) \left(p_f^2 - \frac{C}{A} \vec{p}_i \cdot \vec{p}_f \right)^{-1} \vec{p}_f \quad (1)$$

\vec{p}_i, \vec{p}_f being the c.m. moments of the nuclei A and C. One obtains then the following expression for the transition amplitude of rearrangement (prior-form)⁽⁵⁾:

$$\langle \varphi_f^{\vec{p}_f} | T_{GI} | \varphi_i^{\vec{p}_i} \rangle = -iv \int_{AB} d^2 b e^{i(\vec{p}_i - \frac{A}{C} \vec{p}_f) \cdot \vec{b}} \int d^3 \left\{ \vec{r}_{jA}, \vec{r}_{kB} \right\} \exp(-\vec{p}_f \cdot \frac{A+B}{CD} \sum_{k=1}^K \vec{r}_{kB}) \cdot \quad (2)$$

$$\cdot \varphi_f^x \left(\left\{ \vec{r}_{lC}, \vec{r}_{mD} \right\} z=0 \right) \cdot \left\{ 1 - \prod_{j=1}^K \prod_{k=1}^K \left[1 - \gamma_{jk} (\vec{b} + \vec{s}_{jA} - \vec{s}_{kB}) \right] \right\} \varphi_i \left(\vec{r}_{jA}, \vec{r}_{kB} \right)$$

where φ_i, φ_f denote the intrinsic wave functions of the two nuclei in the initial and final channel. They depend on the set $(\vec{r}_{jA}, \vec{r}_{kB})$ of $A+B-2$ intrinsic nucleon

coordinates, referred to the centre-of-masses of the nuclei A and B, and on the set $(\vec{r}_{iC}, \vec{r}_{mD})$ of intrinsic nucleon coordinates, referred to the c.m. of nuclei C and D, respectively. It is a characteristic feature of the rearrangement collisions that the initial channel coordinate $\vec{r}(b, z)$ is coupled to the intrinsic nuclear coordinates in the final channel.

The profile functions γ_{jk} , depending on the impact parameter vector \vec{b} and on the transverse nucleon coordinates, are to be determined from the free nucleon-nucleon elastic scattering amplitudes:

$$\langle \vec{k}_f | t_{jk} | \vec{k}_i \rangle = -iv \int d^2\vec{b} e^{i(\vec{k}_i - \vec{k}_f) \cdot \vec{b}} \gamma_{jk}(b). \quad (3)$$

when \vec{v} is the relative velocity of the colliding nuclei.

In comparison with the Glauber formula without rearrangement⁽³⁾ eq. (2) reveals two differences: the occurrence of the phase-factor describing the motion of the transferred nucleons, and the dependence of the final channel nuclear wave function on the impact parameter variable. However, the suitable choice of the eikonal axis has assured that in eq. (2) there are no phase-factors dependent on the longitudinal channel coordinates.

In general, the coupling between the channel and nuclear coordinates makes the microscopic treatment quite involved. Therefore we shall confine ourselves to the transfer of a small number of nucleons $K/C = K/D = 0$. In this case the nucleon coordinates in the core nuclei A and D remain unchanged:

$$\begin{aligned} \vec{r}_{iC} &= \vec{r}_{iA} & , l = 1, \dots, A \\ \vec{r}_{iC} &= \vec{r}_{kB} - \vec{r} & , l = A + k, k = 1, \dots, K \\ \vec{r}_{mD} &= \vec{r}_{(k+m)B} & , m = 1, \dots, D. \end{aligned} \quad (4)$$

We write the nuclear wave functions in the initial and final channel as follows:

$$\begin{aligned} \varphi_i(\{\vec{r}_{jA}, \vec{r}_{kB}\}) &= \varphi_A(\{\vec{r}_{jA}\}) \varphi_{Ki}(\vec{r}_{kB}, k=1, \dots, K) \varphi_{Di}(\vec{r}_{kB}, k=K+1, \dots, B) \\ \varphi_f(\{\vec{r}_{iC}, \vec{r}_{mD}\}) &= \varphi_{Af}(\{\vec{r}_{jA}\}) \varphi_{Kf}(\vec{r}_{kB} - \vec{r}, k=1, \dots, K) \varphi_D(\vec{r}_{kB}, k=K+1, \dots, B) \end{aligned} \quad (5)$$

where φ_A and φ_{Af} (φ_{Di} and φ_D) describe the initial and final state of the core nuclei A and D. φ_{Ki} and φ_{Kf} are the wave functions of the initial and final state for the

system of nucleons which undergo transfer.

From (2) and (5) one obtains the following transition amplitude:

$$\langle \varphi_f^{\vec{p}_f} | T_{GI} | \varphi_i^{\vec{p}_i} \rangle = -iv \int d^2 b e^{i(\vec{p}_i - \frac{A}{C} \vec{p}_f) \cdot \vec{b}} \prod_{k=1}^K (d^3 r_{kB} e^{-i\vec{p}_f \frac{A+B}{CD} \cdot \vec{r}_{kB}}) \cdot \varphi_{Kf}^* (\{ \vec{r}_{kB} - \vec{b} \}) \Gamma_K (\vec{b}, \{ \vec{s}_{kB} \}) \varphi_{Ki} (\{ \vec{r}_{kB} \}) \quad (6)$$

where the profile function Γ_K is:

$$\Gamma_K (\vec{b}, \{ \vec{s}_{kB} \}) = \langle \varphi_{Af} \varphi_{Df} | 1 - \prod_{j=1}^A \prod_{k=1}^B [1 - \gamma_{jk} (\vec{b} + \vec{s}_{jA} - \vec{s}_{kB})] | \varphi_{Ai} \varphi_{Di} \rangle \quad (7)$$

We shall elaborate the profile Γ_K in the spirit of the "inert-cores" approximation. Thus we neglect all the virtual excitations of the core nuclei A and D during the collision. In order to do this we may insert between the profile functions γ_{jK} in eq. (7) the unity operators $\sum_i |n_i(A)\rangle \langle n_i(A)|$, $\sum_f |n_f(D)\rangle \langle n_f(D)|$ and then delete the excited states. Assuming, for simplicity, that all elementary profiles are identical one obtains:

$$\Gamma_K = N_{AD} \left\{ 1 - \left[1 - S_{AD}(b) \right]^{AD} \prod_{k=1}^K \left[1 - S_A (\vec{b} - \vec{s}_{kB}) \right]^A \right\} \quad (8)$$

where

$$\begin{aligned} N_{AD} &= \langle \varphi_{Af} \varphi_{Df} | \varphi_{Ai} \varphi_{Di} \rangle \\ S_{AD}(b) &= \langle \varphi_{Af} \varphi_{Df} | \gamma (\vec{b} + \vec{s}_A - \vec{s}_B) | \varphi_{Ai} \varphi_{Di} \rangle \\ S_A (\vec{b} - \vec{s}_B) &= \langle \varphi_{Af} | \gamma (\vec{b} + \vec{s}_A - \vec{s}_B) | \varphi_{Ai} \rangle \end{aligned} \quad (9)$$

For heavy nuclei eq. (8) should be modified in order to account for the Coulomb interaction between ions:

$$\begin{aligned} \left[1 - S_{AD}(b) \right]^{AD} &\longrightarrow \left[1 - S_{AD}(b) \right]^{AD} e^{i \chi_c(b)} \cong \\ &\cong \exp \left[-AD S_{AD}(b) + i \chi_c(b) \right] \end{aligned} \quad (10)$$

$\chi_c(b)$ being the Coulomb phase-shift.

Eq. (8) describes in the microscopic way the interaction of the projectile nucleus A both with the core D and with the K nucleons which undergo transfer. The two contribution may readily be separated by writing:

$$\Gamma_K(\vec{b}, \{ \vec{s}_{kB} \}) = \Gamma_K^{\text{core}}(\vec{b}) + \Gamma_K^{\text{direct}}(\vec{b}, \{ \vec{s}_{kB} \}) \quad (11)$$

where

$$\Gamma_K^{\text{core}}(\vec{b}) = N_{AD} \left\{ 1 - \exp \left[-AD S_{AD}(b) + i \chi_c(b) \right] \right\} \quad (12)$$

$$\Gamma_K^{\text{direct}}(\vec{b}, \{ \vec{s}_{kB} \}) = N_{AD} \exp(-AD S_{AD} + i \chi_c) \left\{ 1 - \prod_{k=1}^K \left[1 - S_A(\vec{b} - \vec{s}_{kB}) \right]^A \right\} \quad (13)$$

The latter should gives basic contribution to the transition amplitude.

3. - FINAL REMARKS

In order to evaluate the cross section of the transfer reactions, some further steps are to be made starting from (6) and (11).

As a first approach we envisage the transfer of uncorrelated nucleons and a simplified independent particle model for the wave functions of the nucleons inside the core nuclei. With a suitable choice of the nucleon wave function, it is easy to calculate the factor N_{AD} (eq. (9)), whereas the standard Fourier-Bessel transform method can be applied to the integrals S_{AD} and S_A to express them in terms of phenomenological scattering form factors.

In this way we are able to put eq. (6) in a computable form. The computer analysis of the resulting integrals involving the zero degree Bessel functions, is in progress. Both the standard numerical methods and suitable analytical methods by series expansion are under consideration.

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