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Laboratori Nazionali di Frascati

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G. Pancheri-Srivastava, M. Ramón-Medrano and N.Y. Srivastava:
SOFT-GLUON CORRECTIONS TO THE NUCLEON STRUCTURE FUNCTION F_3.

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G. Pancheri-Srivastava(*), M. Ramón-Medrano(°) and N.Y. Srivastava(*): SOFT-GLUON CORRECTIONS TO THE NUCLEON STRUCTURE FUNCTION F_3.(+)

ABSTRACT

Q^2-dependence of the nucleon structure function F_3 is analyzed using an approach inspired by the Bloch-Nordsieck method in the contex of QCD. Moments are obtained and compared successfully with the existing data. Detailed comparison with the renormalization group is also made.

1. INTRODUCTION

In this paper, we apply the classic Bloch-Nordsieck[1] (BN) theorem to obtain the Q^2-dependence of the (non-singlet) structure functions through a summation of soft and collinear-hard gluon effects. We have emphasized on several occasions[2] the importance of the soft radiation (be it in QED or QCD) in discussing the asymptotic behaviour of current amplitudes. To accomplish the above, we suggested the BN approach which is direct, intuitive and physically appealing. The observed x-distribution is supposed to be a folding of the "primitive" (scaling) probability of finding a quark in a hadron with the probability of soft and collinear-hard gluon emission summed to all orders. Under this hypothesis we show (sect. 2) that for the moments $\langle x^N \rangle$, the above implies a factorization into two terms. One part depends upon the primitive distribution[3] and is thus not known (a priori), but is independent of Q^2, and the other one is the calculated distribution which gives the relevant Q^2-dependence of the moments.

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(*) Permanent Address: Northeastern University, Boston, Mass. 02115.
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Previously, moments of structure functions have been analyzed in detail by the renormalization group[4,5] (RG) or equivalently through the Altarelli-Parisi[6] (AP) equations. Next order corrections have also been performed[7] and found to give rather large corrections. Thus it is of interest to inquire what one obtains through the BN method of summing soft photons.

In a recent paper, Doria, Frenkel and Taylor[8] claim that a BN cancellation of IR singularities occur only at the leading log level and does not hold for the non-leading terms, at least for qq scattering. In our opinion their result is incomplete. In the coherent state formalism[9], the IR cancellation occurs (even at the leading log level) only when both initial and final states are modified to account for soft quanta. In their work soft quanta are included only for the final states. It is thus an open question whether the inclusion of soft quanta for initial states as well cancels all of the IR divergences.

Of course, for potential scattering (from a color singlet source) all IR divergences do cancel[10] and there is no problem.

The comparison of our algorithm with the renormalization group calculation is rather encouraging. For soft radiation alone, the Q^2-dependence of the BN moments agrees with that obtained through the RG equation, in the one loop approximation. This leads us to suggest a simple formula which takes into account also the collinear-hard gluons. These modified BN moments obey an equation which then coincides to first order in $_s$ with that obtained from the renormalization group or Altarelli-Parisi equation.

Section 2. defines the problem and sets up the algorithm for the separation of the Q^2-dependence of the moments.

In section 3. we develop our approach and obtain the formula for the BN moments.

In Section 4. we discuss the equation obeyed by the moments and compare it with the Renormalization Group result. Section 5. deals with the phenomenological analysis of the moments of the (non-singlet) nucleon structure function F_3[11].

2. SEPARATION OF PRIMITIVE AND RUNNING MOMENTS

Let F(q,P) be some structure function where q and P are the momenta of an electro-weak current and hadron respectively. Let the probability of finding a quark (with momentum p') inside a hadron (P) be given by the (primitive) distribution $dP_o(p'/P)$. Now, this quark can emit gluons and will find itself at the end with momentum p. Let this probability be given by $d\bar{P}(p/p')$. Since the two steps are assumed independent, we have that

$$F(q,P) = \int dP_o(p'/P)\, d\bar{P}\,(p/p')\, \hat{F}\,(q,p), \tag{1}$$

where $\hat{F}(q,p)$ represents the corresponding point-like structure function of a free quark (momentum p) with the above current. The Bjorken variable $x = Q^2/2q.P$, where $q^2=-Q^2 < 0$, is easily, seen to be related to the ratio xP = p. (For a point-like quark $\hat{F}(q,p)$ is proportional to a δ-function and thus vanishes unless $2q.p=Q^2$). Defining the intermediate variable X as XP = p', eq. (1) may be rewritten as

$$F\,(x, Q^2) = \int_x^1 dX\, \frac{dP_o(X)}{dX}\; .\; \frac{d\bar{P}(x/X,Q^2)}{dx}\; dx\,. \tag{2}$$

In writing eq. (2) we have absorbed a possible multiplicative constant present in F(q,p) in the definition of the unknown probability distribution dP_o. By momentum conservation it follows that $P \geqslant p' \geqslant p$ and thus $1 \geqslant X \geqslant x$. Thus,

$$\frac{dF(x,Q^2)}{dx} = \int_x^1 \left(\frac{dX}{X}\right) \frac{dP_o(X)}{dX} \quad \frac{d\bar{P}(x/X,Q^2)}{dx/X} \tag{3}$$

Defining the moments

$$\langle x^n \rangle = \int_o^1 \left(\frac{dx}{x}\right) x^n \frac{dF(x,Q^2)}{dx} \quad , \tag{4}$$

we obtain the factorized form

$$\langle x^n \rangle = \langle X^n \rangle \cdot \langle \bar{x}^n(Q^2) \rangle \ , \tag{5}$$

where in an obvious notation

$$\langle X^n \rangle = \int_o^1 \left(\frac{dX}{X}\right) X^n \frac{dP_o(X)}{dX} \tag{6a}$$

and

$$\langle \bar{x}^n(Q^2) \rangle = \int_o^1 \frac{dy}{y} y^n \frac{d\bar{P}(y,Q^2)}{dy} \quad . \tag{6b}$$

$\frac{d\bar{P}}{dy}$ is properly normalized to a Dirac δ-function, $\delta(1-y)$, when the QCD coupling constant $\alpha_s=0$, so as to obtain the primitive scaling distribution dP_o/dx in this limit. Since the moments $\langle X^n \rangle$, by definition, have no Q^2 dependence, all the observed Q^2 variation is contained in the running moments $\langle \bar{x}^n(Q^2) \rangle$ for which we obtain explicit expressions in the next section. From now on, we shall drop the bars over $\langle \bar{x}^n \rangle$ and call them simply $\langle x^n \rangle$.

The above is true only for non-singlet quark distributions. For the general case, both quark and gluon distributions enter and we have a system of matrix equations from which the primitive moments do not drop out. It is for this reason that the analysis, say of F_2, is more model dependent since the primitive distributions need to be specified. In this work we shall only deal with non-singlet distributions.

3. BN MOMENTS

The use of the Bloch Nordsieck theorem to describe the energy momentum distribution of the soft QCD radiation rests on two conjectures which have been proved valid at least in the leading log approximation. They are that the emission of infrared gluons is finite to all orders in α_s and that it exponentiates as it does in QED, with non-abelian effects appearing only through the energy dependence of the strong coupling constant as obtained via the RG equation. While at present, the infrared finiteness of non leading terms in QCD is under scrutiny for qq scattering, it is believed to be valid for potential scattering and thus for the case at hand to allow for the exponentiation of the leading terms.

Let us first review the photon probability function and the infrared convergence factor β in QED and then generalize the same procedure to QCD.

In QED, the Bloch-Nordsieck theorem and the overall energy-momentum conservation lead to the following soft photon formula for the energy distribution[2]

$$\frac{dP}{d\omega} = \int_{-\infty}^{+\infty} \frac{dt}{2\pi}\, e^{i\omega t - \beta h(E,t)} \tag{7}$$

where

$$h(E,t) = \int_{0}^{E} \frac{dk}{k}\,(1-e^{-ikt}) \tag{8}$$

and

$$\beta = - \int d^2 \vec{n}\, j_{\mu}(n)\, j^{\mu +}(n) \tag{9}$$

with $\vec{n}^2 = 1$, is the single photon spectrum written in terms of the classical current

$$j_{\mu}(k) = \frac{ie}{(2\pi)^{3/2}} \sum_i \eta_i \left(\frac{p_{i\mu}}{p_i \cdot k}\right) \tag{10}$$

Defining the fraction $x = \omega/E$, a standard calculation yields[2]

$$\frac{dP_{\gamma}}{dx} = \frac{\beta\, x^{\beta-1}}{\gamma^{\beta}\,\Gamma(1+\beta)} \quad , \qquad (x \lesssim 1) \tag{11}$$

where $\gamma = e^{C} = 1.781$ is Euler's constant.

Eq. (11) gives the well known soft-photon distribution function. It is worth noticing that the factor $\gamma^{\beta}\,\Gamma(1+\beta)$ is obtained by normalizing the distribution (7) from zero to infinity. This factor differs from 1 by terms of order β^2 which can be interpreted as representing the probability that two or more photons of energy $< E$ contribute to give an energy loss which is greater than E.

In massless QED, it is also necessary to include the collinear hard-photon correction. This can be done replacing the spectrum $\frac{1}{k}$ by $\frac{1}{k}\,(1+(1-k/E)^2/2)$ as given through the Weizsäcker-Williams approximation. A justification for exponentiating this entire (soft+collinear-hard) spectrum is provided through the cancellation of mass singularities[12,13,14]. Thus, we can substitute eq. (8) with

$$\bar{h}(E,t) = \int_{0}^{E} \frac{dk}{k}\,(1-e^{-ikt})\left(\frac{1+(1-k/E)^2}{2}\right) \tag{12}$$

and approximate it for large E (and small x) by an asympotic expansion, i.e.

$$\bar{h}(E,t) \simeq h(E,t) + \int_{0}^{E} \frac{dk}{k}\left(-\frac{k}{E} + \frac{k^2}{2E}\right) \simeq \ln(E\gamma t) - \frac{3}{4} \tag{13}$$

The probability distribution (for $x \simeq 0$) now reads[15]

$$\frac{dP}{dx} \underset{x \simeq 0}{\simeq} \beta \frac{e^{3/4\beta} x^{\beta-1}}{\gamma^{\beta} \Gamma(1+\beta)} \qquad (14)$$

To enlarge the range of validity in x of the above formula we require that the first order result be reproduced as we move out of the soft ($x \simeq 0$) region. This gives finally

$$\frac{dP}{dx} = \beta \frac{e^{3/4\beta} x^{\beta-1}}{\gamma^{\beta} \Gamma(1+\beta)} \cdot \frac{1+(1-x)^2}{2} \qquad (15)$$

The corresponding electron spectrum is given by

$$\frac{dP_e}{dx} = \beta \frac{e^{3/4\beta} (1-x)^{\beta-1}}{\gamma^{\beta} \Gamma(1+\ )} \cdot \frac{1+x^2}{2} \qquad (16)$$

Eq. (16) has the following nice properties:

- it gives the electron spectrum near $x \simeq 1$ due to the summed up soft photon emission.
- it reproduces the correct first order spectrum (for all x) as $\beta \longrightarrow 0$
- it satisfies the Adler sum rule up to order β^2, i.e. $\int_0^1 dx\,(dP_e/dx) \simeq 1+0(\beta^2)$.

The above argument ignores the production of e^+e^- pairs and is thus particularly suitable for $(e-e^-)$ or "non singlet" distributions in general, for which the pair production cancels out.

In QCD, in the leading logarithmic approximation, IR divergences have been shown to cancel for color singlets(9,16,17,18). In this approximation, the non-Abelian nature of QCD manifests itself essentially in replacing the coupling constant α by the running coupling constant $\alpha(k_\perp^2)$ where $k_\perp^2$ is the transverse momentum of the gluon(14). Thus, for QCD, with massless quarks, $\beta h(E,t)$ is replaced by

$$H(E,t) = \frac{C_2(R)}{\pi} \int_0^{E^2} \frac{dk^2}{k_\perp^2} \alpha(k_\perp^2) \int_k^E \left(\frac{dk}{k}\right) \sqrt{\frac{1}{1-k_\perp^2/k^2}} \; \frac{1+(1-k/E)^2}{2} \; (1-e^{-ikt}) \qquad (17)$$

where $C_2(R) = \frac{N_c^2-1}{2N_c}$ for $SU(N_c)$ color.

Eq. (17) tells us that for large E, the logarithmic singularity (in the dimensionless variable Et) and the mass singularity are obtained when $k_\perp \longrightarrow 0$ in the second integral. Thus, we approximate eq. (17) as before in the QED case by

$$H(E,t) \simeq \beta \bar{h}(E,t), \qquad (18)$$

where $\bar{h}$ is as given by eq. (13) but the new β is given by

$$\beta \simeq \frac{C_2(R)}{\pi} \int \frac{d\,k_\perp^2}{k_\perp^2} \alpha(k_\perp^2) \qquad (19)$$

For deep inelastic scattering, if we use the asymptotic freedom formula $\alpha(k_\perp^2) = \frac{1}{b \ln k_\perp^2/\Lambda^2}$ with b $=(11N_c-2N_f)/12\pi$ for N_f (massless) flavors, we get

$$\beta(Q^2) \underset{Q^2 \text{large}}{\simeq} \frac{C_2(R)}{b\pi} \ln(\ln \frac{Q^2}{\Lambda^2}) \tag{20}$$

Eq. (16), with β given by eq. (19), is the expression we propose for describing the probability that in deep inelastic scattering a quark of momentum p' finds itself with momentum p=xp' after emission of soft and hard collinear gluons of total energy p'-p. Near x=1, our expression coincides with the one obtained by many authors(19,20,21) for the Q^2-dependence of the valence densities, i.e.

$$q(x,Q^2) = \frac{\exp\left[(\frac{3}{4} - \ln\gamma)\, C_F\, \xi\right]}{\Gamma(C_F\, \xi)} \quad (1-x)^{C_F \xi - 1} \tag{21}$$

where $C_F\ \xi = \beta(Q^2)$ as defined in eq. (19).

We can now calculate the moments of this "running" distribution, i.e. the moments defined in eq. (6b) and conjectured to be responsible for the entire Q^2-dependence of the moments of the non-singlet structure function F_3. Using eq. (16) we get

$$< x^n > = \int_0^1 dx\, x^{n-1} \frac{dP_q(x,Q^2)}{dx} = \frac{e^{(3/4)\beta}}{2\gamma^\beta} \frac{\Gamma(n)}{\Gamma(n+\beta)} \cdot \left[1 + \frac{n(n+1)}{(n+\beta)(n+1+\beta)} \right] \tag{22}$$

To compare the Q^2-variation of the nth moment with the one obtained through the AP equations(6), we differentiate eq. (22) with respect to β . One obtains

$$\frac{d\langle x^n \rangle}{d\beta} = \langle x^n \rangle B_n(\beta)$$

with

$$B_n(\beta) = \frac{3}{4} + \psi(1) - \psi(n+\beta) - \frac{n(n+1) \cdot (2n+1+2\beta)}{(n+\beta)(n+1+\beta)\left[n(n+1)+(n+\beta)(n+1+\beta)\right]} \tag{23}$$

If we set β =0, we see that $B_n(0)$ coincides exactly with the renormalization group result(4,5,6). If, instead of eq. (16), one uses eq. (21) to calculate the moments, one obtains a result which coincides with the AP (or RG) moments only at large n, thus underlying the fact that eq. (21) is only valid for $x \simeq 1$.

We dedicate the next section to a detailed comparison of the Bloch-Nordsieck function and the renormalization group calculation.

4. BN MOMENTS AND THE RG EQUATION

In order to facilitate comparison with RG or AP equations, we shall write an "evolution" equation for the distribution function of the soft QCD radiation $\pi(x, \beta)$. In the large Q^2 limit, we have

$$\pi(x,\beta) = \int \frac{dt}{2\pi} \, e^{ixt-\beta(Q^2)g(t)} \tag{24}$$

with $\beta\,(Q^2)$ given by eq. (19) and

$$g(t) = \int_0^1 \frac{d\lambda}{\lambda}\,(1-e^{-i\lambda t}).$$

The function $\beta\,(Q^2)$ is a convolution of the running coupling constant $\alpha\,(k_\perp^2)$ with the soft gluon spectrum $dk_\perp^2/k_\perp^2$ and represents the actual expansion parameter at large Q^2. In QCD it is therefore quite natural to observe the evolution of a given distribution as a function of β rather than $\ln Q^2$. We have

$$\frac{\partial\pi(x,\beta)}{\partial\beta} = \int_0^1 \frac{dy}{y}\left[\pi(x-y,\beta) - \pi(x,\beta)\right] \tag{25}$$

To obtain the equation for the quark densities we define the quark distribution in the soft $(x \simeq 1)$ region as

$$Q(x,\beta) = \pi\,(1-x,\beta) \qquad x \lesssim 1$$

Since $\pi(x,\beta) = 0$ for $x \lesssim 0$ (due to the analyticity of g(t) in the lower half plane), eq. (25) can be rewritten for $Q\,(x,\beta)$ as

$$\frac{\partial Q(x,\beta)}{\partial\beta} = \int_0^{1-x} \frac{dy}{y^{1-\epsilon}}\,Q(x+y,\beta) - \int_0^1 \frac{dy}{y^{1-\epsilon}}\,Q(x,\beta) \tag{26}$$

where we have introduced a small parameter ϵ to make the two integrals separately convergent. After the integrations are performed, $\epsilon \longrightarrow 0^+$.

Taking the moments, one gets

$$\frac{\partial M_n(\beta)}{\partial\beta} = \int_0^1 \frac{dz\,z^{n-1}}{(1-z)^{1-\epsilon}} \int_0^1 dy\,y^{n-1+\epsilon} Q(y,\beta) - \int_0^1 \frac{dz}{(1-z)^{1-\epsilon}}\,M_n(\beta) \tag{27}$$

with

$$M_n(\beta) = \int_0^1 dy\,y^{n-1}\,Q(y,\beta)$$

The $\epsilon \longrightarrow 0$ limit in eq. (27) has to be handled carefully, as the following example will show.

If we let $\epsilon \longrightarrow 0$ and take the difference of the two integrals, one is led to

$$\frac{\partial M_n(\beta)}{\partial\beta} = \int_0^1 \frac{dz}{1-z}(z^{n-1}-1)\, M_n(\beta) = \left[\psi(1)-\psi(n)\right] M_n(\beta) \tag{28}$$

However, this equation is only an approximation since we can easily check, from the explicit expression $Q(x,\beta) = \frac{(1-x)^{\beta}}{\gamma^{\beta}\Gamma(\beta)}$, that $M_n(\beta)$ obeys the equation

$$\frac{M_n(\beta)}{\partial\beta} = \left[\psi(1)-\psi(n+\beta)\right] M_n(\beta) \tag{29}$$

The correct procedure thus consists in making a small ϵ expansion of eq. (27) and performing the integrals, after cancelling the $1/\epsilon$ term. What one obtains is

$$\frac{\partial M_n(\beta)}{\partial\beta} = \left[\psi(1)-\psi(n)\right] M_n(\beta) + \int_0^1 x^{n-1} \ln x \, Q(x,\beta)\, dx \tag{30}$$

which coincides with eq. (29) when the explicit form of $Q(x,\beta)$ is used.

It is instructive to compare the above with the corresponding RG or AP equation when calculated with soft radiation only, i.e. ignoring the collinear hard emission. In this approximation, what one gets is exactly eq. (28). The AP equation for the moments $M_n(Q^2)$ of the quark density in fact reads

$$\frac{\partial}{\partial\beta} M_n^{AP}(Q^2) = A_n M_n^{AP}(Q^2)$$

with

$$A_n = \int_0^1 \frac{dz}{1-z}(z^{n-1}-1)\frac{(1+z^2)}{2} \tag{31}$$

To neglect the collinear hard gluons one substitutes A_n with

$$A_n \overset{soft}{=} \int_0^1 \frac{dz}{1-z}(z^{n-1}-1) = \psi(1)-\psi(n) \tag{32}$$

Thus the index of the BN moments agree with the correspondingly calculated RG(AP) index as $\beta \longrightarrow 0$. The difference for $M_n(Q^2)$ is therefore of order β^2 which is as it should be since the BN formula sums soft radiation to all orders whereas in the RG equation the (soft) divergence is introduced and eliminated only to the lowest order.

It is interesting to note that the equation satisfied by the BN moments $M_n(\beta)$ can be recast in a form suggestive of a modified RG equation. In fact, if in eq. (30) we consider n as a continuous variable, we obtain

$$\left(\frac{\partial}{\partial\beta} - \frac{\partial}{\partial n}\right) M_n(\beta) = A_n^{soft}\, M_n(\beta) \tag{33}$$

where A_n^{soft} is the "soft" RG index (which is independent of β) defined in eq. (32) We do not know if the above has any general validity. At any rate, the summation of soft radiation makes the moments non-local in n.

If we accept eq. (33) as providing a reliable equation for including the summed up soft radiation effects, then collinear hard effects can be included by replacing A_n^{soft} with the full RG index A_n.

The solution $M(n, \beta)$ of eq. (33) with A_n in place of A_n^{soft} reads

$$M(n,\beta) = \exp\left\{\int_0^{\beta} d\vartheta\, A(n+\vartheta)\right\}$$

A straightforward calculation gives

$$M(n,\beta) = \frac{e^{\frac{3}{4}\beta}\,\Gamma(n)}{\gamma^{\beta}\,\Gamma(n+\beta)}\sqrt{\frac{n(n+1)}{(n+\beta)(n+\beta+1)}}$$

This simple analytic formula has the following interesting properties:

- for large n or to first order in β , it gives exactly the RG result
- for n=1 moment, it gives
  $M(1,\beta) \sim 1+\vartheta(\beta^2)$
  i.e. the Adler sum rule is satisfied to order β^2.

5. PHENOMENOLOGY OF F_3 MOMENTS

We now present a fit to the moments of the nucleon structure function F_3 based on the expression (22). We shall assume, for lack of a better knowledge, the asymptotic form

$$\beta \simeq \frac{C_2(R)}{\pi b} \ln(\ln Q^2/\Lambda^2) ,$$

(valid for large Q^2) throughout for all Q^2. Clearly, this is a gross approximation and reflects our ingorance of the non-perturbative confinement region.

The data are taken from ref. (11). For this Q^2 region, we shall assume 3 flavors. Thus, for N_c=3, we have

$$\beta \simeq \left(\frac{16}{27}\right) \ln(\ln Q^2/\Lambda^2). \tag{34}$$

We have also chosen Λ=0.75 GeV, the same value which gives the best fit to first-order QCD result. Figure (1) shows the comparison between data and our expression for the moments of F_3 for n=2,3,4 and 5. The agreement seems quite satisfactory down to $Q^2 \gtrsim 0.8$ GeV^2.

Considering that there are no free parameters the agreement is very good.

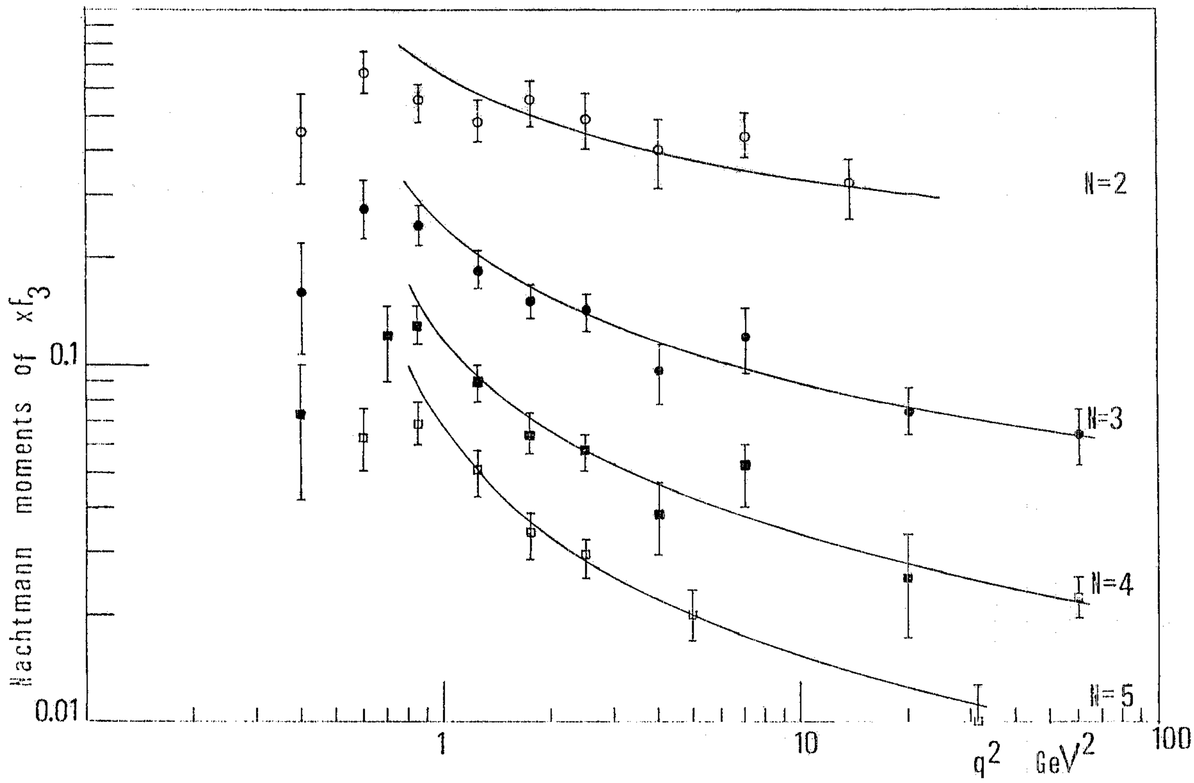

FIG. 1 - Comparison of eq. (22) for n = 2,3,4,5 with data from ref. (11).

6. CONCLUSIONS

The analysis presented here seems quite well supported by the data even for rather low values of $Q^2 \gtrsim \Lambda^2$ and for the lower moments the fits are somewhat superior to the RG result. It appears remarkable that we are able to do as well as we do with the asymptotic formula for β (valid for large Q^2) all the way. It is an interesting open problem to derive theoretically a reasonable low energy expression for β . This is likely to be difficult since it inevitably involves entering the region of confinement. We conjecture that for sufficiently low Q^2, β becomes a constant and probably vanishes. Such is the behaviour in QED and may turn out to be so in QCD as well.

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